ARITHMETICK;

OR,

The Ground of Arts:

TEACHING

That Science, both in whole Numbers and Fractions.

Theorically and Practically

Applied in the Operation and Solution of Queſtions in Numeration, Addition, Subſtraction, Multiplication, Diviſion, the Rules of Proportion, Fellowſhip, Barter, Rules of Practice, Exchange of Coin, Loſs and Gain, Tare, Trett, and other Queſtions relating to Weights and Meaſures, Lengths and Breadths, Equation of Payments, Commiſſion to Factors, Rules of Alligation, and of Falſe Poſition, &c.

Originally Compoſed by Dr. *Record*, and others.

And now peruſed, Corrected, new Methodized, much Improved;

And thereto Added,

I. A New Treatiſe of Decimals, with the Demonſtration of each Rule, and the Relation it has to Vulgar Fractions; Alſo why Decimals are wrought as whole Numbers, &c.

II. Tables of Simple and Compound Intereſt, with the manner of Calculation, and uſe thereof in Reſolving all the moſt neceſſary Queſtions, concerning Intereſt and Diſcount of Money; the Purchaſing or Selling Eſtates, in Land or Houſes, in Preſent, or Reverſion, in Fee or for time Limited, and for Fining off Rent, &c.

III. The eaſieſt Method of Extracting the Square and Cube-Roots of Numbers, whole or broken, and the uſe thereof in many material Inſtances.

By Edw. Hatton, Philomercat.

Scientia non habet inimicum niſi ignorantem.

LONDON,

Printed by *J. H.* for *Charles Harper*, at the *Flower de Luce* againſt St. *Dunſtan's* Church, and *William Freeman*, at the *Bible* againſt the *Middle-Temple-Gate*, in *Fleet-ſtreet*. 1699.

ARITHMETICK

OR

The Sculls of Arts;

TEACHING

The Science, both in Whole Numbers and Fractions,

Theoretically and Practically.

To His Illuſtrious Highneſs,

WILLIAM *DUKE of*

GLOUCESTER,

Knight, and Companion of the moſt Noble Order of the GARTER.

May it pleaſe Your Highneſs,

YOUR Noble *Birth*, and promiſing *Goodneſs* are ſo Perſpicuouſly evident, that I ſhall not (as

(as is more frequently than pertinently done on this occasion) be so vain to Attempt an account of either.

But this Humble Address is to crave the Favour of Your Highness's Patronage for the following Work, which may so far overawe the envious Criticks of this Age, that they dare not pretend faults where there are none, which is all the favour desired from them.

There's not the least doubt but Your Highness is sufficiently Accommodated with the best Instructions in matters of this Nature, as well as all other necessary Arts, both from Men and Books, and I presume to hope, that the variety of well grounded Rules contained in the following Treatise, may

[illegible] one time or other afford Your Highness either Profit or Pleasure.

Arithmetick (give me Leave to tell Your Highness) is one of the most useful Sciences that can (morally speaking) be acquired, as being necessary for all Classes and Conditions of Men whatsoever; For *That* is the Basis whereon are Erected all those Noble, most Useful, and permanent Superstructures in Geometry, Navigation, Fortification, Gunnery, Military Order, and other parts of the Mathematicks, it being impossible any one should excel in *These* who are ignorant in *That*; That whereby all Commerce and Traffick is maintained, and made a principal Ingredient for enriching Empires and Kingdoms; and in brief,

brief, *That* without which no particular person can be Eminent for his Accomplishments, nor any Nation, or Body of Men for *Riches* and *Grandure*.

But that I may not seem Tedious in Commending that which needs no Encomiums; nor Assume too much Liberty by the Prolixity of this Dedication: I crave leave to Subscribe,

Your Highness's most Obedient,

most humbly Devoted, and

most Faithful Servant,

Edward Hatton.

THE PREFACE.

THough the Original Author of the following Treatiſe was one of the moſt Eminent Arithmeticians of his time (as appears by the great variety of Compendious and Excellent Rules therein, and the Eſteem and Credit the Book acquired for near 150 Years together) yet at length the Stile and Phraſe growing obſolete, and ſome Errors, for want of the Authors Correction in Reprinting, having crept in, the Bookſellers (not willing ſo choice a piece of Arithmetick ſhould be loſt for want of a little Poliſhing, the Principal parts being Extraordinary) were pleaſed to recommend the performance thereof to me, and I have taken all the Care I could to do the Author Juſtice, by Expunging what is now uſeleſs, and ſubſtituting in lieu thereof ſuch Expreſſions, Rules, and Examples as are moſt agreeable to the preſent times, moſt uſeful to the Reader that would learn without a Maſter, and moſt Correſpondent to the Authors primitive Deſign.

And becauſe the Author wrote by way of Dialogue, as judging by that means a Learner has the greateſt opportunity of declaring wherein he is Dubious, and of receiving ſatisfaction therein, I have upon that account continued the Diſcourſe in that method, and have likewiſe put in the ſame way of Queſtion and Anſwer, what was added by others and my ſelf thereto.

I have alſo new-modelled the Book, by altering the places of Rules, which were before very irregular, cauſed (I ſuppoſe) by the multiplicity of Additions, and alterations made after the Authors Death. The Rules of Practice *I have put in a new Method, which are not only in better order for finding how to work for any price, but alſo are more Numerous and brief than any I have ſeen, or believe to be Extant; and in*

this Rule I have added the Method of finding the Intereſt of any Summ at any Rate by Practice, and alſo how Factors may know what their Commiſſion of any Summ amounts to, and many thouſands of Alterations, Additions & Improvements more, I have made, which would be too prolix in this place to take notice of.

I have likewiſe joined as an Appendix to the former Work, a New Treatiſe of Decimal Arithmetick, wherein all the neceſſary Rules for acquiring the perfect knowledg thereof are conciſely, pertinently, and clearly made evident from undeniable Demonſtration of each particular; and becauſe Decimals are wrought as Whole Numbers, I have ſhewed why they are ſo, and the relation they have to them, and to Vulgar Fractions, both in themſelves and manner of operation, all which is for the moſt part new, and was never before publiſhed.

In the next place I have ſhewed the uſe of Decimals in General, and why no Fraction ſo eaſie as this is, or can be invented, and have Exhibited uſeful Tables of Intereſt, Simple and Compound, with plain and eaſie Rules for Calculating, Examining, or Enlarging the ſame; as alſo the uſe of thoſe Tables, not only in finding the Simple Intereſt, or Diſcount of any Summ; but alſo the Compound Intereſt: as the amount and preſent worth of any Summ, the amount, preſent worth and Purchaſe of Annuities; wherein are plain and practical Rules for Selling or Purchaſing an Eſtate, either in Land or Houſes, in Preſent or in Reverſion, in Fee, or for a time limited, in Fining off Rent, &c.

And after this I have ſhewed the eaſieſt method of Extracting the Square and Cube-Roots of Numbers, whole or broken, and the uſe thereof in the Solution of Queſtions Geometrical; ſo that I doubt not but the Judicious will agree, That this Arithmetick as now methodized and improved is more compleat, eaſie, and univerſally uſeful than any yet made publick; and I hope the Reader will reap that advantage thereby, which was ſincerely aimed at by his Friend and Servant

May, 31. 1699.

E. Hatton.

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AN INTRODUCTION

Touching the

Uſefulneſs of Arithmetick.

CHAP. I.

Scholar.

SIR,

SUCH is your authority in my eſtimation, that I am content to receive what you ſay, as truth, though I ſee no other reaſon that leads me thereunto; otherwiſe in my opinion, it appeareth but vain, to beſtow any time in learning of that which every Child may and doth at all times, when he doth any thing alone, and much more when he talketh or reaſoneth with others.

Maſter. This is the manner of arguing by all them that defend their blind ignorance, that when they think they have made ſtrong reaſons for themſelves, then have they proved quite contrary. For if *numbering* be ſo common that no man can do any thing alone, and much leſs talk or bargain with other, but he ſhall ſtill have to do with *number*: this proveth not *number* to be contemptible and vile, but rather excellent, and of high Reputation, ſince it is the ground of all mens affairs, ſince, without it no Money can be told, no communication without it can be continued, no bargaining without it can be duly ended. Theſe Uſes, if there were none other, are ſufficient to recommend the Art of *numbering*. But there are innumerable others, far paſſing all theſe, which declare *number* to exceed all praiſe. Wherefore in all matters are Accomptants ſo

much deſired? Wherefore are Auditors ſo much encouraged? What cauſeth Geometricians ſo highly to be valued? Why are Aſtronomers ſo greatly advanced? Becauſe that by *number* they diſcover ſuch things as would otherwiſe never be found out.

Schol. Truly, Sir, if it be ſo, that theſe men by *numbring* do attain their knowledge, at whoſe great works moſt men do wonder, then I ſee I was much deceived, and *numbring* is a more uſefull thing than I took it to be.

Maſt. If *number* were ſo mean a thing as you did eſteem it, then would it not be uſed ſo much in mens communication. Exclude *number*, and anſwer to this queſtion: How many years old are you?

Schol. Mum.

Maſt. How many *days* in a *week*? How many *weeks* in a *year*? What *lands* hath your *Father*? How many *men* doth he keep? How long is it ſince you came from him to me?

Schol. Mum.

Maſt. So that if *number* be wanting, you anſwer all by *Mumm*: And thus you ſee, what rule *number* beareth, and that if *number* be lacking it maketh *men dumb*; ſo that to moſt queſtions they muſt anſwer impertinently.

Schol. This is the cauſe, Sir, that I judged it ſo vile, becauſe it is ſo common.

Maſt. The more common that the thing is, being needfull, the better is the thing, and the more to be deſired. But in *numbring*, as ſome of it is plain; ſo the moſt part is difficult, and not eaſily attain'd. The eaſier part ſerveth all men in common, and the other requireth ſome Learning. Wherefore as without *numbring* a man can do almoſt nothing, ſo with the help of it you may attain to all things.

Schol. I perceive then it were beſt to learn the *Art of numbring* firſt of all other Learning, and then a man need learn no more if all other knowledge attend it.

Maſt. Nay, not ſo; but if it be firſt learned, then ſhall a man be able to learn and attain to other *Sciences*; which without it he could never do.

Schol. I conceive then by your former words, that *Aſtronomy* and *Geometry* depend much on the help of *numbring*: but that other *Sciences*, as *Muſick*, *Phyſick*, *Law*, *Grammar*, and ſuch like, have any dependence on *Arithmetick*, I perceive not.

Maſt.

Maſt. I ſee your judgment in theſe things, by the ordering of your *Sciences*: but I will let that paſs now, becauſe it toucheth not the matter that I intend, and I will ſhew you how *Arithmetick* doth profit in all theſe, according to your ſmall underſtanding; omitting other reaſons more ſubſtantial.

First, (as you reckon them) *Muſick* is not only much aided by *Arithmetick*, but is made and hath its perfection from it: for all *Muſick* ſtandeth by *number* as to *time* and *proportion*: And in *Phyſick*, beſide the Calculation of critical days, with other things which I omit, how can any man judge the Pulſe rightly, or direct the proportions of a Compoſition, that is ignorant of *numbers*? *Muſick.* *Phyſick.*

And as for the *Law*, it is plain, that the man that is ignorant of *Arithmetick*, is neither fit to be a *Judge*, an *Advocate*, or a *Proctor*. For how can he well underſtand another mans cauſe, appertaining to *diſtribution of Goods*, or other *Debts*, or of *Summs of Money*, if he be ignorant of *Arithmetick*? This oftentimes cauſeth a deprivation of Right, when the Judge either delighteth not to hear of a matter that he apprehendeth not, or cannot judge for want of Right underſtanding. *Law.*

Now, as for *Grammar*, methinks you ſhould not doubt in what it needeth *number*, ſince you have learned that *Nouns* of all ſorts, *Pronouns*, *Verbs* and *Participles* are diſtinct by *numbers*: beſides the variety of *Nouns* of *number*, and *Adverbs*. And if you take away *number* from *Grammar*, then is all the quantity of *Syllables* loſt. And many other ways doth *number* help *Grammar*. Whereby was the meaſure of Poeſy, or all kind of Verſe found and made? was it not by number? *Grammar.*

But how needfull *Arithmetick* is to all parts of *Philoſophy*, they may ſoon ſee, that do read either *Ariſtotle*, *Plato*, or any other Philoſopher's Writings. For all their Examples almoſt, and their Probations depend on *Arithmetick*. It is the ſaying of *Ariſtotle*, that he that is ignorant of *Arithmetick*, is qualified for no *Science*. And *Plato* his Maſter wrote a little ſentence over his School-houſe door, *Let none ignorant of Arithmetick enter in here.* *Philoſophy.*

And how needfull *Arithmetick* is to *Divinity*, it appeareth, ſeeing ſo many *Doctors* gather ſo great myſteries out of *number*. And if I ſhould undertake to write all the uſes of *Arithmetick* in civil Acts, as in Government of Common-weals, in time of Peace, and in due proviſion and order of Armies in time of War for numbring of the Soldiers, ſumming of their Wages, Proviſion of Victuals, viewing of *Divinity.* *Armies*

of Artillery, with other Armour; beſide the moſt material point of all for caſting of ground, for encamping of men, and the like: And how many ways alſo *Arithmetick* is conducible to all private affairs of Lords and of Merchants, and all other Occupiers, and generally for all eſtates of men, beſides *Auditors*, *Treaſurers*, *Receivers*, *Stewards*, *Bailiffs*, and ſuch like, whoſe Offices without *Arithmetick* are nothing: If I ſhould (I ſay) particularly repeat all ſuch uſes of the noble *Science* of *Arithmetick*, it were enough to make a very large Volume.

Schol. No, no, Sir, you ſhall not need; for I doubt not but this that you have ſaid, were enough to perſuade any man to think this art to be right excellent: and ſo neceſſary for man, that (as I think now) ſo much as a man is deficient in it, ſo much he lacketh of the foundation of all uſefull knowlege.

Maſt. What, is your mind ſo far changed, by hearing theſe few uſes of *Arithmetick* in general? 'tis probable you would be more of another mind, if you knew all the particular advantages thereof.

Schol. I beſeech you Sir, reſerve thoſe that are left behind unto their proper places: and if you will be ſo good as to lay open at this time this excellent Treaſure, ſo that I may be ſomewhat enriched thereby, if ever I ſhall be able, I will requite your pains.

Maſt. I am very glad of your requeſt, and will do it ſpeedily, ſince you be ſo ready to learn it.

The duty of a Scholar.

Schol. And I ſubmit to your authority, whatſoever you ſay, I take it for truth.

Maſt. That is too much for any man to be believed in all things, without ſhewing of reaſon. Though I might of my *Scholar* ſome credence require, yet except I ſhew reaſon, I do not deſire it. But now ſince you are ſo earneſtly reſolv'd to attain this art, it is beſt to omit no time, leſt ſome other paſſion cool this great heat, and then you leave off before you ſee the end.

Perſeverance in Study.

Schol. Though there be ſome perſons ſo unconſtant, that turn with every wind, which often begin, and never come to the end, I am none of this ſort, as you partly know. For by my good will, what I once begin, till I have it fully ended, I would never leave off.

Maſt. So have I found you hitherto indeed, and I hope you will encreaſe, rather than go back. For better it were never to Eſſay, than not to be a Proficient; but I hope you will not do ſo: therefore tell me briefly: what call you the *Science* that you deſire ſo greatly to learn.

Schol.

Schol. Why Sir, you know.

Maſt. That makes no matter, I would hear whether you know, and therefore I ask you. For it were great ſhame to have ſtudied a Science, and to be ignorant of its name.

Schol. Some call it *Arſemetrick*, and ſome *Augrime*.

Maſt. And what do theſe names betoken?

Schol. That if it pleaſe you, I would learn.

Maſt. Both names are corruptly written: *Arſemetrick* for *Arithmetick*, as the *Greeks* call it, and *Augrime* for *Algoriſme*, as the *Arabians* ſound it: which doth betoken the *Science* of *Numbring*: for *Arithmos* in *Greek* is called *Number*: and of it cometh *Arithmetick*, the Art of *Numbring*: ſo that *Arithmetick* is a Science or Art teaching the manner and uſe of *numbring*.

Schol. This will I remember. But how many things are to be learned to attain this Art fully?

Maſt. There are reckon'd commonly ſeven parts or works of it. *Numeration*, *Addition*, *Subſtraction*, *Multiplication*, *Diviſion*, *Progreſſion*, and *Extraction* of Roots. But, if I ſhall ſpeak exactly of the parts of *numbring*, I muſt make but five of them; for *Progreſſion* is a compound Operation of *Addition*, *Multiplication* and *Diviſion*, and ſo is the Extraction of Roots: but it is no harm to name them as kinds ſeveral, ſeeing they appear to have different Operation. For it matters not ſo much to contend for the number of them, as for the due knowledge and practiſing thereof.

Schol. Then you will that I ſhall name them, as ſeven kinds diſtinct; but now I deſire you to inſtruct me in the uſe of each of them.

Maſt. So I will, but it muſt be done in order: for you may not learn the laſt ſo ſoon as the firſt, but you muſt learn them in that order, as I did rehearſe them, if you will learn them ſpeedily and well.

Schol. Sir, as you pleaſe. Then to begin. *Numeration* is the firſt in order, what ſhall I do with it?

Maſt. Firſt, you muſt know what the thing is, and then after learn the uſe of the ſame.

CHAP.

CHAP. II.

Numeration of Whole Numbers.

Definition of Numeration.

NUmeration is that *Arithmetical* skill, whereby we may duly value, expreſs, write down and read any Number or Summ propounded.

Schol. Why? then methinks you put a difference between the Value and the Figures.

Maſt. Yes, ſo I do; for the *Value* is one thing, and the *Figures* are another, and that cometh partly by the diverſity of *Figures*, but chiefly in the places wherein they be ſet.

Schol. Then muſt I know here three things, the *Value*, the *Figure*, and the *Place*.

Maſt. Right. But yet add *Order* to them as the fourth. And firſt mark, that there are but *ten Figures* that are uſed in *Arithmetick*; and of thoſe ten, one doth ſignifie nothing by it ſelf, which is made like an o, and is called a *Cypher*. The other *nine* are called ſignifying *Figures*, and be thus wrote down.

A Cypher.

1, 2, 3, 4, 5, 6, 7, 8, 9.

And this is their value.

One, Two, Three, Four, Five, Six, Seven, Eight, Nine.

But here you muſt obſerve, that every *Figure* hath two Values; one always certain, that it ſignifieth properly, which it hath of its form, and the other uncertain, which it taketh from its place.

A Place.

A Place is the Room that a Figure ſtandeth in. And look how many Figures are written in one Summ, ſo many Places hath that whole number. And that muſt be called the firſt Place, that is next to the Right hand, and ſo reckoning by order towards the Left hand, ſo that that Place is laſt that is next to the left hand.

Schol. I pray you, how many of theſe Places be there in all?

Maſt. There is no certain number of them, but they are ſometimes more, and ſometimes fewer, according to the ſumm that is expreſſed. For ſo many as the Figures are, ſo many are the Places:

and

and the laſt Place is ſo called, not becauſe it is the laſt of all other, but it is the laſt of that preſent Summ, and it may be the middle Place in another Summ.

Schol. I conceive this very well, as touching the order of reckoning of the Places: but as for the number of them, you ſay there is no certainty. Now it reſteth to declare the Value of the Figures by the diverſity of Places, which you called the Value uncertain. *Value uncertain.*

Maſt. But firſt let me hear whether you know perfectly the certain Value. *Value certain.*

Schol. Yes Sir, as you wrote them ſo I marked them.

Maſt. How write you then Five?

Schol. By this Figure 5.

Maſt. And how Six?

Schol. Thus 6.

Maſt. Write theſe three Numbers each by it ſelf, as I ſpeak them, Seven, Four, Three.

Schol. 7, 4, 3.

Maſt. How write you theſe four other, Two, One, Nine, Eight.

Schol. Thus, 2, 1, 6, 8.

Maſt. Nay there you miſs.

Schol. Sir, true, I was to blame, I take 6 for 9, but I will beware hereafter.

Maſt. Now then take notice of thoſe certain Values every Figure repreſenteth when it is alone written, without other Figures joined to it. As for example, this Figure 9 is Nine, ſtanding alone.

Schol. Then 9 without more Figures of *Arithmatick* betokeneth Nine, whatſoever other Letters be about it.

Maſt. Right; ſo it doth, if it be in the firſt place joyned with other, how many ſoever do follow, as in this example, 3679. You ſee 9 in the firſt place, which doth betoken Nine as it were alone.

Schol. I perceive that, and doth not 7 that ſtandeth in the ſecond place (between 9 and 6 in the third place) betoken Seven, and ſo 3 in the fourth place betoken Three?

Maſt. Their Figures be as you have ſaid, but their Values are not ſo. For, as in the firſt place, every Figure betokeneth his own value certain only, ſo in the ſecond place every Figure betokeneth ten times its own Value certain: as in the example, 7 in the ſecond place is ſeven times ten, and is Seventy. And in the third place, every Figure betokeneth its own Value an hundred times, ſo the 6 in that place is ſix Hundred; and in the fourth place every Figure betokeneth his *A general Rule.*

own

own Value a Thouſand times, as in the aforeſaid number 3 in the fourth place ſtandeth for 3 Thouſand, and in the fifth place every Figure ſtandeth for his own Value ten thouſand times, and in the ſixth place a hundred thouſand times, and in the ſeventh place a Million of times, and in the eighth place ten Millions, ſo that every place exceedeth the former ten times.

Schol. As thus; if I make this number at all adventures, 91359684, here are eight places. In the firſt place is 4, and betokeneth but four; in the ſecond place is 8, and betokeneth ten times 8, that is Eighty; in the third place is 6, and betokeneth ſix Hundred; in the fourth place 9 is nine Thouſand; and 5 in the fifth place is fifty Thouſand: ſo 3 in the ſixth place is three Hundred Thouſand; then 1 in the ſeventh place is one Million; and 9 in the eighth place is ninty Millions. But now I cannot eaſily nor quickly read it in order.

Maſt. You may attain to that by this means. Firſt, put a Point over the fourth Figure, and ſo over the ſeventh. And (if you have ſo many) over the tenth, thirteenth, ſixteenth, and ſo forth, ſtill leaving two Figures between each two Points.

Then begin at the laſt Point, and ſee how many Figures are between it and the end, which cannot exceed three, reckoning it ſelf for one: then pronounce them as if they were written alone from the reſt, and at the end of their Value, ſo many times Thouſands as your numbers have Points. As for example, 91359684, the aforeſaid number.

Put a Point over 9 in the fourth place, and over 1 in the ſeventh place, and then no more (for your places come not to ten) as thus, 91359684.

Now go to the laſt Point over 1, and take it and the Figure 9 that followeth it, and value them alone.

Schol. 91, that is Ninty one.

Maſt. So it is, then reckon for the number of your Points twice a Thouſand.

Schol. That is, Ninty One Thouſand Thouſand.

Maſt. So it is; then take the three other Figures, from one to the next Prick, and value them.

Schol. 359, that is, Three hundred fifty nine.

Maſt. Now add for the one Point that is between them and the firſt place, a Thouſand.

Schol. Three hundred fifty nine Thouſand.

Maſt.

Mast. Then come to the other 3 Figures that remain.

Schol. 684, that is, Six hundred eighty four.

Mast. Now you have valued all. And at the end of the last number you must add nothing, because there remaineth no Point nor number after it, yet prove it in another number, as thus, 2308640891105340.

Schol. 2308640891105340. I have pointed them as you taught me, but I am in doubt whether I have done well or no, because of the Cyphers, for I remember you told me that they do signifie nothing, and therefore I doubt whether I should reckon them for a Figure in setting of the Points; and again, I know not wherefore they serve.

Mast. That I will tell you now. Indeed they are of no value themselves, but they serve to make up the number of Places, and to make the Figure following them to be in a further Place, and therefore to signifie the more value: as in this example, 90, the Cypher is of no Value, but yet it possesseth the first place, and causeth 9 to be in the second place, and so to signifie ten times 9, that is Ninety. *The use of Cyphers.*

Schol. Then I perceive in the example above, I have pointed well enough, for though that Cypher that is pointed signifies nothing, yet must it have the Point, because it came in the thirteenth place. Then will I try to number that Summ.

First, There is 230 thousand thousand thousand thousand.
Secondly, 864 thousand thousand thousand.
Thirdly, 89 thousand thousand.
Fourthly, 105 thousand.
Fifthly, 340.

Mast. You say right, according to the foregoing rule of reckoning each Point or Prick to be 1000; but if you call every Thousand thousand, a Million, as is most proper; then the foregoing Figures are thus numbered.

First, 230 Millions of Millions.
Secondly, 864 Thousand Millions.
Thirdly, 89 Millions.
Fourthly, 105 Thousand.
Fifthly, 340 Units, which will appear more plain to you by the following Table.

Schol. I think, Sir, I perfectly understand what you have thus far discourſed of: is there any thing else to be learned in Numeration, or shall I pass on to the next rule to be learnt in order.

Maſt. I might ſhew you here who were the firſt Inventors of this Art, and the reaſon of all theſe things that I have taught you, but that I will reſerve till you have learned the practice of this Art, leſt I ſhould trouble you with over many things at the firſt.

Three kinds of Numbers. But yet this you muſt obſerve, that there are three kinds of Numbers, one called Digits, another Articles, and the third mixt Numbers.

Digits. A Digit is any number under ten, as theſe, 1, 2, 3, 4, 5, 6, 7, 8, 9.

Articles. And 10 with all other that may be divided into ten parts juſt, and nothing remain, are called Articles, ſuch as are 10, 20, 30, 40, 50, *&c.* 100, 200, *&c.* 1000, *&c.*

Mixt. And that number is called Mixt, that containeth promiſcuouſly Articles, or an Article, and a Digit, as 12, 16, 19, 21, 38, 107, 1005, and ſo forth; and for the more eaſe of underſtanding and remembrance, take notice, That the Digit number is never written with more than one Figure, but the Article and the mixt Number are ever written with more than one Figure. And thus they differ, that the Article hath always this Cypher 0 in the firſt place: and the mixt Number hath ever there ſome Digit.

Schol. I now apprehend this, and can eaſily diſtinguiſh theſe three kinds of Numbers.

Maſt. If you remember now all that I have ſaid, you have learned ſufficiently this firſt kind of *Arithmetick* called Numeration. But as you have learned to gather and expreſs the Value of a Summ propounded and ſet down before you, ſo muſt you practiſe and write down with proper Figures and in due Places, any number only named, or recited to you, or of your ſelf imagined; as for a proof. How do you write down this Summ, five thouſand, two hundred, fifty and ſeven?

Schol. I am at a ſtand, whether I ſhould begin at the firſt, or at the laſt. For reaſon, methinks, ſhould cauſe me to begin at the firſt, and yet if I write it as you ſpeak it, I muſt begin at the laſt.

Maſt. When you know your places perfectly, you may begin where you pleaſe; but it is more eaſie to begin with the laſt, that is to ſay, as I did ſpeak them, yet for more certainty, you may begin at the firſt, repeating my words backwards, thus, ſeven, fifty, two hundred, five thouſand: or elſe ſounding them all by their Digit or Value, as thus: ſeven, five, two, five; for that way is eaſieſt: But then you muſt look whether there be any Cypher in your Summ, that

that it may be ſet in its place: as if the laſt Valuer of your Summ (as you ſpeak it) be above 9, then is there a Cypher in the firſt place. And if it be an hundred, or above, then is there two Cyphers, one in the firſt place, and another in the ſecond, and ſo forth.

But becauſe this cannot be fully explained without many words; I have here at the end of Numeration, added a Table for your exerciſe.

The Numeration Table in common Engliſh Numbers.

The Denominators of the place or value uncertain.	*Units.*	*Tens.*	*Hundreds.*	*Thouſands.*	*M. of Thouſands.*	*C. of Thouſands.*	*Millions.*	*M. of Millions.*	*C. of Millions.*	*M. of Millions.*	*X. M. of Millions.*
The Names of Digits, Values certain.											
Nine.	9	9	9	9	9	9	9	9	9	9	9
Eight.	8	8	8	8	8	8	8	8	8	8	8
Seven.	7	7	7	7	7	7	7	7	7	7	7
Six.	6	6	6	6	6	6	6	6	6	6	6
Five.	5	5	5	5	5	5	5	5	5	5	5
Four.	4	4	4	4	4	4	4	4	4	4	4
Three.	3	3	3	3	3	3	3	3	3	3	3
Two.	2	2	2	2	2	2	2	2	2	2	2
One.	1	1	1	1	1	1	1	1	1	1	1
Cypher.	0	0	0	0	0	0	0	0	0	0	0
The Order of Places.	Firſt.	Second.	Third.	Fourth.	Fifth.	Sixth.	Seventh.	Eighth.	Ninth.	Tenth.	Eleventh.

A Table of the Value of Latin Numbers.

i	I	1	One	xx	XX	20	Twenty.
ii	II	2	Two	xxx	XXX	30	Thirty.
iii	III	3	Three	xl	XL	40	Forty.
iiii	IIII	4	Four	l	L	50	Fifty.
iv	IV	4	Four	lx	LX	60	Sixty.
v	V	5	Five	lxx	LXX	70	Seventy.
vi	VI	6	Six	lxxx	LXXX	80	Eighty.
vii	VII	7	Seven	xc	XC	90	Ninety.
viii	VIII	8	Eight	C	C	100	One Hundred.
ix	IX	9	Nine	CC	CC	200	Two Hundred.
x	X	10	Ten	D	D	500	Five Hundred.
xi	XI	11	Eleven	M	M	1000	A Thousand.

The first Table hath eleven Places, and in each of them are set all the Digits, whose certain Value is written on the right hand of the Table, and the Value uncertain on the left hand; so that by this Table you may learn both how to express any Number you please, if it exceed not eleven places, that is to say, Ninety Thousand Millions, and so may you by help of it, value all Summs proposed under the said Number.

For example; Take the Summ that I proposed before, which was five thousand two hundred fifty and seven. And if you will express it, take the first number (as I speak it) which is five Thousand, whose Valuer or certain Value is 5, and his uncertain Value, or Denomination is a Thousand. First, you must seek at the right hand of the Valuer 5. Then seek along under the Title of Denomination toward the left hand, till you find Thousands, and under it, at the foot of the Table, is the number of the place, that is in the fourth, wherein you must write your Digit, or Valuer 5.

Afterward come to the second part of the number two hundred, whose Valuer is 2, and its Denomination a Hundred. Seek two at the right hand of the Table, and go along under the Denomination toward the left hand, till you come under Hundreds, then look to the foot of the Table, and there you shall see the number of the place, that is to say, the third, wherein you must set your Digit 2.

Then

Then do ſo by your other two numbers that remain, and you ſhall find 5 in the ſecond place for your fifty, and 7 in the firſt place for your ſeven. And thus you may do with other numbers.

Schol. Maſter, I thank you heartily. I perceive you ſeek to inſtruct me moſt plainly and briefly. For this rule is as plain, as I can deſire. And though it ſeem ſomewhat long, yet I perceive it to be a ſure way.

Maſt. So it is; and though it be long, yet it is neither too long, nor too plain for young Learners that want practiſe: for this Table is inſtead of a Teacher to them that want one. But now I hope I have ſaid enough of Numeration: which after you have well practiſed, then may you learn forward.

Schol. Yet I pray you in one thing to tell me your judgment. Why do men reckon the order of the places backward, from the right hand to the left? *Why Numbers are written backward.*

Maſt. In that thing all men do agree, that the *Chaldees*, which firſt invented this Art, did ſet theſe Figures as they ſet all their Letters, for they write backward as you term it, and ſo do they read. And that may appear in all *Hebrew*, *Chaldee* and *Arabic* Books, which are not only written from the right hand to the left, and ſo muſt be read; but alſo the right-hand ſide of the Book is the beginning of it, whereas the *Greeks*, *Latins*, and all Nations of *Europe*, do write and read from the left hand toward the right: and all their Books begin at the left ſide.

Schol. That reaſon hath ſatisfied me.

Maſt. But it doth not well ſatisfie me, becauſe I ſee the *Chaldees* and *Hebrews* do not ſo uſe their own Numbers, as at another time I will declare. But this plain reaſon may beſt ſatisfie you at preſent, that in pronouncing of Numbers we keep the order of our own reading, from the left hand to the right: and again, we do ever name the greater Numbers before the ſmaller; ſo that the leſſer places, containing the leſſer Numbers, are ſet on the right hand, and the greater places containing the greater Numbers toward the left hand.

Schol. This reaſon is to me ſo plain, that it ſeemeth now againſt reaſon to make a doubt of that order. So that now for Numeration I am ſatisfied: hoping that practiſe ſhall make me fully ready and expert in it. And in the mean time I deſire you to teach me the other kinds of Arithmetick. For the ſecond Table is ſo eaſie, that you need not explain it.

Maſt.

Mast. That is well said; but what should you learn next? can you tell?

Schol. I remember you said, that *Addition* was next.

Mast. Right, and what that is, you must first know.

CHAP. III.

Addition of Whole Numbers.

ADdition is the gathering together of two numbers, or more, into one Summ. As if I have 106 Books in the *Latin* Tongue, and 136 in the *Greek*, and would know how many they be in all, I must write these two Numbers one over another, so that the first Figure of the one may be under the first Figure of the other, and the second under the second, and so forth in order.

When you have so done, draw under them a right line, then will they stand thus.

160
136
6
160
136
96
160
136
296

Now begin at the first places toward the right hand always, and put together the two first Figures of these two Numbers, and the Summ of them write under the line. As in saying 6 and 0 is 6, write 6 under 6, as thus:

And then go to the second Figures, and do likewise: as saying 3 and 6 is 9, write 9 under 6 and 3, as here you see.

And likewise do so with the Figures that be in the third place, saying 1 and 1 make 2, write 2 under them, and then will your whole Summ appear:

So that now you see that 160, and 136, do make in all 296.

Schol. What? this is very easie to do, methinks I can do it already.

There came through *Cheapside* two droves of Cattle; in the first was 848 Sheep, and in the second was 186 other Beasts.

Those two Summs I must write as you taught me, thus: Then if I put the two first Figures together, saying, 6 and 8, make 14: That must I write under 6 and 8 thus:

848
186

Mast. Not so: and here you are twice deceived. First, in going about to add together two Summs of sundry things, which you ought not to do, except you seek only the Number of them, and care not for the things: for the Summ that should result to that Addition, would be a Summ neither of Sheep nor of other Beasts, but a confused Summ of both. Howbeit sometimes you may have Summs of divers Denominations to be added, of which I will tell you by and by: but first I will shew you where you were deceived in another point, and that was in writing 14, which came of 6 and 8, under 6 and 8, which is impossible: for how can two Figures of two places be written under one Figure and one Place?

Schol. True, but yet I did so understand you.

Mast. I said indeed, that you should write that under them that did result of them both together: which is always true, if that Summ do not exceed a Digit: But if it be a mixt Number, then must you write the Digit of it under your Figures as you have said before: and if it be an Article, then write 0 under them, and in both Cases you must keep the Article in your mind; and therefore when you have added your second Figures which possess the place of Tens, you must put that one thereto, which you kept in your mind; for tho' it were ten indeed, yet in that place, it is but as one, because that every one of that place is ten, for that it is the place of tens. And in like manner, if you have in the second place so great a Number that it amounteth to above 9, then write the Digit, and reserve the Article in your mind, ever adding it to the next place following, and so of all other places, how many soever you have. And if you have a mixt number when you have added your last Figures, then write the Digit under the last Figures, and the Article in the next place beyond them: so shall your Number resulting of Addition, have one place more than the Numbers which you have added together.

Reason of Addition.

A place.

Schol. Now I understand you, and the reason of this is, (I presume) because that no one place can contain above 9, which is the greatest Figure that is, and then all Tens or Articles must be put to the next place following: for every place (I see) exceedeth the other place next before him by 10.

Now, if it please you, I will return to my Example of Cattle. But I remember you said I might not add Summs of sundry things together, and that I may see by reason.

Mast. True, but if you only seek a bare Summ, and have no respect to the thing, then were it better to name the Summ only, without any thing:

thing: as in saying 848, without naming Sheep or any thing else. And likewise 186, naming nothing.

Now let me see how you can add those two Summs.

Schol. I must first set them so that the two first Figures stand one over another, and the other each one over that of the same place; then shall I draw a line under them both: and so likewise of other Figures, as followeth.

Then must I add 6 to 8, which make 14, that is a mixt Number, therefore must I take the Digit which is 4, and write it under 6 and 8 keeping the Article 1 in my mind thus:

	848
	186
	4

Next that, I come to the second Figures, adding them together, saying 8 and 4 make 12, to which I put the one reserved in my mind, and that maketh 13, of which Number I write the Digit 3 under 8 and 4, and keep the Article in my mind, thus:

	848
	186
	34

Then come I to the third place of Figures, saying, 1 and 8 make 9, and 1 in my mind maketh 10. Sir, shall I write the Cypher under 1 and 8?

Mast. Yes.

Schol. Then of 10 I write the Cypher under 1 and 8, and keep the Article in my mind.

Mast. What needeth that, seeing there follow no more Figures?

Schol. Sir, I had forgotten, but I will remember better hereafter. Then seeing I am come to the last Figures I must write the Cypher under them, and the Article in a further place after the Cypher thus:

	848
	186
	1034

Mast. So now you see, that 848, and 186 added together, amounteth to 1034.

Schol. Now I think I am perfect in Addition.

Mast. That I will prove by this Example. There are two Armies of Soldiers; in the one are 106800, and in the other 9400. How many are there in both Armies?

Schol. First, I set them one over another, beginning with the first Number on the right hand thus:

	106800
	9400

But the lower Number will not come so far to the left hand as the other Number.

	106800
	9400
	0

Mast. That is no matter.

Schol. Then do I add 0 to 0, and there amounteth 0, that must I write under the first place, as in the Margent.

Mast.

Mast. Well said.

Schol. Then likewise in the second place, I add 0 to 0, and there ariseth 0, which I write under the second place thus:

Then I come to the third place, saying, 4 and 8 make 12, of which I write the Digit 2, and keep the Article 1 in my mind, thus:

Then I add 9 to 6, which make 15, to that I add the Article 1 that was in my mind, and it is 16, I write 6 under 6 and 9, and keep 1 in mind, thus:

Mast. Why do you not write both Figures, seeing you are come to the last couple of Numbers?

Schol. Nay, reason sheweth me, that I must add that Article which is in my mind unto the next Figure of the upper Summ, though there be no more in the lower Summ.

Mast. That is well considered: then do so.

Schol. Then I say, 0 in the upper Summ, and one in my mind maketh 1: that write I under 0. Then followeth there yet one more in the upper Summ, which hath none to be added to it, for there is none in the lower Summ, nor in my mind: therefore I think I must write that under as it is.

Mast. You say right.

Schol. Then doth my whole Summ appear as in the Margent, thus:

106800	
9400	
00	
106800	
9400	
200	
106800	
9400	
6200	
106800	
9400	
116200	

Mast. If you mind this, you have learned perfectly the common Addition of all Summs which are of one Denomination: so that you observe this also, that in Addition you must have two Numbers at the least; or else how can you say that you do add?

And forget not, that (if you have many Numbers to add together) you may have oftentimes an Article of a greater Value than 10, sometimes 20, sometimes 30, sometimes more, and so to 100 or more. Therefore as you did with the Article 10, so do with them, reserving them in your mind, and adding to the Number next following so many, as their Value is: that is to say 2 for 20, 3 for 30, 5 for 50, 10 for 100, 12 for 120, and so forth. So that if the Article be 100, then must you set down the 0, and keep 10 in mind, to be carried to the next row of Figures or Place, if any such happen to be. For your better understanding, take this Example for all.

I would add these thirteen Summs into one, which I set after this manner: then I begin and gather the Summ of the first row of Figures, which come to 107, (for I take 9 there ten times, and that is 90) then 9 and 8 is 17, that is in all 107, of which Summ I write the 7 under the first row of Figures, and then because 100 is ten tens, I keep ten in my mind, which ten I must add unto the next row of Figures, which are in the second or tens place; ——— which second row of Figures, (when they are added together with that ten that I had in my mind) make in all 129, of which Summ write the Digit 9 under the second row, and then (because 120 containeth twelve tens) I keep 12 in mind to be added to the third place or row of Figures; which being added together, make in all 60, the Cypher o I set down under the row of Figures in the third place.

And the Figure 6 I keep in mind to be added to the row of Figures in the fourth place, which (when they are added together) make 29. The Figure or Digit 9 I set down under the fourth place. And because I have no more places of Figures to the left hand, I set down the 2 also that I have in my mind to the left hand the 9 in the fifth place; so these Summs do make in all 29057.

4889	
4599	
2290	
3699	
2299	
4099	
1099	
3298	
299	
699	
499	
899	
389	
4889	
4599	
2290	
3699	
2299	
4099	
1099	
3298	
299	
699	
499	
899	
389	
29057	

But (for your more ease in work) when you have an Addition of so many Summs to be added together, it were best to part that Summ into two or three parts, and add them severally, and so put their Summs together.

Schol. This seemeth somewhat hard, by reason of so many Numbers together.

However, I think (if I do often prove the same example, either by working of it alone, or else by parting it as you said) that I shall be able to do so with any other Summ.

Mast. So you will: for it is practice that maketh a man quick and ripe in all things: but because, as well in great Summs as in small there may chance to be some error, I will teach you how you shall prove whether you have done well or no: but you must first learn to add Numbers of divers Denominations together.

Mast.

And now I will teach you how to add the same: which thing can never be, but when the one Denomination is such that it containeth the other certain times. And yet you shall add them to the other, not after the manner (as you did them that were of one Denomination) but as I will shew you; but first take the following Tables to your Assistance. *Addition of number of divers Denominations.*

Table I. *Of* English *Money.*

4 Farthings	make	1 Penny.
12 Pence		1 Shilling.
6 Shillings 8 pence		1 Noble.
10 Shillings		1 Angel.
13 Shillings 4 pence		1 Mark.
20 Shillings		1 Pound.

Also observe that in a Pound Sterling there are,

960 Farthings.
480 Half pence.
240 Pence.
120 Two Pences.
80 Three Pences.
60 Groats (or Four Pences.)
40 Sixpences.
20 Shillings.
3 Nobles.
2 Angels.

Thus much for Money. Now I will shew you in like manner the distinction of Weights. *Weights.*

By a Statute made *Anno* 11 *H.* 7. there ought to be but one sort of Weight, as 24 Barley-corns dry, and taken out of the midst of the Ear, do make a Penny Weight, 20 of these Penny Weights make an Ounce, 12 Ounces a Pound *Troy* Weight, by which is weighed Bread, Gold, Silver, Pearl, and such like. But commonly there is used another Weight called *Averdupoize*; in which 16 Drachms is 1 Ounce, 16 Ounces make 1 Pound, 28 Pound 1 quarter of a Hundred, 4 Quarters 1 Hundred, or 112 lb. *Troy weight. A Penny weight. An Ounce. A Pound Troy.*

Howbeit there are in some things other names, as in Sheeps Wooll, 28 Pound is not called a Quarter, but a Todd: and 14 Pound is not named *Wool. Weights. Todd.*

Stone. named half a Quarter but a Stone, and the 7 Pound half a Stone. As for other names because they differ in many places, and agree in few, I let them pass.

Sack of Wool. But a Sack of Wool by the Statutes is limited to be 26 Stone.

Moreover this Weight is used by the Apothecaries in their Physical Composition, and Mixture of Medicines, wherein the least is a Grain; *viz.*

The Apothecaries weights.

And	20 Grains	make	A Scruple	℈.
	3 Scruples		A Drachm	ʒ.
	8 Drachms		An Ounce	℥.
	12 Ounces		A Pound	lb.

Measures for Liquor. Now of Weights are made measures both for Grain and Liquor. For a Pound in *Troy*-Weight maketh a Pint in measure; so that 8 Pound or 8 Pints do make a Gallon; half a Gallon is named a Pottle; and half a Pottle is called a Quart, which containeth two Pints: Now above a Gallon the next measure is a Firkin: then the Tertian or Kilderkin or half a Barrel, and a Barrel: And by these measures are sold commonly Ale, Beer, Wine, and Oyl, Butter and Soap, Salmon, Herrings, and Eeles.

A Pint.
Gallon.
Pottle.
Quart.
Firkin.
Tertian.
Kilderkin.
Barrel.

But as these be unlike things, so the measures of their Vessels do differ, for the measures of them all, are as followeth:

Ale measures.

Of Ale	The Firkin,	containeth	8
	The Kilderkin,		16 Gallons.
	The Barrel,		32

Beer measures.

Of Beer	The Firkin,	containeth	9
	The Kilderkin,		18 Gallons.
	The Barrel,		36

Soap measures both Firkin, Kilderkin, and Barrel should be equal to Ale measure.

Moreover the Statutes do limit the Weight of every of those three Vessels being empty.

A Barrel,	to weigh empty	26	Pounds.
Half a Barrel,		13	
A Firkin,		6½	

Note,

Note, The Gallon of Ale or Beer contains 282 ſolid Inches.

Herrings are ſold by the ſame meaſures that Ale and Soap be ſold by. *Herrings.*

Herrings are ſold by the tale, 120 to the Hundred, ten thouſand to the Laſt.

Salmon and Eeles have a greater meaſure.

Salmon and Eeles	The Butt,	holdeth	84	Gallons.	*Salmon and Eeles.*
	The Barrel,		42		
	Half Barrel,		21		
	The Firkin,		10½		

Now as for Wine Veſſels they are ſeldom ſmaller than Hogſheads, which are of 63 Gallons: every Hogſhead is two Barrels: yet there are many other Wine Veſſels, but of them all ſee this Table. *Wine meaſures.*

Of Wine and Oyl	The Rundlet,	holdeth	18½	Gallons.
	The Barrel,		31½	
	The Hogſhead,		63	
	The Tertian,		84	
	The Pipe,		126	
	The Tun,		252	

And the Gallon contains 231 ſolid Inches.

But you muſt know that there be other kinds of Tertians: for there are Tertians (that is to ſay) Thirds of Pipes, of Hogſheads, and of Barrels, as well of other things as of Wine. *Tertians.*

Alſo Malmſeyes, and Sack, *&c.* the half Tun is not called a Pipe, but rather a Butt. *A Butt.*

And as of Weights there cometh the Liquid meaſures (whereof I ſpake laſt) ſo of the ſame cometh dry meaſures, as Pecks, Buſhels, Quarters, and ſuch like, whereby are meaſured Corn and like Grains, alſo Salt, Lime, Coals, and the like. And this is the order and quantity of them *Dry meaſures.*

A Peck is the meaſure of two Gallons, each Gallon 272½ Inches.

A Buſhel	containeth	Four Pecks.	*A Buſhel.*
A Quarter		Eight Buſhels.	*A Quarter.*
A wey		Five Quarters.	*A Wey.*

Theſe

These are the common names and measures, but in divers places there be divers sorts.

Measure of length, breadth and thickness.

But now remaineth yet another kind of measures, whereby men measure Length, Breadth and Thickness, and those are, an Inch, a Foot, and such other, whose names and quantities this Table sheweth.

An Inch.	3 Grains of Barley in length	make	an Inch.
Foot.	12 Inches		a Foot.
Yard.	3 Foot		a Yard.
Ell.	3 Foot and 9 Inches		an Ell *English*.
Pearch.	5 Yards and a half		a Pearch.

Acre.

1 Pearch in breadth, 40 in length, do make a Rodd of Land, which some call a Rood or quarter of an Acre.

Moreover 40 Rods in length make a Furlong. 8 Furlongs make an *English* Mile, which containeth 320 Perches.

So that an *English* Mile grounded upon the Statutes, is in length 1760 Yards, 5280 Foot, and 63360 Inches.

Somewhat greater than the *Italian* Mile of 1000 Paces, and 5 Foot to a Pace.

Addition of divers Denominations.

If you have a Summ of divers Denominations, then set every Denomination by its self, with some note over of its Denomination. Then write your other Summs so under that first, that every one be set under the other of the same Denomination: as for Example, If your Denominations be Pounds, Shillings, and Pence, write Pounds under Pounds, Shillings under Shillings, and Pence under Pence: and not Shillings under Pence, nor Pence under Pounds.

Schol. Methinks I need not to be warned of it, for it were against reason so to confound Summs: but yet if you had not spoken of it, I should perhaps have been deceived in it.

Mast. If you say it is plain, I will speak no more of it, but with an example make the matter to appear evident.

First, one man oweth me 22 *l.* 6 *s.* 8 *d.* another oweth me 5 *l.* 16 *s.* 6 *d.* and another oweth me 4 *l.* 3 *s.* I would know what this is altogether: Therefore I must first set down my first Summ, and then the other, every one under his Denomination agreeing to the greatest Summ, as here you see, with a line under them.

l.	*s.*	*d.*
22	6	8
5	16	6
4	3	0

Then

Then I begin at the leaſt Denomination (which muſt always be ſet next the right hand) and add them together: and if the Summ will make 1 or 2 or 3 of the next Denomination, then muſt I keep it in my mind till I come to that place, and under that firſt place I muſt note the reſidue (if there remain any of the ſame Denomination) but if there remain none, then need I to write under it nothing. And this is all that you muſt obſerve in this Addition: for all other things are like to the manner of Addition before mention'd: Therefore the chiefeſt point of this Addition is, to know the values of common Coins and rated Summs; as how many Shillings be in a Pound, how many Pence in a Shilling, how many Drachms are in Ounce, &c. which you learn from the foregoing Tables of Coin, Weight and Meaſures.

Therefore let us return to that former example which I propoſed of the Debtors: which Summs when I had ſet orderly, they ſtood thus with a line under them.

Then to add them into one Summ, I muſt begin at the right hand where the ſmalleſt Denomination is, and add them together, firſt ſaying 6 and 8 make 14. Now ſeeing theſe 14 are Pence, which contain one Shilling and two Pence: the two Pence I ſet down under the line of Pence, and the one Shilling I keep in my mind to carry to the next row being the place of Shillings.

	l.	s.	d.
	22	6	8
	5	16	6
	4	3	0
			2

Then I add the Shillings together, ſaying 1 in my mind and 3 make 4, and 6 make 10, and 6 make 16, and 1 in the ſecond place which ſtandeth for 10, make 26, which is 1 l. 6 s. The 6 s. I ſet down under the place of Shillings, as appeareth in the example. And the 1 pound I keep to carry to the pounds.

	22	6	8
	5	16	6
	4	3	0
		6	2

Then I come to the pounds, adding them all together, ſaying 1 that I carry and 4 make 5, and 5 make 10, and 2 make 12. The Figure or Digit 2 I ſet down right under that place or row of pounds where I added, and the Article 1 I carry to the next place, ſaying, 1 in my mind and 2 is 3, which 3 I ſet down directly under the 2. And then appeareth my whole Summ thus.

	22	6	8
	5	16	6
	4	3	0
	32	6	2

And

And thus muſt you do with all ſuch like Summs whatſoever, whether they be Money, Weight or Meaſure, and, if you practiſe diverſ Summs, you will be well acquainted with the nature of Addition.

And that you may be exerciſed therein, take the following Examples of Weight and Meaſure.

Troy Weight.

lb.	℥	dw.	gr.	
14	10	17	23	} Add
90	6	12	10	
4	11	10	3	
12	1	0	11	
112	6	0	23	Summ.

Averdupoize Weight.

Ton.	C.	Q.	lb.	℥.	dr.	
17	19	1	20	15	15	} Add
92	1	3	16	4	10	
15	6	0	7	10	5	
72	12	2	13	12	4	
197	19	3	24	11	2	Summ.

Liquid Meaſures, as Beer,

Bar.	Fir.	Gall.	
37	3	8	} Add
12	1	2	
75	2	6	
19	2	7	
145	2	5	the Summ.

Long Meaſures, as Cloth, &c.

Yards.	Qr.	Na.	
176	1	2	} Add
734	3	2	
97	1	3	
17	2	1	
1026	1	0	Summ

And as the Pounds, Shillings, and Pence were added, ſo are theſe Examples, having reſpect to the Tables foregoing, to know how many Grains make a Penny weight, how many Penny weight an Ounce, *&c.* Alſo how many Drachms make an Ounce, how many Ounces is a Pound Averdupoize, *&c.* Or how many Gallons is a Firkin of Beer, how many Firkins a Barrel; or how many Nails is a quarter of a Yard, *&c.*

For by thoſe Tables, for inſtance, 24 Grains is a Penny weight *Troy*; therefore for every 24 in the Grain's place, 1 muſt be carried to the Penny weights place: and 20 penny weights is an Ounce, therefore for every 20 in the penny weights place, carry 1 to the Ounces place, and ſo of all the reſt, the Tables ſhew you how to add each Series of Figures with reſpect to the next Denomination toward the Left hand.

Schol. All this I believe I underſtand, and preſume I am now perfect in Addition.

Maſt. There is only one thing behind, which is the proof of the Work of Addition; which when I have ſhew'd you the method of, we will paſs on to Subſtraction. *Proof of Addition.*

The Rule for proving any Summ given to be Added, is to ſeparate it into 2 parts at pleaſure; and if the Summ of both thoſe parts is equal to the total found, then is the Work right, otherwiſe not. *For inſtance,*

The Summ of the 4 Numbers in the laſt Example of Long Meaſure is 1026 01 00; which we will prove by dividing the 4 Numbers into 2 parts, as you ſee by the line in the middle, and the Summ of the 2 uppermoſt lines *viz.* 176: 1: 2 and 734: 3: 2 is yards 911: 01 qr. And the Summ of the 2 lower lines, *viz.* 97: 1: 3 and 17: 2: 1 is yards 115, now the Summ of thoſe 2 Summs, *viz.* 911: 01: 00 and 115: 0: 0 being 1026 yards, 01 qr. equal to the Total of the 4 Lines, proves the Work to be true.

CHAP. IV.

Subſtraction of whole Numbers.

NOW *I have learned the two firſt kinds of* Arithmetick: *and (as I remember)* Subſtraction *doth follow, which (me thinks) muſt be contrary to* Addition. *Subſtraction.*

Maſt. So it is indeed: for as Addition increaſeth one groſs Summ, by bringing many into one: ſo contrariwiſe, Subtraction diminiſheth a groſs Summ by drawing of others from it: *So that* Subſtraction *is nothing elſe but an* Art *to draw and abate one Summ from another, that the remainer may appear.*

Schol. What do you call the Remainer?

Maſt. That you may perceive by the name.

Schol. So I partly do: but yet it is good to ask the truth of all ſuch things, leſt in truſting to my own conjecture, I be deceived.

Maſt. It is the ſureſt way. And, as I ſee cauſe, I will ſtill declare things unto you ſo plainly, that you ſhall not need to doubt. However, if I do overpaſs it ſometimes, (as the manner of men is to

forget the ignorance of them to whom they ſpeak) then do you put me in remembrance your ſelf.

Remainer. And as for this word that you laſt aſked me, take this deſcription: The Remainer is a Summ left after one Subſtraction is made, which declareth the exceſs or difference of the two other numbers, as if I would abate or ſubſtract 14 out of 18, there would remain 4 which is called the Remainer, and is the difference between thoſe two numbers 14 and 18.

Schol. I perceive then what Subtraction is: now reſteth to know the order how to work it.

Maſt. That you ſhall do by this means. Firſt, you muſt conſider, that if you ſhould go about to Subſtract or Deduct, you muſt have two Summs propoſed: the firſt, which is your groſs Summ, (or Summ total) and it is generally ſet higheſt: and then the ſubtrahend (or Summ to be ſubſtracted) which muſt be ſet under the firſt, and that in ſuch a manner, that the firſt figures be one juſt over another, and ſo the ſecond and third, and all other following, as you did in Addition: then muſt you draw under them a line, and ſo are your Summs duly ſet to begin your working.

Then begin you at the right hand (as you did in Addition) and take the lower number out of the higher, and if there remain any thing, write that right under them under the line: and if there remain nothing (by reaſon that the two Figures were equal) then write under them a Cypher: And ſo do you with all the other Figures, evermore taking the lower out of the higher, and write under them the Remainder, till you come to the end: And ſo will there appear under the line what remaineth of your groſs Summ after you have deducted the other Summ from it, as in this Example.

I received of your Father 48 *l.* of which I have laid out for you 36 *l.* now I would know what doth remain: And therefore I ſet my number thus in order. Firſt, I write the greateſt Summ, and under it the leſſer, ſo that the Figures at the right ſide be even one under another, and ſo the others thus.

48
36
—

Then I take 6 out of 8, and there reſteth 2, which I write under 8 thus.

48
36
—
2

Then I go to the ſecond Figures, and take 3 out of 4, and there remaineth 1, which I write under them, and then the whole operation appeareth thus.

48
36
—
12

Whereby I know, that if I deduct 36 out of 48 there remaineth 12.

Schol.

Schol. Now I will prove it in a greater Summ, and I will ſubſtract 2367924 out of 3468946, thoſe Summs I ſet in order thus:

3468946	
2367924	

Then I begin at the right ſide, and deduct 4 out of 6, and there reſteth 2, which I write under them. Then I go to the ſecond figures, and take 2 out of 4, and there remaineth 2, which I ſet under them alſo: then I take 9 out of 9 and there reſteth 0, which I write under the Nines (for you ſay, that if the Figures be equal, ſo that nothing doth remain, I muſt write the Cypher under them.)

Maſt. It was well remembred: now go on.

Schol. Then I come to the fourth place, and take 7 out of 8, and there remaineth 1, which I write under them alſo. Then in the fifth place, I take 6 out of 6, and there reſteth 0. So I write under them the Cypher. Then in the ſixth place, 3 from 4 there remaineth 1, which I write under them, and likewiſe in the ſeventh and laſt place, 2 taken from 3 there is left 1, which I write under them: ſo have I done my whole work, and my Summs do appear thus. Whereby I ſee, that (if I take 2367924 out of 3468946) there remaineth 1101022.

3468946	
2367924	
1101022	

Maſt. This is well done. And that you may be ſure to know fully the Art of Subſtraction, let me ſee how you can ſubſtract 32984732 out of 8250003456?

Schol. Firſt, I ſet down the greateſt Summ, and after that I will write under it the leſſer number, beginning at the right ſide, and then my Figures will ſtand thus:

8250003456	
32984732	

Then I take 2 from 6 and there reſteth 4, which I write under them. Then I Subſtract 3 from 5, and there remains 2, which I write under them. Then I take 7 out of 4, but that I cannot, what ſhall I do now? Note.

Maſt. Obſerve well what I tell you now, how you ſhall do in this caſe, and in all other the like: If any Figure of the Lower Summ be greater than the Figure of the Summ that is over it (ſo that it cannot be taken out of the Figure over it) then muſt you put 10 to the uper Figure, and then conſider how much it is, and out of that whole Summ draw the lower Figure, and write the reſt under them. Can you remember this?

Schol. Yes, I believe I ſhall. Now then in my example where I ſhould have taken 7 out of 4, and could not, I put 10 to that 4,

which maketh 14, from which I take away 7 and there reſteth 7, which I write under them.

Maſt. You have done well: But now muſt you mark another thing alſo: that (whenſoever you do ſo put Ten to any Figure of the upper number) you muſt add one ſtill to the Figure or place that followeth next in the lower line: as in the example there followeth 4, to which you muſt put 1, and make it 5, and then go on as I have taught you.

	8250003456
	529847 32
	018724

Schol. Then ſhall I ſay, 4 and 1 (which I muſt put to it for the 10 that I added to 4 before) make 5, which I ſhould take out of 3, but that cannot be; therefore I muſt put 10 to it, and then it will be 13, from which I take 5, and there reſteth 8 to be written under them: and becauſe of that 10 added to the 3, I muſt add 1 to 8 that followeth in the lower line, and that maketh 9, which I ſhould take out of 0 and cannot; therefore I put thereto 10, and that maketh 10, from 10 I take 9, and there remains 1, which I write under them.

Thus I add 1 likewiſe to the next Figure below, which is 9, and that maketh 10, and that 10 ſhould I take out of the Figure above, but I cannot, for it is 0, therefore I put 10 to it, and ſo take 10 out of 10, and there reſteth 0 to be written under them.

Then I come to the next Figure which is 2, and to it I add 1, which maketh 3, that 3 I cannot take out of nought, therefore of that nought I make 10, and thence I take 3, ſo there remaineth 7 to be written under them: likewiſe put 1 to 5, which make 6, that 6 I cannot take out of 5, therefore I add 10 to that 5 and make it 15, from which taking 6 there remaineth 9, which I write under them. Now have I done with all the lower Figures, and what ſhall I do more?

	8250003456
	529847 32
	8197018724

Maſt. You ſhould have added one to the next Figure following (if there had been any) becauſe you added 10 to the laſt Figure before of the uper line: but being there is no Figure more in the lower line, you muſt deduct that one from the number next to that you laſt Subſtracted from.

Schol. Then ſhall I ſay, becauſe I borrowed 10 to the 5, I muſt take 1 from 2, and there reſteth 1 to be written under that in the ninth place. Now I have no more to ſubſtract, for there is not any Figure remaining beneath, neither any unite to be added, becauſe

cauſe I borrowed not 10 to the Figure laſt before: and yet is there 8 remaining in the upper line, which I think (by reaſon) ſhould be ſet at the end of the Figures in the loweſt row, which is under the line, becauſe there was nothing taken from it.

Maſt. That is well conſidered, and reaſon teacheth ſo indeed.

Schol. But Sir, I beſeech you, ſhall I always when any number ſo remaineth alone, as thus 8 did, write it under the line ſtraight againſt its own place?

Maſt. Yes, whether they be one or many: and this well remembred, you have ſufficiently learned Subtraction; However, becauſe of certain things that might deceive you, if you did not take good heed to your working, I will propoſe to you another example of many numbers to be ſubſtracted, as thus: I received of a friend of mine to keep 2869 Crowns, of which at one time I delivered him again 500, at another time 368, at another time 440, at another time 80, and at another time 64, now I would know how many reſt behind.

Therefore firſt I ſet down my groſs Summ 2869 *Crowns received*, and underneath it I ſet all the parcels thus, and under them a double line.

Then take the Total of the Summs paid, out of the Crowns received, and the Remainer ſhews what you have yet in your hands. *For inſtance*,

The Summ of the Crowns paid back are 1452, which I take from 2869, the Crowns received; and the Remainer (as you have learn'd) is 1417, and and ſo many Crowns remain yet in my hand.

2869	*Crowns received.*
500	
368	
440	*Paid, which Add.*
80	
64	
1452	*Paid in all.*
1417	*Reſt behind.*

Schol. And is this all that I ſhall learn of Subſtraction?

Maſt. Yes, ſaving that (as you have ſeen in Addition) there are numbers of divers Denominations, in which the working is not much unlike: yet (without ſome inſtructions be given of it) it might ſeem to a learner more difficult than indeed it is. Therefore I will briefly ſhew you the uſe of it only by an example or two.

A certain man owed to me 14 l. 12 s. 8 d. *of which he paid me at one time* 4 l. 6 s. 8 d. *at another time* 3 l. *at another* 2 l. 3 s. 4 d. *and laſt of all* 6 s. 8 d.

Now

Now I would know what remaineth unpaid yet? therefore I ſet my Summs every one in their due place; As *Pounds* under *Pounds*, *Shillings* under *Shillings*, *Pence* under *Pence*, thus.

li.	*s.*	*d.*
14	12	8
4	6	8
3	0	0
2	3	4
0	6	8

Schol. Sir, I pray you, why do you write 2 *l.* for it is more common to ſay, 40 *s.*

Note, *how the Pen differeth from the common order of Counting.*

Maſt. We muſt here uſe the Denomination that is greateſt in any Summ, ſo that we may not write according as we uſe to ſpeak, ſaying, 16 *d.* 18 *d.* or likewiſe 7 *groats*, 8 *groats*, 24 *s.* 40 *s.* 48 *s.* and ſuch other: but we muſt write every Denomination in any Summ by it ſelf.

Namely, *Shillings* and *Pounds*: So muſt we write for the laſt Summs now named, 1 *s.* 4 *d.* 1 *s.* 6 *d.* 2 *s.* 4 *d.* 2 *s.* 8 *d.* 1 *l.* 4 *s.* 2 *l.* 8 *s.* and ſo on.

Schol. So that we may not write in Arithmetick, *Pence*, when the Summ amounteth to *Shillings*, nor *Shillings*, when the Summ maketh *Pounds*. Now (if it pleaſe you) end your example.

Maſt. When my Summs are ſo ſet as I ſhewed, then (according to the rules of Addition) I gather all the particular Summs, which are paid me, into one Total Summ, directly to be ſet under them between the two lines, not medling with the 14 *l.* 12 *s.* 8 *d.* owing me: therefore I muſt begin with the ſmalleſt Denomination, ſaying, 8, 4, and 8. is 20 *Pence*, which maketh one *Shilling* and 8 *Pence*, the 8 *d.* I ſet down under the place of *Pence*, and the one *Shilling* I keep in mind to carry to the next Denomination of *Shillings*; Then I come to the *Shillings*, and ſay, 1 that I bring or have in mind, and 6 is 7, and 3 is 10, and 6 makes 16, which becauſe it containeth not one *Pound*, I ſet directly under the place of *Shillings*. Then I come to the *Pounds*, whoſe parcels are 2, 3, and 4. that is in all 9, that 9 I ſet down directly under the *Pounds*: And ſo the Total or whole Addition of all the particulars paid me, amounteth to 9 *l.* 16 *s.* 8 *d.*

li.	*s.*	*d.*
14	12	8
4	6	8
3	0	0
2	3	4
0	6	8
9	16	8
4	16	0 *Reſt.*

Now

Now for the work of Subſtraction, I muſt take that total Summ out of the higheſt number, that is to ſay, from the *14 l. 12 s. 8 d.*

Therefore to perform the work I ſay, 8 *d.* out of 8 *d.* remaineth or reſteth nothing, therefore in the place of the reſt or remain right under the Denomination, I ſet down 0. Then coming to the Shillings, where I find 16, which ſhould be taken out of 12, but I cannot: therefore I imagine to borrow 1 out of the next Denomination, that is, of the *14 l.* and put that one pound ſo borrowed unto *12 s.* which maketh *32 s.*

Now *16 s.* out of *32 s.* reſteth *16 s.* which *16 s.* I ſet down directly under the place of Shillings.

Laſtly, coming to the pounds, ſaying 1 pound that I borrowed, and 9 make 10, then 10 out of 14, there reſteth 4.

So doth my whole reſt or remain appear to be *4 l. 16 s. 0 d.*

This I account the eaſieſt way for a young beginner to practiſe, though it be ſomething long.

Schol. Is there any ſhorter way for this work alſo?

Maſt. Yes, for you may add together the particular Summs as they are ſet in order, beginning with the pence, ſaying, 8, 4, 8, make *20 d.* which *20 d.* you ſhould take out of the 8 *d.* above the line, but you cannot, therefore ſhall you borrow 1 of the next Denomination, that is to ſay, 1 of the Shillings, and put it to the 8 *d.* that maketh *20 d.* now *20 d.* out of *20 d.* reſteth 0, which Cypher I ſet down directly in the pence place. Then one ſhilling that I borrow'd or had in mind, and 6 make 7, and 3 make 10, and 6 make 16, the 16 out of 12 I cannot take, therefore of the next Denomination I borrow 1 *l.* and put it to *12 s.* which maketh *32 s.* then *16 s.* out of *32 s.* reſteth *16 s.*

l.	*s.*	*d.*
14	12	8
4	6	8
3	0	0
2	3	4
0	6	8
4	16	0

Laſtly, I come to the pounds, ſaying, 1 *l.* in mind, or that I borrowed, and 2 make 3, and 3 is 6, and 4 is 10, then 10 out of 14, there reſteth 4.

So doth my remainer or reſt appear as before to be *4 l. 16 s. 0 d.* but in this way you have not the total of the Summs paid any-where put down, and therefore may uſe this or the laſt way, as you find moſt proper in your buſineſs.

There will likewiſe be occaſion to ſubſtract Numbers of other Denominations, as Weight, Meaſure, *&c.* For admit I receive from *Barbadoes* the Cotton Wool following:

At

	C.	Q.	lb.
At one time 6 Bags, being ——	13	2	19
At another 4 Bags, —— ——	9	1	12
In all 10 Bags, —— —— ——	23	0	3
And out of this I have ſold,			
At one time 3 Bags, being ——	5	3	20
At another 4 Bags, —— ——	8	2	12
At another 2 Bags, —— ——	4	3	27
In all 9 Bags, —— —— ——	19	2	3
Reſts 1 Bag —— ——	3	2	0

The queſtion is, how much is bought in all? how much of that is ſold? and what remains unſold?

And for the doing this, you muſt firſt add together the Weight and Number of Bags bought or received. Secondly the Weight and Number of Bags ſold, and deducting thoſe ſold from thoſe bought, the Remainer ſhews how much Cotton reſts unſold?

For finding the Totals bought and ſold, you muſt (as you learnt in Addition for every 28 in the pounds place carry 1 to the Quarters of Hundreds place, for every 4 Quarters carry one to the Hundreds. So the Total received is 23:0:3 and the Total ſold is 19:2:3. Now for ſubſtracting 19:2:3 from 23:0:3 do thus:

Say 3 from 3 there reſt 0, 2 Quarters from 0 you cannot have, but 2 from 4 Quarters that you borrow, and the Remainer is 2. Then ſay 1 borrow'd and 9 Hundred is 10, from 3 you cannot, but from 13 reſts 3, and 1 borrowed, and 1 C. is 2 from 2 reſts 0. So there reſts unſold 1 Bag containing 3:2:0.

For this is a general Rule in Subſtraction. That for as many Units of any one Denomination as you carry one to the next toward the Left hand in adding; ſo many Units muſt you borrow in that reſpective Denomination, to add to the number from whence Subſtraction is to be made in caſe it is too little; as,

Example 3.
Troy *Weight.*

	℥.	*dw.*	*Gr.*
Bought	47	13	20
Sold	9	19	23
Remains	37	13	21

Example 4.
Cloth Meaſure.

	Yards.	*Qr.*	*Na.*
Bought	147	1	0
Sold	99	3	2
Reſts	47	1	2

Schol.

Schol. Then if there be no other thing to be learned in Subſtraction, I may go forward to Multiplication, for that you reckon'd to be next in order.

Maſt. We have done indeed with the Art of Subſtraction, as touching the working.

But yet before we go to Multiplication, I will inſtruct you how to examin your work, whether it be well done or not. For the performance whereof, if you obſerve what I ſaid in Addition, you may eaſily perceive what is to be done for the proof of Subſtraction, which is beſt done by the help of Addition thus.

Draw under the loweſt number (which is your Remainer) a line, and then add this Remainer and the Summ that you did ſubſtract together, and write that Total under the lower line; and if the Summ thereof be equal to that from whence you made Subſtraction, then is it well wrought, or elſe not: as you may ſee in example the 4th.

	Yards.	*Qr.*	*Na.*
The Remainer ——	47	1	2
Being added to ——	99	3	2
The Summ is ——	147	1	0

Which is the ſame with that from whence ſubſtraction was made.

CHAP. V.

Multiplication of Whole Numbers.

Multiplication is an operation whereby two Summs produce a third: which third Summ contains ſo many times the firſt, as there are Units in the ſecond. And it ſerveth inſtead of many Additions.

As for Example: When I would know how many are 30 times 48. If I ſhould add 48 thirty times, it would be a long work. Therefore was Multiplication invented, which will do that at once that Addition ſhould do at many times.

Schol. I perceive the Uſefulneſs of it partly, but I ſhall not ſee the full profit of it till I know it intirely. Therefore Sir, I pray you, teach me the working of it.

Multiplication of Digits.

Maſt. So I will; but becauſe that great Summs cannot be multiplied but by the Multiplication of Digits, therefore I think it beſt to ſhew you the way of multiplying them. As when I ſay, 9 times 8, or 8 times 9, *&c.* And as for the ſmall Digits under 5, it were but folly to teach any rule, ſeeing they are ſo eaſie that every Child can do it: but for the Multiplication of the greater Digits, do thus:

The difference.

Firſt, ſet your Digits one right over the other, then from the uppermoſt downwards, and from the nethermoſt upward, draw ſtraight lines, ſo that they make a croſs, as you ſee here. Then look how many each Digit lacketh of 10, and write that againſt each of them at the end of the lines, and that is called the difference: as if I would know how many are 7 times 8, I muſt write thoſe Digits, thus:

Then I look how much 8 doth differ from 10, and I find it to be 2: that 2 do I write at the right hand of 8, at the end of the line, thus:

After that I take the difference of 7 likewiſe from 10 that is 3, and I write that at the right ſide of 7, as you ſee in this example.

Then I draw a line under them, as in Addition, thus.

Laſt of all, I multiply the two differences ſaying two times 3 make 6, that I muſt ever ſet under the differences, below the line: then I muſt take one of the differences from the other Digit (not from his own) as the lines of the Croſs ſhew, and what is left I muſt write under the Digits, as in this example, if I take 2 from 7, or three from 8, there remaineth 5: that 5 I muſt write under the Digit, and then there appeareth the Product of 7 times 8 to be 56. And ſo likewiſe of any other Digits, if they be above 5, for if they be under 5, then will their difference be greater than themſelves, ſo that they cannot be taken out of them. And again, ſuch little Summs every Child can multiply, as to ſay 2 times 3, or 4 times 5, and ſuch like.

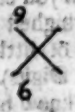
Schol. It is true Sir. And ſeeing I believe that I underſtand the multiplying of the greater Digits, I will prove by an example, how

I

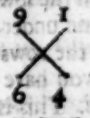
I can do it. I would know how many are 9 times 6.

Mast. It is all one in value to say 9 times 6, or 6 times 9: but yet the order is best to put the less Summ first, saying, 6 times 9, and so of all other Summs.

Schol. Then would I know how many are 6 times 9: therefore I set the Digits thus, and make the cross, thus:



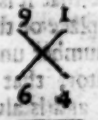
Then I set their differences from 10 at the right side, the difference of 9, which is 1, against it, and the difference of 6, which is 4, against it also, as in this example.



And under them draw a line, then do I multiply the differences together, saying: 1 time 4 maketh 4, that 4 I write under them, thus:

4

Then take I one of the differences from the other Digit, as 1 from 6, or else 4 from 9, and both ways there resteth 5, which I write under the Digit, thus:



And so appeareth the Product of 6 times 9, to be 54. Thus I see the manner of Multiplication of Digits.

54

Mast. Now might you go straight to the Multiplication of great numbers, but first both for your ease and surety in working, I will draw you here a Table, whereby shall appear the Products of all the Digits, and this is it that followeth.

The Multiplication Table.

1	1	2	3	4	5	6	7	8	9
	2	4	6	8	10	12	14	16	18
		3	9	12	15	18	21	24	27
			4	16	20	24	28	32	36
				5	25	30	35	40	45
					6	36	42	48	54
						7	49	56	63
							8	64	72
								9	81

In which Table when you would know the Product in any Multiplication of Digits, ſeek your firſt or laſt Digit in the lowermoſt or ſtant row of Figures from 1 to 9; and from it go towards the right hand, till you come under the number of your ſecond Digit, which is in the higheſt row, and then the number that is in the meeting of the rows of little ſquares (which come directly from both your propounded Digits) is the Product or Number produced of them. As if I would know by this Table the Multiplication of 7 times 9; ſeek firſt 7 in the lower Figures, and then go toward the right hand, till you come under 9, of the higheſt row, and in that common Angle, where the rows of little ſquares directly proceeding from 7 and 9 do meet, you have 63, which 63 is the Product of 9 by 7.

To multiply greater Summs.

Schol. This is very plain. And ſo may I find the Product of any Digits: but now how ſhall I do in greater Summs?

Maſt. When you would multiply any Summ by another, you muſt obſerve that it is the beſt way to ſet the greateſt number higheſt, which is the place of the number that muſt be multiplied: and likewiſe the leſſer number under it, for that is the place of the Multiplier or Multiplicator, that is to ſay, the number by which the Multiplication is made, and is always put before this word, Times; as when I ſay 20 Times 70. And the number that followeth this word Times, is that which muſt be multiplied.

Multiplier.

Times.

Therefore when I would multiply one number by another, I muſt write the greateſt higheſt, and the leſſer under it, as in Addition. And under them I muſt draw a line. As for example. If I would multiply 264 by 29, I muſt ſet them thus,

264
29

Of which number thus ſet down to be multiplied, may be formed a queſtion, as thus, There are 29 men, and each man hath 264 Lambs. The queſtion is, how many Lambs they have in all.

To the performance whereof, I muſt multiply every Figure of the higher row by every Figure of the lower row: and that which amounteth I muſt ſet under the line, as thus:

First I multiply 4 by 9, ſaying, 9 times 4, maketh 36, as the Table before of Digits doth declare, of that 36 I muſt write the 6, that is the Digit under the 9, and the Article 3 I keep in mind to carry to the next place.

264
29
6

Then

Then I come to the second Figure of the higher row, which is 6, and say 9 times 6 make 54, and with the 3 in my mind make 57, the 7 I set down under the 2, and 5 I keep in mind.

264
29
7656

After that I come to the next Figure, which is 2, and multiply it by 9, and that maketh 18, and with 5 that I have in mind, maketh 23: wherefore becauſe it is the laſt figure of the Multiplicand, I ſet it down in order as you ſee.

And ſo have I ended the firſt Figure of the Multiplier. Wherefore I give it now a fine daſh with my Pen.

Then begin I with the next Figure, and multiply it into all the higher Figures as thus.

First, 2 times 4 make 8, that 8 I always write under the ſecond place of the laſt product.

Then do I multiply by the ſame 2, the ſecond Figure of the higher row, which is 6, ſaying two times 6 make 12, wherefore I write the Digit 2 under the third place of the laſt product, and the Article 1 I keep in mind.

Then do I multiply the laſt Figure of the higher Summ by that ſame 2, ſaying, two times 2 is 4, and with the 1 that I have in mind maketh 5, which I write under the fourth place of the laſt product. And ſo have I ended the whole Multiplication: wherefore I alſo give the 2 a daſh with my Pen, thus: and ſo I do ever as ſoon as I have diſpatched any Digit by which I multiply: and the Summs ſtand thus.

264
29
2376

264
29
2376
528

Then I muſt draw a line under the two products of the Multiplication, and muſt add them into one Summ, as in the Example you may ſee.

264
29
2376
528
7656

Where in the firſt place I find but 6, and therefore write I it under the line. Then in the ſecond place 8 and 7 make 15, whereof I write 5, and keep one in my mind, and ſo forth as you learned in Addition. And ſo appeareth the whole Summ to be 7656, which ariſeth from the Multiplication of 264, by 29, and that is the juſt number of the Lambs that 29 men had.

Schol. If there be no more to be obſerved in it, then I can do it, I ſuppoſe, as by this Example I ſhall prove.

There

There is a peice of ground which containeth 1365 yards in length and 236 yards in breadth: I would know how many yards square there is in all this piece of ground: which numbers I set down with the greater above, and the lesser under, as you see.

Then I multiply 5 by 6, saying, 6 times 5 make 30, of which I write the Cypher in the first place, and the Article 3 I keep in mind to carry to the next place.

Then I by the same 6, multiply the second Figure of the higher Summ, which is 6, saying, 6 times 6 make 36, and 3 in my mind make 39, of which I write the 9 under the second place, and the Article 3 I keep in mind.

Then I multiply the third Figure, which is 3 by the same 6, and that maketh 18, and 3 in my mind make 21. The 1 I set down, and keep 2 in mind.

Then come I to the last Figure of the higher Summ, and multiply it by 6, saying, 6 times 1 make 6, and 2 in my mind make 8, that 8 do I write under the fourth place of the Multiplicand. And so have I ended the first Multiplier, and dash it slightly with my Pen.

Then I begin with the second Figure in the Multiplier, and say first, 3 times 5 make 15, of which I set the 5 under the second place, because the Multiplier is there, and the Article 1 I keep in mind.

Then I come to the second Figure that is 6, and multiply it by 3, which maketh 18, and with 1 in mind, maketh 19, the 9 I set down under the third place, and 1 I keep in mind.

	1365
	236
	30
	1365
	236
	90
	1365
	236
	190
	1365
	238
	8190
	1365
	238
	8190
	5
	1365
	238
	8190
	95

Then

Then I come to the third Figure, which is 3, and multiply it by 3, saying, 3 times 3 make 9, and with 1 in mind, make 10, the Cypher I set under that fourth place, and the Article 1 I keep in mind.

And then coming to the laſt Figure 1, I multiply it by 3, and it maketh 3, and with the one in mind, it maketh 4, which 4 I ſet in the fourth place, and then have I ended two of the Figures in the Multiplier, and the Summs ſtand as you may ſee, and then the 3 in the Multiplier I Cancel.

Then I come to the third Figure in the Multiplier, and multiply it into every Figure of the higher Summ, and firſt I ſay, 2 times 5 make 10, of which I ſet the Cypher under the hundreds place of the Multiplier, and the Article 1 I keep in mind.

And ſo multiplying the ſecond Figure 6 by that ſame 2, there amounteth 12, and 1 in my mind maketh 13, whereof I write the Digit 3 under the fourth place of the firſt product, and the Article 1 I keep in mind:

Then I multiply by the ſaid 2 the third figure of the higher Summ, which is 3, and that maketh 6, and the one in mind make 7, which 7 I ſet down under the fifth place, as appeareth by the example.

Then I come to the laſt place, and multiply that 1 by 2, and there amounteth 2, which I ſet under the ſixth place, and then doth the Summ ſtand thus.

And ſo I have ended the whole Multiplication.

1365	
238	
8190	
095	
1365	
238	
8190	
4095	
1365	
238	
8190	
4095	
0.	
1365	
238	
8190	
4095	
30.	
1365	
238	
8190	
4095	
730	
1365	
238	
8190	
4095	
2730	

But now (as you taught me) to know what this whole Summ is, I muſt add all thoſe parcels together, and then under the line will appear, as you may ſee, the groſs or total Summ is, 322140 Where by I know there is ſo many yards ſquare in that piece of ground.

1365	
8190	
4095	
2730	
322140	

Maſt. This is well done.

Schol. Then me thinketh I could call it well done, when I know, whether I had well done or no.

Maſt. The ſureſt proof is by Diviſion, and therefore I will reſerve that proof by Diviſion, till you have learned the Art of Diviſion.

But firſt, for your further inſtruction in Multiplication, *I will with one example try your skill therein, and ſo make an end: And the* Queſtion *is this. I would know how many days it is ſince* the Incarnation of our Lord and Saviour Jeſus Chriſt, *unto this year* 1699 *inclusively, which to perform, you muſt* Multiply *this preſent year* 1699, *by the days in one whole year, which are* 365.

Schol. Now for that you have given me ſo much light into the Queſtion, you ſhall ſee I will handſomely finiſh the work, for according to your former inſtructions, I ſet them down with a line under them thus.

Years ſince our Saviour.	1699
Days in a Year.	365
	8495
	10194
	5097
Days in 1699 Years	620135

Here you ſee I Multiply 1699 by the 5 in the Units place of the Multiplier, and the products is (working as in the 2 foregoing Examples) 8495.

Then the product of 1699 by 6, the ſecond Figure in the Multiplier is 10194.

And the 1699 by 3, the third Figure in the Multiplier produceth 5097, which being wrought as before and placed as in the Margent above, I find the Summ to be 620135 days.

Maſt.

Maſt. I commend you for your diligence, the work is very perfectly done, and declareth the number of days ſince our Lord and Saviour his Incarnation, unto the end of 1699 years, beſides 424 days, for leap years.

Schol. This is wonderfull, me thinks, that ſuch great matters may ſo eaſily be attained by this Art, which heretofore I ever thought had been impoſſible, as infinite ſorts of People are of that mind.

Maſt. Truth it is, that knowledge hath no greater Enemy than Ignorance, for this is one of the leaſt of ten thouſand things that may be done by this Art, as hereafter you ſhall be able to juſtifie.

Schol. The manner of Multiplication I perceive, if there be no more in it.

Maſt. Yes, there are other forms and helps for eaſe and ſhorter work of Multiplication, but I will omit them till you have a little taſted Diviſion, where alſo the like help in Diviſion may be uſed: and ſo therefore under one example for both will I ſhew you both eaſe in Multiplication, and alſo in Diviſion.

But ſince the other forms and workings do nothing differ from theſe works in effect, but only in ſetting of the numbers, I will paſs them at preſent. And now I will inſtruct you in Diviſion, if you think your ſelf ſufficiently to underſtand what I have taught you.

Schol. Yes Sir, I thank you, and hope you will be as good as your promiſe to teach me to prove this Work after I have learnt Diviſion, which pray let us proceed to.

CHAP. VI.

Diviſion of Whole Numbers.

Schol. WEll Sir, then I pray you to inſtruct me in Diviſion. But methinks by the name of it, that it ſhould be all one with Multiplication: for I call that Diviſion, when any thing is parted into divers parts.

Maſt. You take it as it is taken commonly; but if you obſerve well, you ſhall perceive that it is quite contrary to Multiplication, and doth

not part one thing or few things into many, but contrary ways, it bringeth many parcels into few, but yet so, that these few taken together, are equal in value to the other many: for by Division Pence are turned into Shillings, and Shillings into Pounds: As for Example, of 120 Shillings, it maketh 6 Pounds, so are 120 turned into 6, which is a smaller number: but then if you consider the Denomination, you shall see that they are such, that one of the latter is equal to 20 of the first, and so in value the Summs are one, though in number they do differ, and the latter Summ is the lesser, and so it is always in Division.

Scho. I think I shall better understand the reason of the name when I know the use of the work, therefore now would I gladly learn that.

Division what it is.

Mast. Division is a distributing of a greater Summ by the Units of a lesser: Or, Division is an Arithmetical producing of a third number, in respect of two propounded numbers; which third number shall so often contain an Unit, as the greater of the two propounded numbers does contain the lesser. So that as Multiplication did seem to serve instead of many Additions, so Division serves instead of many Substractions: Because that third number briefly expresseth how many times the lesser of your two propounded numbers may be substracted from the greater: as in Practise will more largely appear. Therefore (as you may perceive) unto Division are required three numbers: The first, which should be divided called the *Dividend* and that must (generally) be the greater: The second, is that by which the other must be divided, and that is (generally) the lesser, and is called the Divisor: And the third, answereth to the question, and therefore is called the Quotient.

A general rule for placing the figures.

The Greatest must be first written, and the second so set under it, that the last figure of the lowet number be right under the last of the higher, contrary to the work of other kinds of Arithmetick: for in them the two first Figures were set one under the other; but in Division, the last Figures must be set so, except it chance so that the last Figure of the Divisor be greater than the last of the Dividend, for then you must set the last of the Divisor under the last save one of the higher number, as for example:

If you would divide 365 (which are the Summ of the days of a year) by 28, which are the days of a common Month, then should you set them thus.

365
28

But

But if you would divide thoſe 365 days by 52, which is the number of weeks in one year, then ſhould you ſet them thus.

365
52

Likewiſe if I would divide the ſame 365 by 4, which is the Summ of the quarters of years, then muſt I ſet them thus.

365
4

Schol. Sir, this I underſtand, but how ſhould I do to divide the one by the other?

Maſt. You muſt begin with the laſt Figure next the left hand, and ſee how many times the laſt Figure of the Diviſor may be taken out of the laſt Figure of the Dividend, and that ſhall you note within a crooked line toward your right hand: As for example. I would divide 365 by 28, then I ſet thoſe two Summs thus.

365 (
28

And I look how many times I may find 2 (which is the laſt Figure of the Diviſor) in 3, (which is the laſt of the number to be divided) and conſidering that I can take 2 out of 3 but once, I make a crooked line at the right hand of the numbers, and within it I ſet 1, and that is called the Quotient number, as I told you. *Quotient Numbers.*

Then becauſe that when 2 is taken out of 3, there remaineth 1, I muſt write that 1 over 3, and deface or Cancel the 3 and the 2, then will the Figures ſtand thus.

1
365 (1
28

Then I come to the next Figure of the Diviſor, and take it likewiſe ſo many times out of the Figures that be over it, *viz.* the 1 that remain'd and the 6, that is 16, and what doth remain, I write over them, and Cancel them, as in this Example.

Therefore now I take once 8 out of 16, and there remaineth 8, which I muſt ſet over the 6, and cancel or croſs out the 16, and the 8 of the Diviſor: and then will the Figures ſtand thus. And ſo I have once wrought.

8
365 (1
28

Schol. So I perceive that you take the lower Figure, not only out of the other that is right over him, but out of that with the other alſo that remaineth before, and are written toward the left hand.

Maſt. You ſay right: for you muſt ſo take the Diviſor out of the upper Number, that there remain not over it ſo great a Summ as it ſelf is: for then were the Figure put in the Quotient too little.

But yet again here muſt you obſerve, that when you ſeek how many times the laſt Figure of the Diviſor may be found in the number over him, that you look alſo whether you may as often find all the Figures following in thoſe that are above them (conſidering all the remainders, if there be any) if not, make your Quotient leſs by one, and then prove again, and ſo till you find a proper Quotient, and by that Quotient you muſt always multiply your Diviſor, and then Subſtract that product from the number to be divided that ſtandeth directly over it, as you have ſeen me do.

When you have thus wrought once, then muſt you begin again, and write your Diviſor a new, nearer toward the right hand by one place, as in this example, you ſet 2 under 8, and 8 under 5, thus.

```
  28
 365 (13
 288
   2
```

Then (as before) ſeek how many times you may take your Diviſor out of the number over him.

Schol. That I may do here 4 times.

Maſt. Truth it is that you may find 2 four times in 8: but then conſider whether you can find the Figure following ſo many times in the other that is over him. Can you find 8 four times in 5?

Schol. No, neither yet once.

Mark how to conſider this kind of Remainer.

Maſt. Therefore take 2 out of 8 once leſs.

Schol. That is three times.

Maſt. Well then, 3 times 2 make 6: if I take 6 out of 8, there remaineth 2: which 2 with the 5 following make 25, in which Summ I find 8 three times alſo, and therefore I take 3 as a true Quotient, and write it within the crooked line of the Quotient before the 1, thus:

```
  28
 365 (13
 288
   2
```

Then I ſay, 3 times 2 make 6, 6 out of 8 reſteth 2, therefore I cancel the 8, and write over it the 2 that doth remain, thus:

Then I take 8 as many times out of 25, ſaying, 3 times 8 makes 24, and if I take 24 out of 25, there remaineth 1, ſo then I cancel 25 and 8; and over the 5 ſet 1 thus:

Or you might (after you find 3 to be a fit Quotient) have multiplied the whole Diviſor 28, by that at once, which giveth 84, which being ſet under 28, and duely Subſtracted from 85, of the number divided, giveth 1, the remainer of the whole Diviſion, as before you had. Work which way you liſt, here you may ſee alſo the form.

And now I have done with the dividing; for I cannot find my Diviſor 28 any more in the upper Summ.

Schol.

Schol. No, except you would part the 1 that remaineth into 28 parts.

Mast. That is well said; and so we must do in such cases, when there remaineth any thing; but I will let that pass now, and will make you perfect in Division of Whole Numbers, and will hereafter teach you particularly of broken Numbers called Fractions. Now if you do perceive the order of Division, then do you divide this Summ 136280 by 452.

Schol. First I set down the number that should be divided; then I set the Divisor under the last figures of that number which will stand thus:

136280
452

Mast. Can you take the last of your Divisor (which is 4) out of 1, which is the last of the upper number?

Schol. I had forgotten, the last of the Divisor cannot be taken out of the last of the upper number, because it is the greater, therefore I must set the Divisor one place more forward toward the right hand, thus:

136280
452

And then I must look how often I may find the last figure of the Divisor (that is 4) in 13, which I may do 3 times, therefore I say, 3 times 4 is 12, which I take out of 13, and there remaineth 1. Then I make at the right hand of my Summs a crooked line, and write before it my Quotient 3, and I cancel 13 and 4, and over the 3 I set the 1 that remaineth, and then the figures stand thus:

1
136280 (3
452

Then I multiply the same Quotient into every figure of the Divisor, and deduct the Summ that ariseth out of the numbers over them, as first I say, 3 times 5 make 15, which I take from 16, and there resteth 1, I cancel therefore 16 and 5, and write over the 6 the 1 that remaineth, thus:

11
136280 (3
452
116

Then I say likewise, 3 times 2 make 6, which I take out of 12, and there resteth 6, therefore I cancel the 12 and the 2, and then I write the 6 that remaineth, thus:

136280 (3
452
1160

Then I should set forward the Divisor into the next place toward the right hand, thus:

136280 (3
452
452

Mast. But you may see that over the 4 is no figure, therefore you must set the Divisor yet forwarder by another place.

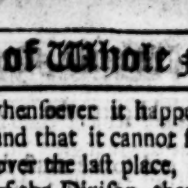
And

And obſerve, that whenſoever it happeneth, that you ſhould ſet forward the Diviſor, and that it cannot ſtand here, becauſe there is no number uncancell'd over the laſt place, or if there be any, it is leſſer than the laſt figure of the Diviſor, then muſt you remove the Diviſor yet one place farther toward the right hand, and write in the Quotient a Cypher, and if you ſhould by chance need to do ſo oft times, for every time write a Cypher in the Quotient. The reaſon of this I will ſhew you hereafter.

Schol. Then I muſt ſet my Summs, thus:

And becauſe I removed the Diviſor, ſo that I overskipped one place, I muſt write a Cypher in the Quotient: and then muſt I ſeek a new Quotient, as in this example, I muſt ſay, how many times 4 is there in 6? (and ſince it can be but once) therefore I write 1 in the Quotient: and then ſay, 1 time 4 taken out of 6, remaineth 2, I cancel the 6 and 4, and write 2 over them, thus:

Then ſay I again, once 5 out of 28, remaineth 23: I let the 2 ſtand as it did, and over that 8 I ſet 3, cancelling the 8 and the 5 under it, as in the Margent.

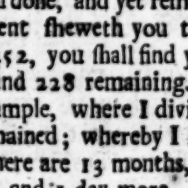


Maſt. You might as well have ſaid, once 5 out of 8, and ſo remaineth 3, but now go forward.

Schol. Then once 2 out of 0 cannot be: What ſhall I do now?

Maſt. Borrow of the next number that is behind, for there is 230, and do as you learned in Subſtraction in like caſe.

Schol. Then I muſt borrow 1 out of the 3 coming next, and make that 0 to be 10, and then I take 2 out of 10, and there reſteth 8: and becauſe I borrowed one of the 3, I muſt cancel the 3, and write 2 over it, then doth the figure ſtand thus:



Maſt. Now have you done, and yet remaineth 228, and your Quotient ſheweth you that if you divide 136280 by 452, you ſhall find your Diviſor in your greater number 301 times, and 228 remaining.

And in the other example, where I divided 365 by 28, the Quotient was 13, and 1 remained; whereby I knew that in a year, which containeth 365 days, there are 13 months, reckoning 28 days, or 4 weeks, juſt to a month, and 1 day more.

Schol. Why then do we call a year but 12 months?

Maſt.

Maſt. Of that at a more convenient time I will fully inſtruct you; but now it is not convenient to entangle your mind with other things than do directly pertain to your matter. Therefore if you remember what you have heard, you have learned a ſhort manner of Diviſion, which I would have you often to practiſe, ſo that you may be perfect in it, and hereafter I will ſhew you certain other proper points touching it.

Schol. Then I pray you tell me how I ſhall examin and try my work whether I have done well or no, that though no man be by to tell me, yet I may perceive it myſelf.

Maſt. The moſt certain working is by Multiplication, thus: Multiply the Quotient by the Diviſor, and if the Product ariſing be equal to the Summ that was divided, then have you well divided: elſe not. *The Proof of Diviſion by Multiplication.*

But obſerve this, that if there remained any thing after the Diviſion, that you muſt add to the Summ that ariſeth from the Multiplication. As in our firſt example, the Quotient was 13, and the Diviſor was 28; now multiply the one by the other, and the Product will be 364; to that if you add the 1 that remained after the Diviſion, then will it be 365, which was the Summ given to be divided; and therefore I know that I have well done.

Schol. Now I will prove the ſame in the ſecond example, whoſe Diviſor was 452, and the Quotient 301; theſe I multiply together, and they produce 136052: to which if I add the 228 that remained, then will it be 136280, which was the whole Summ to be divided; and therefore I perceive that I have performed the work right.

Maſt. This is the ſureſt way to examin Diviſion by Multiplication; and contrarywiſe, the ſureſt proof of Multiplication is by Diviſion.

And therefore, according to my promiſe, I will ſhew you how you may prove Multiplication by Diviſion.

When you have ended Multiplication, and would know whether you have well done or not, ſet the Product uppermoſt, and divide it by the Multiplier: and if the Quotient be the ſame number that ſhould be multiplied, then have you well wrought, elſe not, as in that example where we multiplied 264 by 29, the Product was 7656. *Proof of Multiplication by Diviſion.*

Now if you will know whether that Multiplication be true, you muſt divide that 7656 by the Multiplier 29, and the Quotient will

will be 264, and that is a proof that you have well wrought.

Schol. By your patience I will prove that, and first set down the Product, and the Multiplier, not after the rule of Multiplication, but after the rule of Division, for now that number is become the Divisor, that was before the Multiplier; I should set them therefore thus:

	7656
	29

Then I seek how many times 2 in 7, that may be three times, and one remaineth; but then may not 9 be found so often in 16: therefore I must take a lesser Quotient, that is to say 2; then I say, twice 2 maketh 4, which I take out of 7, and there remaineth 3, then I cancel 7 and 2, and over 7 I write 3, and in the Quotient I set 2, so the figures stand thus:

3	
7656	(2
x9	
1	

Then I say further, two times 9 make 18, which I abate out of 36, and there resteth 18: then I cancel 3, and over it I set 1, and likewise I cancel 6 and 9, and over them I set 8: so that thus stand the figures.

38	
7656	(2
x9	

Then I set forward the Divisor by one place, and seek a new Quotient; that is to say, how many times 2 are in 18, which I find to be 9 times; but then I cannot find 9 so many times in 5; and therefore I take a lesser Quotient, as to say 8; but yet that is too great; for if I take 8 times 2 out of 18 there remaineth but 2, and cannot find 8 times 9 in 25; therefore yet I take a less Quotient, that is 7, which is also too great; for if I take 7 times 2 out of 18, there resteth 4, but now I cannot take 7 times 9 out of 45, therefore yet I seek a lesser Quotient, as to say 6, then I say 6 times 2 make 12, that I take out of 18 and there remaineth 6, so I cancel 18 and the 2, and write 6 over 8, thus:

x6	
38	
7656	(26
x99	
x	
1	
x6	
381	
7656	(26
x99	
x	

Then I say 6 times 9 maketh 54 that I take out of 65, and there remaineth 11, and the figures stand thus:

Then I must write down the Divisor again, and seek a new Quotient, which will be 4: for though I may find 2 in 11 five times, and 1 remain, yet I cannot find 9 so often in 6, therefore I set the figures thus:

1	
x6	
381	
7656	(264
x999	
xx	

And

And the 4 in the Quotient I multiply into the figures of the Divisor, saying, four times 2 makes 8, which I take out of 11, and there rests 3, therefore I cancel the 11, and the 2, and set 3 over the first place of 11, thus:

```
  1
 163
 381
 7656 (264
 2999
  xx
```

And then I say, 4 times 9 maketh 36, which I take from 36, and there remaineth nothing, so that the Quotient of this Division also (where 7656 is divided by 29) is 264; which sheweth, that if 264 be multiplied by 29, the product will be 7656. And thus I perceive now how both Multiplication is proved by Division, and Division by Multiplication.

Most. Now have I ended the five common kinds of Arithmetick: For as touching Mediation, Duplation, Triplation, (and such other) they are no several kinds of Arithmetick, but are contained under the other. For Mediation is contained under Division and is nothing else but dividing by 2; and so are Duplation and Triplation contained under Multiplication: for Duplation is nothing else but multiplying by 2, and Triplation is multiplying by 3, of which I will only propose an example, for the Rules you have heard already.

If you would mediate or divide into two this Summm 4531010, you must set 2 for the Divisor, and work as you learned before, as thus: *An example of Mediation.*

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4531010 (
2
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Then I find 2 in 4 two times, therefore my Quotient must be 2: so I cancel 4 and 2, and remove the Divisor forward thus, as the work requireth, and as before in Division hath been declared.

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4531010 (2265505
xxxxxxx
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Which Mediation or Division by 2 being finished, you will have for your Quotient 2265505, which is the half of 4531010, as you may try by Duplation; for double that Quotient, or multiply it by 2, and the same number will arise from the work.

Here now (according to my promise) I will teach you certain easie forms both of Multiplication, and of Division. And first of Multiplication.

If you would therefore multiply any Summ by 10, you need to do no more but add a Cypher before its first place; as for example, 36 multiplied by 10, make 360. *Easie forms of Multiplication.*

hath

Likewiſe if you would multiply any Summ by 100, put two Cyphers at the beginning. So if you would multiply any Summ by 1000, add three Cyphers to the beginning of it.

Schol. This I underſtand, and alſo the reaſon of it.

Maſt. I will omit all reaſons till our next meeting, when I ſhall tell you the reaſon of all other parts of Arithmetick alſo: and as to our matter now, look, as I have told you, that you both remember it, and alſo often practiſe it.

And now you have learned how to multiply eaſily by 10, 100, 1000: ſo in like manner may you do with any other of like nature.

But now if you will multiply by 20, 30, 40, and ſo forth, or by 200, 3000, and ſuch like, where there is one Cypher in the firſt place, or many orderly in the firſt places, you muſt take away thoſe Cyphers, and multiply the Summ only by the other figure or figures (if they be many) and then at the beginning of the Summ that is produced, you muſt ſet ſo many Cyphers as you took away.

Example of 2873, which I would multiply by 300. Firſt, I omit the 2 Cyphers from the Multiplier, and I multiply the Summ by three only that is left, and it amounteth to 8619: before which I put the two Cyphers that I before omitted or took away, and then it is 861900. And that is the true Product, when 2873 is multiplied by 300.

Schol. And if there were two or more figures beſide the Cyphers, I muſt only take away the Cyphers, and multiply by the other figures, as I learned before: As if I would multiply 93648 by 25000, I ſhould take away the three Cyphers, and multiply the ſame by 25, and then at the beginning of that Product ſhould I add the 3 Cyphers again?

Maſt. Yes, but if it chance the number that ſhould be multiplied, or both the Summs, as well the number that ſhould be multiplied, as the Multiplier, to have Cyphers in their firſt places, evermore omit the Cyphers and work by the reſt. But remember to reſtore as many Cyphers to the Product as you abated before. As in this example; 30200 ſhall be multiplied by 206, I ſhall only take away two Cyphers from the greater number, and then multiply 302 by 206, and afterward add the two Cyphers again. But if I would multiply the ſame 30200 by 2060, I ſhall not only take away the two Cyphers from the number that ſhould be multiplied, but alſo I may take away the one Cypher from the Multiplier, and then I muſt add three Cyphers to the Product: but take heed that you take away no Cypher that cometh between two ſignificant figures: as in the laſt example,

ple, you must not take away that in the fourth place of the higher number, neither that in the third place of the Multiplier: but thus you may do: if one Cypher or more come in the last of your Summs, you may multiply the other figures, and overskip them; but so as you give every figure its due place: as thus, I will multiply 3026 by 2004, therefore I set them thus:

3026
2004
12104

And thus I multiply them. First, 4 times 6 make 24, I set them under the first place, and keep the 2 still in my mind. Then I say again, 4 times 2 make 8, and the 2 that is in my mind maketh 10, I set down the Cypher 0, and keep the Article 1 in my mind. Then 4 times 0 is 0, and the 1 in my mind maketh 1, I set down the figure 1, and say again, 4 times 3 is 12, I set down 2, and keeping the one still in my mind (having no more places of the upper number to multiply it withal) I put it down next 2 in the fifth place.

But now when I come to the next place (being a Cypher 0) I let it go, because it multiplieth nothing, and likewise the second Cypher.

But then when I come to the 2, and multiply it into the 6 of the upper number, you must take heed (according as I taught you in Multiplication) that the first number be set right under the Multiplier, 2 places extraordinary toward the Left hand, as under 2 instead of under 0, and the other orderly toward the Left hand, according as you may see in this example, which being finished, with the addition thereof gathered together, will stand as in this example.

3026
2004
12104
6052
6064104

Which is indeed wrought so much the sooner and shorter by omitting the two Cyphers: which otherwise, (if the same example were wrought at length) it would have had two workings more, as by the same example here also set down doth appear.

3026
2004
12104
0000
6052
6064104

Scho. Sir I thank you, for I see great ease in this way of Multiplication: and if you can shew me such like in Division you will greatly further me.

Mast. Yes, I will teach you some easie ways in Division also, and the first this: if you would divide any Summ by 10, you shall only with your Pen make a Dash between

Easie forms of Division.

the first figure of your Summ and the second, and then you have done; for the whole number toward the left hand of the line ſtandeth for the Quotient, and the figure that is before the line, is the Remainer. As for example, 3648 divided by 10, where 364 is the Quotient, and ſheweth that ſo many times 10 are in 3648, and the 8 after the line is the Remainer, which cannot be divided into 10, but by breaking it into Fractions, wherewith I will not meddle yet.

3648
10

And ſo likewiſe if you would divide any Summ by 100, with your Pen you ſhall cut off the two firſt figures, and if you would divide by 1000, you muſt cut off the three firſt figures, and ſo of any other Diviſor, whoſe laſt figures is 1, and the other Cyphers; look how many Cyphers the Diviſor hath, and ſo many figures at the beginning you muſt cut off with the line, and they ſtand always for the Remainer, becauſe they are leſs than the Diviſor, and cannot be divided by it, and the other figures that ſtand toward the Left hand the line ſtand for the Quotient.

But now if your Diviſor have any other figure in his laſt place than 1, and in all his other places have Cyphers, look how many Cyphers they be, cut away ſo many of the firſt figures of the number that ſhould be divided, and divide the reſt that followeth the line by that figure that is in the laſt place, as if it were the whole Diviſor.

Example of 64284, which I would divide by 300, here I muſt cut away the two firſt figures, (for ſo many Cyphers my Diviſor hath) and muſt divide the reſt by 3, which is the figure in the laſt place of the Diviſor. Firſt, therefore I cut off the two firſt figures, and the Summ ſtandeth thus:

64284
300

Then I divide 642 by 3, and the Quotient is 214. And this is a ready way to turn Shillings into Pounds: for ſince 1 Pound doth contain 20 Shillings, I muſt divide the whole number of Shillings by 20. Therefore I eaſily ſee that my Diviſor hath one Cypher, and therefore I cut away one figure from the beginning of the whole Summ of Shillings, and then I mediate or divide by the other figures or ſumm that followeth.

Schol. I will put an example.

If you would divide 64287 Shillings by 20; that is to ſay, if I would turn ſo many Shillings into Pounds, I muſt cut away the firſt figure, that is 7, and divide the reſt, that is, 6428 by 2, ſo ſhall the Quotient be 3214, whereby I know that 64287 Shillings make 3214 Pounds, and 7 Shillings remaining.

Maſt.

Mast. Now prove by Multiplication whether you have well done or no.

Schol. The Quotient is 3214, which I multiply by the divisor 2, and the product is 6428.

Mast. Hereby you may perceive not only that you have well done, but also how by Division you may turn Shillings easily into Pounds: and contrariwise by Multiplication you may turn Pounds into Shillings.

But here you shall see amongst divers men divers forms of such Division: but if you observe what I have told you, you may perceive easily all the ways. For some men do not cut away so many of the first Figures of the Summ that they would divide, as there are Cyphers in the first place of the Divisor: but they set all their Cyphers orderly under the first places of the Number that they would divide: and then with the other Figure or Figures, (if there be many) they divide the rest of their Summ.

Example. If they would divide 725931 by 3400, they set their Summs thus:

```
725931
34 00
```

And then they divide orderly till they come to the Cyphers: for there they stay and end their work, as in this example.

They seek how often 3 may be found in 7, which is two times, and 1 remaining: therefore they set 2 in the Quotient, and cancel 3 and 7, and over 7 they set the 1 that remaineth thus: Then go forward, saying, two times 4 make 8, which they take out of 12, and there remaineth 4, then they remove the Divisor forward, and seek how often 3 may be found in 4, which is but once, and 1 remaineth, then set they 1 in the Quotient, and cancel 3 and 4, and over them they set that 1, then take they once 4 out of 15, and there resteth 11. Or else more easily: Take once 4 out of 5, and there resteth 1: so they cancel the 4 and 5, and set 1 over them, then set they the Divisor again, and seek how many times 3 are in 11, which they find three times, and 2 remaining: so they set 3 in the Quotient, and cancel 11 and 3, and over them set 2, then they multiply 4 by 3, which maketh 12, that they Deduct out of 29, and there resteth 17, of which the 7 must be set over the 9, and the 1 over the 2 thus:

```
  1
 12
 1417
725931(213
344400
 33
```

And now are the two Cyphers next ensuing, so that the divisor can no more be set forward, and therefore is the division ended, and the remainer is 1731. Now the Quotient which is 213, doth shew

shew that if you divide 725931 by 3400, you shall find it therein 213 times, and there remaineth 1731: so you shall find it, if you work as I taught you, by cutting away the two first Figures, because of the two Cyphers. But this must you observe (as you may perceive by this last Example) that if there be left any other Remainer in the Summ that was behind the line, that Remainer must be set to the latter end of the first Remainer, which was cut away with the line: as if you would divide 725931 by 3400, after the form that I taught you, then will your Summs appear thus.

Note.

```
      | 1
      ---
   12 |
  141 | 7
 7259 31 (213
 3444 00
  33
```

So that 17 which remaineth after the line, must be set to the 31 (that was cut away with the line) in higher places, as you see here: where that 17 with the 31, do make 1731.

Schol. Sir, is there no other form of Division in practice but this?

Mast. Yes verily, there are other forms in practice, but because I love brevity, I will declare only one, which I first learned, and is practised by that worthy Mathematician, my ancient and special loving friend Master *Henry Bridges*, and many others. Wherein not any one Figure is defaced or cancelled. As if I should divide 72 by 6, first place them thus.

```
6) 72
```

Then inquire how oft 6 may be had in 7, and seeing 6 is but once in 7, set 1 in the Quotient line, thus:

```
      (1
6) 72 (12
   6
   --
   12
   12
   --
    0
```

Then multiply the Divisor 6, by the Quotient 1, and set the Product 6 under 7, then draw a line under 6, and deduct 6 out of 7, setting the remainder 1 under 6, thus:

Then bring down the next Figure of the Dividend, and set it with the Remainer 1 under the line, which makes 12, and as before enquire how oft 6 is in 12, and finding it to be twice set 2 in the Quotient, as you see:

And multiply 6 by that new Quotient 2, setting the Product 12, under the other 12, and deducting it out of the upper number, there resteth nothing.

And since the Units of this Product do stand under the Units of the Dividend, the Division is ended, otherwise you shall proceed as before, bringing down the next Figure: dividing, multiplying, deducting, &c.

Schol. This is very easie; but if there be greater numbers propounded, is the operation the same?

Mast.

Mast. If the numbers be never so great, the work is the same without any difference, as shall appear by this Example.

Divide 7890 by 33. 33)7890(2

First set them thus: then see how oft 33 is found in 78, which is twice, and therefore set 2 in the Quotient, by which multiply the divisor 33, and set the Product 66 under 78, and Substract it out of it thus.

Then bring the next Figure 9 down, and set it with the Remainer 12, it maketh 129, and enquire how often 33 is contained in 129, and I find it but thrice, (though at the first it made a shew of more) therefore set 3 in the Quotient, and multiplying 33 by 3, set the Product under 129, deducting that product out of the number above, and proceed as before.

33)7890(239 2/3
66
129
99
300
297
3

Then shall you find the Divisor, 9 times in the Remainer 300, therefore putting 9 in the Quotient, multiply, and substract as before, and at the last you shall find only 3 remaining, which must be set above a line after the Quotient, and the Divisor under, as above appeareth.

Schol. Is there no more difficulty in the whole Rule?

Mast. Not any, although your Number be never so great.

And here I will make an end of Division, (saving that I do request you to exercise your self well herein by many Summs, till you have attained some proficiency therein.)

For the reasons and conclusions thereof are so many, and so available for all sorts of men whatsoever; that if I should speak of the infinite uses thereof, I should rather want words than matter. And therefore recommending it to your judgment hereafter, upon your further progress in the Art, I will here end this Rule, representing unto you one example, or simple question of Division and Multiplication, instead of many, which is this.

A question of shooting in Ordnance.

There are four Brass pieces: The first of them at a shot spendeth 9 pounds of Powder, the second spendeth 5 pounds, the third 4 pounds, and the fourth 2 pounds. They are all appointed against the battery of a Hold, and there is allowed by the Master Gunner, 700 pounds of Powder to be spent by these four Pieces, in this assault. The question is twofold. The first, how many shot each piece shall justly make with this 700 pounds of Powder? And lastly, how many

pounds

pounds of Powder ought justly to be allowed to each piece for his true proportion?

Schol. Why Sir, you make me smile, to hear, that these two demands may be simply resolved by Multiplication and Division.

Mast. Truly they may, and that you may by and by work your self with a little labour: First, add together their quantities of Powder, that is 9 pounds, 5 pounds, 4 pounds, and 2 pounds, all which make 20: divide the 700 pounds of Powder by that 20, and your Quotient giveth 35, as here appeareth, which sheweth most certainly that they shall make just 35 shots each.

9	
700	
200	(35
0	

Schol. Sir, all this have I done, and I see it is so, but whether it be true or not I cannot tell.

Mast. To try the truth of the same, multiply the first Piece that spends 9 pounds by 35, and you shall see his allowance, which is 315 pounds of Powder. Multiply also the second piece that spends 5 pounds by 35, and you shall find 175 pounds his allowance; then 4 by 35, and you shall find 140 pounds his allowance. Lastly, multiply 2 by 35 and you shall find 70 pounds his allowance. All which four particular Summs you shall add together by Addition, as here appeareth, and it maketh just 700 pounds, and so is the question truly resolved.

315
175
140
70
700

Schol. Truly Sir, these excellent conclusions do wonderfully more and more make me in love with the Art.

Mast. It is an Art, that the further progress you make therein, the more you thirst to go on forward. Such a Fountain, that the more you draw, the more it springs: and to speak absolutely, in a word (excepting the study of Divinity, which is the Salvation of our Souls) there is no study in the World comparable to this, for delight and profit; for the knowledge hereof is well known immediately to have flowed from the wisdom of God into the heart of man, whom he hath created the chief image and instrument of his praise and glory.

Schol. The desire of knowledge doth greatly incourage me to be studious herein: and therefore I pray you cease not to instruct me further in the use hereof.

Mast. I will. And therefore for the further use of Multiplication and Division, I will briefly shew you the Art of Reduction.

CHAP.

CHAP. VII.

Reduction of Whole Numbers.

Reduction, what it is.

REduction is the bringing all Numbers of a greater Denomination into those of a lesser Denomination. And contrariwise all Summs of a lesser Denomination may be brought into those of a greater Denomination.

Schol. What call you a great Denomination, and lesser Denomination?

Great Denomination what.

Mast. That I call a great Denomination, which doth contain under it many other lesser: as a Pound (in respect to Shillings) is a great Denomination: for it is greater than Shillings, and containeth many of them. And Shillings (in comparison to Pounds) are a lesser Denomination, because they are lesser than Pounds, and many of them are contained in one of the other: and so likewise of other things: what thing soever is compared to another, if it be greater and containeth many of them, it is a gross or great Denomination, but if it be lesser (so that many of them are in the other) then are they called the lesser Denominations: whereby you may perceive that one Denomination may be called a great Denomination, and also a lesser in divers comparisons. For Shillings compared to Pounds, are a lesser Denomination: but compared to Pence they are a greater Denomination.

Lesser Denomination.

Schol. Now I understand the name, I pray you teach me the use.

Mast. The use is easily learned, if you remember what you have been taught before. For if you will reduce any Summ of a greater Denomination into a Summ of a smaller Denomination, you must consider how many of that lesser Denomination do make one of the greater Denomination, and by that Number or Numerator you must multiply the Summ. As if you would reduce 20 Pounds into Shillings, you must consider that in a Pound are included 20 Shillings, therefore multiply the one 20 by the other 20, and there will arise 400, whereby you may know that in 20 Pounds are contained 400 Shillings. Likewise if you would reduce 30 Shillings into Pence, considering that in a Shilling are 12 Pence, you must multiply 30 by 12, and it will be 360, whereby you may find that in 30 Shillings are contained 360 Pence. And thus you may reduce any greater

To reduce great Denominations to lesser.

Denomination into a leſſer, by Multiplication, if you know how many of the leſſer do make the greater: of which I have given you a brief Table for the moſt accuſtomed kinds of Money, Weights, Meaſures, and Time, and ſuch like; whereby you may know how often each leſſer Denomination is contained in the greater, when you ſhall need it for the aforeſaid kind of Reduction. And alſo the ſame will ſerve you, if you would reduce any Summ of a leſſer Denomination, into a Summ of a greater Denomination. For in ſuch Reduction you muſt conſider (as in the other form) how many of the ſmaller do make the greater: and by that number you muſt divide the other Summ, and the Quotient will declare how many of the greater Denomination are comprehended in that leſſer Summ. As for example; If you would know how many Shillings are contained in 3240 *d.* conſider that 12 *d.* do make 1 *s.* you muſt therefore divide that 3240 by 12, and your Quotient will be 270, whereby you know that ſo many Shillings are in 3240 *d.* But if you would know further how many Pounds are in theſe 270 Shillings, ſeeing that every Pound containeth 20 Shillings: divide that 270 by 20, and it will be 13 and 10 remaining, whereby you know, that in 3240 *d.* (or 270 Shillings) are 13 *l.* 10 *s.* For the Remainer muſt always be named by the name, or Denomination of that Summ that was divided, which in this place were Shillings. And thus may you do with any other Denomination.

Leſſer Denominations reduced to greater.

Here I would expreſs the Values of ſeveral ſorts of Coins of divers Countries, but for three cauſes I omit the ſame. As firſt, Becauſe they are not currant by the ſtatutes of this Realm. The ſecond cauſe is, by reaſon they are ſo uncertain, that they are never long at one rate. And thirdly, Becauſe they are ſo different in ſo many places, that it were matter enough for a large Volume, to ſpeak ſufficiently of them all. However, becauſe you ſhall not be altogether ignorant of them, I will ſhew you the values of ſome that are moſt in uſe in the courſe of Exchange.

The chief uſes that can be made of teaching you to reduce Foreign Coin, is to enable you to know upon occaſion, how many Pounds Sterling are contained in any number of *French* Crowns, *Flemiſh* Pounds, *Spaniſh* Pieces of Eight, (by ſome called Dollars) Ducats of *Venice*, and the like. For when a Bill of Exchange is drawn on any perſon to pay any number of Crowns, Dollars, Ducats, *&c.* the value of the Piece is always expreſſed, which is ſometimes higher, ſome-

sometimes lower, as a *French* Merchant draws a Bill on a *London* Merchant to pay 1000 *French* Crowns here; he must at the same time advise what he values a *French* Crown at; for the circumstances of Trade alters the value of the Piece, as sometimes he says, pay so many Crowns at 54 *d.* sometimes 53 *d.* or the like, Sterling *per* Crown. So that what I would have you to understand, is to know how many Pounds Sterling, any number of *French* Crowns, *&c.* at a value expressed does amount to; which you will the better know how to perform, when you have learnt the Rules of Practice; and therefore I shall be more concise here: thus,

In 1000 *French* Crowns at 54 *d. per* Crown, are 225 *l.* Sterling. Which is done by multiplying 1000 by 54, and the Product is 54000 Pence, which Pence divided by 12 gives 4500 Shillings, which divided by 20 (the Shillings in a Pound) the Quotient is 225 Pounds Sterling or *English*.

Schol. Sir, I heartily thank you for this usefull Discourse, and believe I perfectly understand you, which will be evident by this Example of like nature. I would know how many Pounds Sterling are in 1500 Pieces of Fight or Dollars, supposing each Piece of Eight is valued at 51 Pence *English*.

To perform which, I multiply 1500 by 51 (according to the compendious way you taught me at the latter end of Division) and I find the Product 76500 Pence, which divided by 12, gives 6375 Shillings, which divided by 20, gives 318 Pound, and 15 Shillings remains.

```
      1500
        51
      ----
        15
        75
      -----
12 ) 76500 d.
      6375 s.
      -------
      l. 318:15
```

Mast. Very right. and I will (since you are so perfect) only propose this farther. To reduce 4738 Mill-rees of *Portugal* at 6 *s.* 8 $\frac{3}{8}$ *d. per* Mill-ree into Pounds Sterling.

Schol. According to the Rules you taught me before, I must multiply the 4738 by 80 (the Pence in 6 *s.* 8 *d*) and the Product is 379040 Pence; which I can easily reduce into Pounds, by dividing by 12, and the Quotient thence arising by 20, but then I know not what to do with the $\frac{3}{8}$.

```
  4738
    80
  ----
379040
```

Mast. The $\frac{3}{8}$ are three Eight parts of 1 penny (as you will know when we come to Fractions) so that if you multiply the 4738 by 3, and divide the Product by 8, the Quoti-

ent added to the 379040 Pence before produced, the Summ reduced into Pounds is the anſwer. See the Work.

4738 } Multiply.
3 }

8) 14214

Quotient = 1776 — 6 rem.
Add = 379040

Of which — 380816 Pence.

8 *d.* rem.
A Twelfth is = 31734 Shillings.

And a Twentieth-1586 *l.* 14 *s.*
And the Anſwer = 1586 : 14 : 8$\frac{6}{8}$

Note, that the 6 remaining in dividing by 8, is 6 eight parts of a Peny or $\frac{3}{4}$.

But the moſt brief ways of working theſe kind of Queſtions we muſt let alone till we have ſome diſcourſe of the Rules of Practice in their proper place, in the mean time I ſhall proceed to other Rules to be firſt learnt; and next of all to ſhew you how to reduce Weight and Meaſure.

Schol. That I would gladly learn Sir, if your time permit now to inſtruct me therein.

Maſt. The Tables relating thereto I gave you in Addition, which will be alſo uſefull to you here; that you may know what to divide or multiply by, in reducing things from one Denomination to another, wherein you muſt very well obſerve, the Diſcourſe at the beginning of Reduction, which we had concerning reducing things from a lower to a higher Denomination, and the contrary.

For inſtance, If you would reduce 357 ℔. 10 ℥. 13 *dw.* 21 *Grains* into Grains.

Having ſet down the Pounds, Ounces, Penny Weights, and Grains given, as you ſee, putting over them their proper Marks.

Firſt

First multiply the 357 Pounds by 12 the Ounces in a Pound, adding the 10 Ounces, and the Product is 4294 Ounces; which multiply by 20, the Penny Weights in an Ounce, adding the 13 d.w. *viz.* fay 0 times 4 is (●) by 5 of the 13 *pw.* given, is 3, put 3 under in Units place of the Product, then multiply by the 2, faying 2 times 4 is 8, and 1 (in tens place of the 13 *pw.* given) is 9, put 9 down to the Left hand the 3, *&c.* as in Multiplication, the whole Product is 85893 in 357 lb. 10 ʒ. 13 *dw.* which 85893 multiply by 24 the Grains in a Penny Weight (adding 1 in the 21 Grains given, when you multiply by 4, and 2 in the 21, when you multiply by 2) ſo the Anſwer is 2061453 Grains.

gr.	lb.	ʒ.	*dw.*
357	10	13	21
12			
714			
358			
4294 Ounces.			
20			
85893 Penny Weight.			
24			
343573			
171788			
2061453 Grains			

Schol. I preſume, Sir, the ſame method of multiplying and adding is to be obſerved in reducing Pounds, Ounces, and Drams of Averdupoize weight into Drams, as in 146 Pounds, 15 Ounces, and 13 Drams. I firſt multiply 146 by 16, adding the 15 Ounces, and the Product is 2351 Ounces; which I multiply by 16 the Drams in an Ounce, and adding the 13 Drams given, the Product is 37629 Drams for Anſwer.

lb.	ʒ.	*dr.*
146	15	13
16		
881		
147		
2351 = Ounces.		
16		
14109		
2352		
37629 = Drams.		

Maſt. Very well; and let me ſee you do this other Queſtion of Averdupoize Weight. Reduce 7 Tun, 13 Hundred, 2 Quarters, and 25 Pounds into Pounds. *Schol.*

Schol. To do that, I place the numbers thus, (putting the marks over as you taught me.) Then I multiply 7 by 20 the Hundreds in a Ton, and the Product, adding the 13 C. is 153 Hundreds, which I multiply by 4 the Quarters in a Hundred Weight; and that Product, adding the 2 Quarters, is 614 Quarters of a Hundred, which I multiply by 28, the pounds in 1 Hundred (or 112) and the Product, adding the 25 pounds, as you taught me before, is 17217 pounds the answer.

Ton.	*C.*	*Qr.*	*L.*
7	13	2	25
20			
153 Hundreds			
4			
614 Quarters.			
28			
4917			
1230			
17217 Pounds.			

Mast. You have done the last Operation very well, according to the general Rules of Reduction before given you: but I will shew you a way something shorter for reducing, as now practised among men of business, which is not only done with putting down fewer figures, but is much easier in the Operation, thus: Having reduced the Tons and 13 C. into 153 C. as before, put that down 4 times, placing them exactly in the manner they stand in the Examample above, and the half Hundred, or 56 pounds, and the 25 above being 81 pounds, place the 1 in Units, and 8 in Tens place, as you see, and the Summ is the Answer in pounds.

Ton	*C.*	*Qr.*	*L.*
7	13	2	25
20			
153 Hundreds.			
153			
1531			
1538			
17217 Answer.			

Schol. I thank you Sir, for this brief way now shew'd me, and if there is nothing more to be learnt in Reduction, I would go on to the next Rule to be learnt.

Mast. But first I will shew you how to reduce Weight, Measure, &c. from a lower Denomination or Name to a higher, and shall choose 2 or 3 of the results of the foregoing Examples, which will not only prove the truth of them, but of those we are now going upon.

As

As first, Suppose you would know how many Pounds, Ounces, Penny Weights, and Grains are in 2061453 Grains?

You must divide the Grains given by 24, the Grains in a Penny Weight, and the Quotient is 85893 Penny Weight, and 21 Grains remains.

Then dividing the 85893 p. w. by 20, the Penny Weights in an Ounce, the Quotient is 4294 Ounces, and 13 Penny Weight remains.

Lastly, Divide the 4294 Ounces by 12, the Ounces in a Pound, and the Quotient is 357 Pound, and 10 Ounces remains. So that the Answer is 357 ℔. 10 ℥. 13 *dw.* 21 *gr.* which proves the truth of the first Example of Reduction of Weight foregoing. See the whole Operation.

```
                20)      12)
24) 2061453 |(8589 3 (4294 (357 lb.
    .....   |  ...   |  ...
    192       8        36
    ---       --       --
     141      05       69
     120       4       60
     ---      --       --
      214      18       94
      192      18       84
      ---      --       --
       225      09       10 ℥. or Ounces rem.
       216       8
       ---      --
        93      13 Penny Weight remains.
        72
        --
        21 Grains remains.
```

Schol. This I understand: and to shew you that I do, I will reduce 37629 Drams into Pounds Averdupoize, which will also prove the truth of the second Question in Reduction of Weight. In order to which, I place the Numbers as follows; and having divided the Drams by 16, because 16 Drams is an Ounce, the Quotient is 2351 Ounces.

16)

16)
16) 37629 (2351 (146 = Pounds.
32 16
56 75
48 64
82 111
80 96
29 15 Ounces rem.
16

13 *Drams* remains.

And 13 Drams remains, (for what remains is always of the ſame name with the Dividend as you taught me before.)

Then I divide the 2351 Ounces by 16, the Ounces in a Pound, and the Quotient is 146 Pounds, and 15 Ounces remains. So the Anſwer is 146 *lb.* 15 *ou.* 13 *dr.* and is an Indication of the truth of my work, being the ſame, as in the ſaid ſecond Example of Reduction of Weight.

Maſt. I now ſee, that you underſtand the nature of Reduction both by Multiplication and Diviſion, but there is one other thing I will ſhew you before we leave this Rule: and that is to caſt up the value of any Quantity of Wares and Merchandize at any rate propoſed, and the Rule is this.

General Rule.

Multiply the number of Pounds, Ells, Yards, Ounces, *&c.* given, by the price of one Yard, Ell, Ounce, *&c.* and the Product is the Anſwer in the reſpective Denomination to that of the price given, or your Multiplier, which, if Farthings, Pence or Shillings, muſt be reduced into Pounds, as before taught.

Example. What coſt 747 Ells of Linnen, at 3 *s.* 9 *d. per* Ell?

Here the 3 *s.* and 9 *d.* being reduced, make 45 *d.* whereby (according to the General Rule) I multiply the 747 Ells given, and the Product is Pence, which I reduce into Pounds as before. See the Work.

3 *s.*

s.	d.
3	9
12	

45 Pence.

747 } multiply
45 }

3735
2988

33615 (280 | 1
s.
l. | s. d.
24 or 140 | 1 3

96
96

015
12

3 Pence rem.

This Example ſhall ſuffice, in regard the three firſt Examples of Reduction are much of the ſame nature, and alſo that I ſhall ſhew you variety of ſuch Queſtions in the Rules of Practice, anſwer'd after a brifer method than this.

Schol. Sir, What you have been pleaſed to impart to me, I perfectly underſtand, which makes me have ſuch a ſincere Love to, and Admiration of the Art of Arithmetick, and the Profeſſors thereof, eſpecially your ſelf, whoſe Candor and Benignity I have ſo largely experienc'd, that I cannot ſufficiently expreſs my Gratitude.

Maſt. Your being ſo ſenſible of the Excellencies of this Art, and ſo tractable and willing to learn the ſame, pleaſes me far better than the Reception of Gold would do for the inſtruction of idle, and dull-headed Learners, who ſee no pleaſure in any thing but vain Sports, which can never profit them; fooliſhly neglecting at the ſame time, thoſe commendable Studies, which would not only make them valued by others, but would be of real Worth and Advantage to themſelves; I will therefore now paſs forward with you to the next thing to be learnt in order, which is *Progreſſion*, or the relation that Numbers have one to another.

CHAP. VIII.

Of Progreſſion, or the Relation of Numbers.

What Progreſſion is.

ALthough moſt Writers have defined *Progreſſion* a compendious kind of Addition, yet truly it is not ſo: for *Progreſſion*, (as the very nature of the word doth inform any man) is a going forward and proceeding in Numbers, and that regularly and orderly, whoſe place is aptly choſen to be very near, or next after the expoſition of the four principal parts of Arithmetick: for in it are all the four former parts exerciſed and practiſed.

You ſhall alſo underſtand there are infinite kinds of Progreſſion, but for you (as yet) two are ſufficient to be exerciſed in, of which the one I call *Arithmetical*, and the other *Geometrical*.

Arithmetical Progreſſion, what.

Arithmetical Progreſſion *is an expreſſing or placing down of many numbers in ſuch order that between every two next numbers, expreſſed or placed down, the difference, or exceſs is equal and alike.*

Schol. Sir, I thank you, for that you have both opened unto me what Progreſſion is, and alſo why it is here placed.

But I pray you make plain your Definition with an Example or two.

Magiſt. You cannot want Examples, ſeeing all reaſonable Creatures naturally uſe the order of one kind of *Arithmetical Progreſſion* (which therefore is alſo named natural) whenſoever they diſtinctly count or number any multitude by ſaying, 1, 2, 3, 4, 5, 6, whereby the proceeding from number to number, and every one exceeding another that is next before by an equal difference (which here is 1) declareth the ſame to be Arithmetical Progreſſion. And for your better information, I ſet it down in this manner.

The common Exceſs. 1 1 1 1 1

The Progreſſion. 1 2 3 4 5 6

Schol.

Schol. This is moſt evident. And I think I am able to tell you now any Progreſſion Arithmetical propounded, what is that common exceſs or difference whereby it proceedeth, if this order be kept in it.

Maſt. What ſay you of 3, 6, 9, 12, 15?

Schol. They exceed each other by 3; and that I ſet down in ſuch order, as you did your Example of natural Progreſſion in this manner.

The common Exceſs. 3 3 3 3

The Progreſſion. 3 6 9 12 15

Maſt. And do you not alſo perceive, that the whole Table of Multiplication may be made by the order of Progreſſion Arithmetical; if you will begin at the firſt number of any of them on the Left hand, and ſo proceed toward the Right hand: or any of the firſt number of the upper row, and go directly downward?

Schol. I pray you let me conſider the thing a little, and I will anſwer you.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

By this trial, I perceive it very well, for the common Exceſs or Difference between any two next numbers, is continually as much as the firſt number of every row, either from the Left hand toward the Right, or from any of the uppermoſt rows downward.

To know the total Summs of an Arithmetical Progreſſion.

Maſt. Now, if of any ſuch Progreſſion, you would ſpeedily know the total Summ; firſt, tell how many numbers there are (which numbers we call Places or Terms) and if they be odd, write their Summ down by it ſelf: as in this Example, 2, 4, 6, 8, 10, 12, 14, where the numbers are 7, as you may ſee; therefore ſet down 7 in a place alone, then add together the firſt number and the laſt, as in this Example: Add 2 to 14, and that maketh 16, half of which multiplied by the number of places, gives the total of all the Numbers; as in this example 8 multiplied by 7 maketh 56, and that is the Summ of all thoſe figures.

Schol. I will work that by another Example. I would know how much this Summ is, 5, 8, 11, 14, 17, 20, 23, 26, 29. I tell the places, and there are 9; then I put the firſt number 5, and the laſt 29, together, and they make 34. I take half of that, which is 17, and multiply by 9, and it maketh 153. That you ſay is the Summ of all the numbers.

Maſt. So you will find it if you try.

Schol. How ſhall I try it?

Maſt. By your common Addition, for if you add all the Terms or Places together, you ſhall ſee the ſame Summ. And that manner of Addition proveth all kinds of Progreſſions Arithmetical.

Schol. Then I can ſumm any Progreſſion, if the numbers of the Parcels be odd. But what if they be even, as in this Example, 1, 2, 3, 4, 5, 6, 7, 8?

Maſt. When the number of the Parcels is even, then add the firſt Summ to the laſt, and by the half of the number of the Places, multiply that Summ as in our Example, the Parcels are 8, that I note, then adding the firſt Summ to the laſt, their Summ is 9, that multiply by half of the Parcels or Terms, that is, by 4, and it produceth 36, which is the Summ of the numbers given.

A general Rule.

But if you will take one rule for theſe both, do thus. Multiply half of the one by all the other, and the Product is the Anſwer. For ſometimes it chanceth that the number of the Parcels be odd, ſo their half cannot be taken: and ſometimes the Summ of the firſt number, and the laſt is an odd number, ſo that half of it cannot be taken; but they will never be both odd.

Schol. This I underſtand, if there be no more belonging to it.

Maſt. I will, before I leave this kind of Progreſſion, propoſe ſome Queſtions and ſhew you the reſolution thereof.

Schol. I think my ſelf very much obliged to you, not only for giving me general Rules, but for illuſtrating the ſame by Examples.

Maſt. The Propoſitions are theſe: 1. *To know the laſt number without proceeding by continual Addition, till you come unto it, ſo that the common Exceſs, the firſt number, and the number of the Places be known.*

2. *The firſt number of the Progreſſion, and the laſt being known, with the common Exceſs to find the number of the Places.*

3. *The Exceſs being given, and the firſt and laſt, to know the quantity of any middle number, whoſe place is given from the firſt or laſt.*

4. *The total Summ being given, and the firſt and laſt, to find out the number of the Places.*

5. *The total Summ of any Arithmetical Progreſſion being given, and the firſt and laſt, to find out the common Exceſs.*

6. *The total Summ being given, and the mutual Exceſs, with the number of the Places, to give the firſt or laſt number of the ſame Progreſſion.*

Many more conſiderations I could propound to you in theſe Arithmetical Progreſſions, but theſe are ſufficient, to give you occaſion to think, that Rules of Knowledge and Arts are infinitely capable of Improvement.

Schol. I ſhould think it a happineſs, did I but well underſtand that which is already invented and publiſhed; therefore I pray you to proceed to the Rules anſwering to theſe Propoſitions.

Maſt. I will for every of theſe 6 Propoſitions give you a Rule and an Example, unleſs the plainneſs and eaſineſs need no further exemplifying.

For the Solution of the firſt, multiply the Exceſs by a number leſs by 1 *than the number of the Places, and to the Product add the firſt number, ſo you ſhall have the laſt number, which is ſought for.* 1 Propoſit.

As for Example; A Merchant buyeth 50 pounds of Spices, and agreeth to pay for the firſt pound 4 pence, for the ſecond 7 pence, for the third 10 pence, for the fourth 13 pence, *&c.*

The queſtion is, How much he muſt pay for the laſt pound, and then how much the 50 pound cometh to?

Schol.

Schol. According to the Proposition, I multiply 49 (which is less by 1 than the number of the Places) by the Excess, which is 3: to the Product 147, I add the first number which is 4, it maketh 151 pence, the price of the last pound. Now I add 4, the price of the first pound, to 151 the price of the last pound, it maketh 155, which I multiply by half the number of places, which is 25, and the Product is 3875 pence — the total Summ, or Price of the 50 pounds of Spices, as appeareth.

49 - Places less 1
3 Excess.

Add { 147
4 — first Numb.

151 the last.

151 last } Add
4 first }

155 — Summ.
25 half the numb. of Places.

775
310

d. 3875 — Total Summ:
which amounteth to

l.	*s.*	*d.*
16	2	11

Mast. It is truly wrought.

Schol. Then I entreat you to proceed to your second Proposition.

2 Propositio. Mast. *The second Rule is this. From the last substract the first, the Remainder divide by the common Excess, to the Quotient add 1, and you have the number of the places, which you would know, as in this Progression.*

6, 11, 16, 21, 26, 31.

If I know only 6 and 31, and that they encrease by 5, then according to the Rule, from 31 I substract 6, there remaineth 25, which 25 I divide by 5 (the common Excess) the Quotient is 5, which added to 1 maketh 6: and so many are the Places as you see.

Schol. This Rule is so easie, that I were much to blame, if I could not remember it.

3 Propositio. Mast. *The third Proposition may always thus be solved. Multiply the Excess by a number less by one, than the distance of the Place from the*

the first, or the last number given: the Product add to the first, if the distance be reckon'd from the first, and the first also known; or subtract from the last, if the distance be counted from the last, and that which cometh forth, either in that Addition to the first or Subtraction from the last, is the number sought. As for Example;

8 15 22 29 36 43 50 57

And for the better considering the manner of this question, I will note over every place its distance from the first, and under every place his distance inclusively from the last, thus.

1 2 3 4 5 6 7 8
8 15 22 29 36 43 50 57
8 7 6 5 4 3 2 1

Now if the Excess, whereby this Progression standeth, be known to be 7, and the first number given, being 8, I would know what number standeth in the fourth place. I multiply 7 by 3 (which is less by 1 than the number of the place proposed) that produceth 21, to which I add 8, the first number, so cometh 29: which is the number in the fourth place, as you see in the example: or if in the third place from the last you would know what number in this example should stand, the last number being known to be 57, and the common Excess 7, then by 2, which is less by 1 than the place propounded, I multiply 7, that giveth 14, which subtract from 57 the last number, and the remainer is 43, which belongs to the third place, reckon'd from the last.

The Rule for the fourth is thus. Add the first and the last together, and by the Summ divide the total Summ of the Progression, then double the Quotient, and that will be the number of the Places. 4 Propositi-on.

Schol. As if I had received 85 pounds of certain men, but of how many I have forgotten, yet I remember, that the first gave me 7 pound, and the last 27 pound, and every payment after other, did rise by a like Summ. And the man for whom I received this Money, conditioned with me, that of every Payment I should have Twelve pence for my labour: now unless I can by Art find the truth of this case, I am like to lose the most part of my Reward. *A question of Money.*

Mast. I perceive you can handsomely frame an Example, which should concern your own gain: I pray you let me see how you would do justice in this point.

Schol. I add the first and the last together, that maketh 34; by which I divide 85, thus:

1 27 17
85 (2 —
34

Why

Why how now? Sir, here is a remainer of 17, in which 34 cannot be had: ſo that now I am at a loſs, for doubling of my Quotient, and farewell then both my Juſtice, and a good lump of my Gains.

Maſt. The 17 remaining is half of 34 the Diviſor, ſo the Quotient is 2 and a half. And that you may try, if you double that which ſo remaineth, for then it will be equal to your Diviſor, as if you double 17, it maketh 34, and your Diviſor alſo was 34, this noteth the remainer to be half of one.

Schol. Now I am glad of this hard Example: For with it I have a general rule for the Fraction that may happen in this work. So that the Quotient being 2 and a half, I double that, it maketh 5, therefore my gain is 5 Shillings. And to be ſure (by your leave) I will try it, for I will multiply half of 34 (which is the firſt and laſt number) by 5, thus:

17	
5	
85	

5 Propoſition. Maſt. *The fifth Propoſition hath this Rule to ſolve it: By the fourth Rule find the number of the Places, that being done, from the laſt ſubſtract the firſt, and the reſidue divide by a number leſs by 1 than the number of the Places, and the Quotient will ſhew the Exceſs, which is ſought for.*

Example. An Example hereof ſhall be this. If you had diſburſed 685 pounds to a certain number of men, you neither can tell how many they were, or how much the ones Money exceeded his next before, but you are ſure that the Exceſs was equal between every two next: and alſo you remember that the firſt had 19, and the laſt 118 pounds, how would you find the number of the Summs paid, and the Exceſs continually obſerved in the Succeſſion of their Payments?

Schol. Your Rule doth plainly bid, firſt to find the number of the Places, which I will do according to the fourth Rule; I add 19 and 118 together, thus:

118	
19	
137	

By this 137, divide 685.

13	
685	(5
537	2
	10

Seeing there is no Fraction but a whole number, being 5, I double that, and then muſt the number of the Places be 10. Now from the laſt I ſubſtract the firſt, as 19 from 118, thus: And there remaineth 99.

This

This 99 I divide by a number leſs by one than the number of the Places, and ſeeing the Places were 10, I divide 99 by 9, thus:

And the Quotient is 11, and ſo was the Exceſs if I have followed your Rule right.

Maſt. You have wrought every part of this Queſtion both well in order, and truely in the practice of your Rules.

118
19
99
99
0
99 (11
99

Your ſixth Rule is this. By the number of the Places divide the total Summ, double the Quotient, and that will be the firſt and laſt joyned in one Summ. Then by a number leſs by one than the number of the Places, multiply the Exceſs, that Product ſubſtract from the firſt doubled Quotient, and half of the remainer is the firſt number. The laſt number you may diverſly find out, as by the firſt of our 6 Rules, or by ſubſtracting this firſt number from the Summ, which here contained both the firſt and laſt joyntly, or (thirdly) by continual adding the Exceſs. 6 Propoſitions.

Schol. I pray you make this ſomewhat more plain with an example.

Maſt. If every Month in the year (counting them now as 13) you gained clearly 40 Shillings more than you did the Month next going before, and at the years end you find the whole gain 5720 Shillings, but you remember not how much either the gain of the firſt Month or the laſt was, by this Rule it may be found out. Example of Gain.

Schol. So that here you ſeem to apply the 13 Months to 13 Places, the 40 Shillings every one more than the other next before it to be the common Exceſs, and 5720 Shillings to be the total Summ.

Maſt. It is true: by 13 then I divide 5720 in this manner.

I double the Quotient, ſo I have 880 for the firſt and the laſt Summ joyned together, by 12, which is leſs by one than the number of the Places, I multiply 40 (the common Exceſs) ſo cometh 480.

This 480 I ſubſtract from 880, and there remaineth 400: half whereof is the firſt number which we deſired to know: that is 200.

This 200 deducted from 880 leaves 680 the laſt Number.

5
5
5720 (440
5333
55
40
12
480
880
480
400

Schol. I thank you moſt heartily for theſe ſix Rules. Now if it be your pleaſure I would hear and learn ſomewhat of Progreſſion Geometrical.

§. 2. *Progreſſion Geometrical.*

Progreſſion Geometrical.

PRogreſſion Geometrical is when the numbers encreaſe by a like proportion, that is, if the ſecond number contains the firſt ſo often as the third containeth the ſecond; and ſo the fourth the third, and the fifth the fourth, *&c.* as in theſe Examples.

2	6	12	24	48
1	3	9	27	81
2	10	50	250	1250

Here in the firſt Example you ſee, that every number containeth the other, that goeth next before it, two times; and in the ſecond Example three times, and in the third Example five times: for as Numbers in Arithmetical Proportion encreaſe by the Addition of ſome number to the firſt, ſecond, *&c.* So thoſe in a Geometrical Proportion are, when the firſt, ſecond, *&c.* numbers are multiplied by ſome common Multiplier. Now if you will know how to find eaſily the Summ of any ſuch numbers, do thus: conſider by what number they be multiplied, whether by 2, 3, 4, 5, or any other, and by the ſame number multiply the Summ in the laſt Progreſſion.

Schol. I pray you work it by this example, 1, 8, 32, 128, 512, 2048; which I have framed by proceeding from 2, and continually multiply by 4.

To find the total Summ in any Geometrical Progreſſion.

Maſt. Then I muſt multiply the laſt Summ (which is 2048) by the common Multiplier 4, as here, and it will be 8192. Now I abate from this Summ the firſt number of the Progreſſion, which here is 2, then reſteth 8190, which Summ I muſt divide by 1 leſs than the number was that I multiplied by. Seeing then I multiply by 4, I muſt divide by 3, ſo dividing 8190 by 3, the Quotient will be 2730, which is the Summ of all the Progreſſion. And now to prove whether you can do the ſame, I give you theſe numbers to add by this Rule, 3, 15, 75, 375, 1875, 9375, 46875.

Schol. I cannot well tell by what number this Progreſſion doth increaſe.

Rule.

Maſt. In any ſuch doubt do thus: Divide the ſecond number by the firſt, and the Quotient will ſhew you the number that encreaſeth the Progreſſion.

Schol.

Schol. Then that number in this Example is 5, for so many times is 3 in 15.

Mast. So it is. Now work as I taught.

Schol. The last number is 46875, which I multiply by 5, and it produceth 234375, from which I abate the first number of the Progression, that is 3, and there resteth 234372, which I divide by 4, for that is one less than 5, and the Quotient is 58593, which is the whole Summ of the Progression.

Mast. If you remember this well, you have learned the Art of Progression both Arithmetical, and also Geometrical, which you may prove either by substracting of each number alone from the Summ, and so will there nothing remain: or else by adding together of all the Terms, for the total of the Progression.

A Mercer hath 12 yards of Sattin, which he valueth at 16 Shillings the yard, and selleth the same 12 yards to another man to be paid as followeth: to wit, for the first yard to have one Shilling, for the second yard two Shillings, for the third yard four Shillings, for the fourth yard eight Shillings, *&c.* Doubling each number, till the twelfth and last yard. The Question is, who hath made the better Bargain, the Buyer or Seller. *A Question of Sattin.*

First you may set down the number of the yards, as you see here in this Example, from 1 to 12. And against each number, the number, of Shillings due to be paid, as the order of Multiplication by 2, teacheth.

Then for the summing of this Progression, I consider that the increase proceeded by the Multiplication of 2, and therefore after I have drawn a line under the 12, I multiply the last Summ by 2 also, and it produceth 4096: from whence I abate the first number of the Progression, which is 1, and then resteth 4095: which I should divide by one less than I did multiply by, but seeing it is 1, I need not to divide it: for 1, (as I have said before) doth neither multiply nor divide: therefore I take that Summ 4095 for the whole Summ of the Shillings, which by Reduction amounteth to 204 Pounds, 15 Shillings, and so much hath the Mercer for his 12 yards of Sattin: which is 17*l.* 1*s.* 3*d.* per yard. But I think you will buy none so dear.

1	
1	1
2	2
4	3
8	4
16	5
32	6
64	7
128	8
256	9
512	10
1024	11
2048	12
4096	

Schol. No Sir, I do not intend it at present.

A Question of an Horse.

Mast. Then what say you to this Question; If I sold unto you an Horse having 4 Shooes, and in every Shooe 6 Nails, on this condition, that you pay for the first Nail one half penny, for the second Nail two, for the third Nail four, and so forth, doubling unto the last Nail. Now I ask you how much would the price of the Horse come to?

Schol. First, to know the number of the Nails, I must multiply 6 by 4, and it maketh 24. Then I will write the number of the Nails every one in order from 1 to 24, and against each number of the Nails the Summ of half pence duly, as the order of Multiplication by 2 teacheth.

Then I do (according to the Rule of summing up the Progression) consider that the encrease of this Summ proceedeth by the Multiplication of 2, as the last Example did. And therefore multiplying the last Summ by 2, it produceth 16777216, from which I abate the first number which is 1, and then resteth 16777215, which I should divide by one less than I did multiply: but seeing that it is 1, I need not to divide it, therefore I take the number 16777215 for the whole Summ of the half pence, which by Reduction I find to be 699050 shillings, and 7 pence half penny, that is 34952 pound, 10 shillings, 7 pence half penny.

pence	
1	1
2	2
4	3
8	4
16	5
32	6
64	7
128	8
256	9
512	10
1024	11
2048	12
4096	13
8192	14
16384	15
32768	16
65536	17
131072	18
262144	19
524288	20
1048576	21
2097152	22
4194304	23
8388608	24
16777216	

Mast. That is well done, but I think you will buy no Horse of the price.

Schol. No Sir, unless his qualities were extraordinary.

Mast. Well then, answer me to this Question.

A Lord delivered to a Bricklayer a certain number of loads of Brick, whereof he required him to make twelve Walls of such sort, that the first Wall should contain two thirds of the whole number, and the second two thirds of that which was left; and so every other two thirds of what remained: and so did the Bricklayer: and when

when the twelve Walls were made, there remained one load of Brick.

Now I ask you how many Load went to each Wall, and how many Load was in the whole?

Scho. Why Sir, it is impoſſible for me to tell.

Maſt. Nay, it is very eaſie, if you obſerve that every Wall ſhould receive two thirds of the Summ that was left. Now take away two thirds from any Summ, and you muſt needs grant, that that which remaineth, is one third, as of 9, from which if you take two thirds, there will remain 3, which is one third of 9. Likewiſe from 3 baſe two thirds, and there remains 1.

Scho. This is true, and now I perceive the leaſt Wall had but two load of Brick.

Maſt. And by the ſame reaſon may you know how many load every Wall had, according to the work following, and likewiſe what the whole Summ of Bricks was, for if you make 12 Summs, multiplying by 3, ſtill from the laſt remainer, as you may ſee here on the left ſide of the Table, there will appear all the remainers of the whole Wall: and if you multiply the laſt of thoſe 12 Summs by 2 alſo, then will that be the Summ of the loads which was delivered to the Bricklayer.

The Remainder of every Wall.		*Loads due to each Wall.*
1	12	2
3	11	6
9	10	18
27	9	54
81	8	162
243	7	486
729	6	1458
2187	5	4374
6561	4	13122
19683	3	39366
59049	2	118098
177147	1	354294
Loads delivered in all—		531440

Again,

Again, if you double every Remainder, as you see is done at the right side of this Table, those numbers will shew the Summ of loads that went to each Wall, whereby you may perceive that each Wall was three times so great as the next less.

Schol. Now it appeareth cleer enough, and I see that Arithmetick is a very excellent Art.

Mast. You will say so, when you know more of the use of it: for this is nothing in comparison to other things that may be performed by it.

Schol. Then I pray you cease not to instruct me further in this admirable Science.

CHAP. IX.

The Golden Rule, or Rule of Proportion Direct, called, The Rule of Three.

The Rule of Proportion.

Mast. BY order of the Science (as some have taught it) there should follow next the Extraction of Roots of Number, which because it is somewhat hard, and chiefly usefull in Geometry, I will let it pass for a while, and will teach you the Rule of Proportion, which for its excellency is called the Golden Rule, and is by three numbers known to find a fourth, which shall bear such proportion to the third, ar the second number does to the first, when rightly stated: as thus;

The Golden Rule.

If you pay for your board for three Months, sixteen Shillings, how much shall you pay for eight Months?

The Rule for placing the three Numbers given.

Rule. First, Put down that number that has no other of the same Denomination with it, as in this question, 16 *s.*.

Second, Place the other two, that are always of one Denomination, on each side that, in such order, that the number on which the middle number depends, stand toward the left hand.

So

So the three numbers in the question will stand thus:

mo. *s.* *mo.*
3 . 16 : : 8

Here according to the Rule, 16 *s.* must ſtand in the middle, having no other of the ſame Denomination with it. Then 3 muſt ſtand toward the left hand that, becauſe it depends on the 16 *s.* which is the Summ paid for 3 Months Board; and conſequently 8 muſt ſtand toward the right hand the 16.

The Rule for finding the Anſwer is,

Multiply the two numbers toward the right hand together, and divide the Product by the number next the left hand, and the Quotient is the anſwer. As in the queſtion above. *Rule.*

mo. *s.* *mo.*
3 . 16 : : 8.
8
s.
3) 128 (42⅔
12
08
6
2 rem.

Hereby I know, that if three Months boarding comes to 16 Shillings, 8 Months boarding will come to 42 Shillings, and two thirds, or 8 pence, and ſo of any other like queſtion.

Here note, that the firſt number and the third be of one Denomination, and alſo the ſecond and fourth for which you ſeek: or elſe be of ſuch Denominations, that you in working may bring them into one; as if a man ſhould ask me this queſtion.

If twelve weeks expence be 14 *French* Crowns at 6 Shillings the piece, how many pounds is that in one year? Here you ſee no two numbers of one Denomination, but yet in working you may turn them into like Denomination: as thus; turn the one year into 52 Weeks, and the fourth Summ will be *French* Crowns, by the order of the working. *A queſtion of Expence.*

Then to know this queſtion, multiply the third Summ 52, by the ſecond 14, and the Product will be 728: that divide by your firſt number

12,

12, and the Quotient will be 60 Crowns, and 8 Crowns remaining: which if you turn into Shillings, they will be 48 Shillings, which if you divide by your firſt number 12, the Quotient will be 4, or 4 Shillings; put thoſe 60 *French* Crowns, which make 18 Pounds with the 4 Shillings for the Summ that anſwereth to the queſtion, and it is the juſt Expences of a year: and the work will ſtand thus:

Weeks.	*Cr.*	*Weeks.*
12 .	14 ::	52 .
	14	
	208	
	52	
12)	728 (60 *Cr.*	
	72	
	08 remains.	

Multiply { 6 *s. per* Crown.

12) 48 *s.* (4 *s.*
48

0 remains.

Schol. This I believe I can do

Moſt. But if the queſtion be asked thus: If in 8 weeks I ſpend 40 Shillings, how long will 105 Shillings ſerve me? Here you ſee, that 8 Weeks anſwers to 40 ſhillings. Therefore you muſt put 8 Weeks in the middle, and 40 ſhillings, join'd thereto in the queſtion, muſt ſtand to the left hand, and 105, on which the anſwer depends, muſt ſtand to the right hand. Now multiply 105 by 8, and it will be 840, which if you divide by 40, will yield 21, which is the fourth number, and ſheweth how many Weeks 105 ſhillings will ſerve, if you ſpend 40 ſhillings in 8 Weeks. So that if 40 ſhillings ſerve for 8 Weeks, 105 will ſerve for 21 Weeks.

s.	*w.*	*s.*
40 .	8 ::	105
		8
40)	840 (21	
	80	
	40	
	0 rem.	

Note, That if any of the 3 numbers given be of more Denominations than one, they muſt be reduced to the loweſt that is named in the Queſtion.

And the firſt and third muſt always be reduced to one Denomination, if they are not ſo before you begin to multiply and divide.

As if 1 *C.* 2 *Qr.* of Sugar coſt 4 *l.* 4 *s.* what will 16 Hundred coſt at that rate? See the Work according to the Rule.

1 C.

C.	Q.	L.	S.	C.
1 :	2.	4:	4::	16
4		20		4
Quarters=6.		84	: :	64 Quarters.
		64		
		336		
		504		
6)		5376		(896 Answer.
6		48		or l. 44: 16: 00
		57		
		54		
		36		
		36		

And for your further aid and instruction, to make you better acquainted with this Golden Rule, I have here proposed six Questions, and their Answers, which I think most convenient to forward your understanding herein. The first four are all branches of one Question, sprung out of the best tree (for a young learner to taste of) that groweth in this *Ground* of *Arts*: for that few Questions in the Rule of three, can be proposed, but it must be comprehended under the reason of one of these four. *Note.*

The Questions.

If 15 *Ells of Cloath cost* 7 Pound 10 Shillings, *what comes* 27 *Ells to at that rate?* Answer, 13 Pounds 10 Shillings.

If 27 *Ells cost* 13 Pound 10 Shillings, *what are* 15 *Ells worth?* Answer, 7 Pound 10 Shillings.

If 27 *Ells cost* 13 Pound 10 Shillings, *how many Ells shall I have for* 7 Pound 10 Shillings? Answer, 15 *Ells.*

If I sell 15 *Ells for* 7 Pound 10 Shillings, *how many Ells are to be delivered for* 13 Pound 10 Shillings? Answer 27 *Ells.*

If 8 *Pound of any thing cost* 16 Shillings 6 Pence; *what money is to be received for* 49 Pound? Answer, 5 Pound, 1 Shilling 0¾. pence fere.

If 4 *Pound of any thing cost* 7 Pence, *what money will* 8765 *pound of that commodity cost?* Answer, 63 pound, 18 shillings, 2 0¾.

Of all which Questions, I omit the work on purpose, that you may whet your wit thereby at convenient leisure, to climb each branch, and gather the fruit of them, before I proceed to give you instructions in the backer Rule of three, whose order is quite contrary to this that you have learned.

Schol. I thank you heartily for the six Questions, which I will (God willing) practise at convenient times; I pray you proceed therefore to the Backer or Reverse Rule immediately,

CHAP. X.

The Golden Rule, or Rule of Proportion Backward, or Reverse.

IN the last kind of Proportion, the fourth Number is always so much greater or less than the third; as the second is greater or less than the first in the order of writing them down.

But in this Reverse Rule of Proportion the greater the third Number is, the less is the fourth.

The Rule for stating the Question, is

The same with that given to the Direct Rule foregoing.

The Rule for performing the Operation.

Multiply the middle Number and that toward the Left hand together; and divide the Product by that next the Right hand, and the Quotient is the Answer. For instance,

How many Yards of Anterine of ¾ quarters of a yard broad will Line a Piece of Broad Cloath, that is 18 yards long, and 1 yard and 3 quarters, or 7 quarters broad?

Lbr.

Q. br.	Yds. lon.	Q. broad.
7 .	18 .	3
	7	
3)	126 (42	Yards long, for Anſwer.
	12	
	06	
	6	
	0 Remains.	

A General Rule.

The Quotients and Remainers in all Queſtions, both in the Rules of Proportion, Direct and Indirect or Reverſe; are of the ſame Denomination or Name with the ſecond Number, after the 3 are rightly ſtated according to the Rule preceding.

Schol. Sir, methinks this Rule is not only uſeful for Taylors, but alſo for their Cuſtomers, to prevent being Cheated.

Maſt. It is alſo uſeful for moſt other Tradeſmen. As for Example,

How many Yards of Matting, of $\frac{3}{4}$ Yards Broad will Cover a Gallary of 4 Yards and a Quarter broad, and Twenty Yards long? See the Work above.

Q. br.	Long.	Q. broad.
17 .	20 .	14
	20	
14)	340 (24	$\frac{4}{14}$ or $\frac{2}{7}$
	28	
	60	
	56	
	4 Rem.	

Or thus: How many Inches in length of a Board that is 5 Inches broad will make a Square Foot (which is 12 Inches long and 12 broad, or 144 Inches?) See the Work.

I. br.	I. lon.	I. broad.
12.	12.	5
	12	
5)	144	(28 $\frac{4}{5}$ Ingh. for Anſw.
	10	
	44	
	40	
	4	

Maſt. Now what ſay you alſo to this Queſtion? I lent my Friend 400 Pound for 7 months, how much Money ought he to lend me again for 12 months to recompence my courteſie ſhewed him? can you anſwer to this?

Schol. Yes Sir, I ſuppoſe I can, for I will ſet down my Numbers thus: where I multiply 7 into 400, and it maketh 2800, which I divide by 12, and the Quotent is 233 Pound, and there is 4 Pound remaining of my Diviſion, what ſhall I do therewith?

mo.	*l.*	*mo.*
7.	400.	12

Maſt. Turn the 4 Pound into Shillings, and then divide it by 12 as you did before.

Schol. Well Sir, it is done: ſo have I 6 Shillings for my Quotient, and yet remaineth 8 Shillings upon my Diviſion.

Maſt. You muſt alſo reduce that 8 Shillings into Pence, which maketh 96, and divide that alſo by your Diviſor.

Schol. So have I done, and I find 8 Pence for my Quotient, and nothing is left.

Maſt. This you muſt always do when any thing remaineth upon your Diviſion, whether it be Money, Weight, Meaſure, or any kind of thing whatſoever. This Rule is ſo profitable for all ſorts of men, that for this Rule only (if there were no more but it) all men were bound highly to eſteem Arithmetick.

By this rule may a Captain in War, work many things, as Mr. *Digges* in his *Stratiocos* doth declare. And I wiſh you and every of my Countrymen whatſoever, that are inclin'd to Military Affairs, to be familiarly acquainted with this Excellent Art, which he ſhall find not only at the Sea, but alſo in the Camp and Field-ſervice, abundantly uſeful, either in Fortification, paying of Souldiers wages, charges of Ordnance, Powder, Shot, Munitions, and Inſtruments whatſoever, as for example.

Queſtion of an Army. If it ſhould chance a Captain which hath 40000 Souldiers were incloſed with his Enemy, that he could have no freſh Purveyance of Victuals, and that the Victuals he had would ſerve that Army but only three Months, how many men ſhould he diſmiſs to make the Victuals to ſuffice the reſidue 8 Months?

Schol. As you taught me, I ſet the Numbers thus: ſaying, if three Months require 40000, how many will 8 require?

Mon.	*Men.*	*Mon.*
3.	40000.	8.

To know this; I multiply the firſt number 3 into the ſecond 40000, and it produceth 120000, which Summ I divide by 8, and there will be

be in the Quotient 15000, which if I Subſtract from 40000, the remainder will declare that he muſt diſmiſs 25000.

Maſt. Now anſwer me to this Queſtion: If 136 Maſons in a Month be able to build a Fort to preſerve the Souldiers from the Enemy, and ſuch Expedition is required that I would have the ſame finiſhed in eight days: how many workmen ſay you is there to be appointed? *A Queſtion of a Fort.*

Schol. As you taught me, I ſet the Nummbers thus: ſaying, If 28 Days require 136 Maſons, what number by the like proportion will 8 Days require?

Da.	*Maſ.*	*Da.*
28.	136.	8.

To know this I multiply the firſt Number 28 into 136, and it produceth 3808: which I divide by 8, and my Quotient is 476: which is the juſt Number of Maſons that ſhall perform this work. And now methinks theſe Queſtions are very eaſie.

Maſt. Truly if you take pleaſure herein, you ſhall find this Art not only eaſie, but wonderful delightful and profitable. Now therefore one Queſtion more I will propoſe, and ſo leave off this Rule in whole Numbers until we come to the uſe of it in broken Numbers: for had you the underſtanding of broken numbers perfectly, not only in this Rule, but in all others the queſtion that in appearance ſeemeth to be harder to reſolve, may thereby be wrought as ſoon or ſooner than this.

Schol. Your words do greatly incourage me to be ſtudious to attain whole Numbers: and might I once attain to be a Practitioner in broken Numbers, I ſhould think my ſelf very happy.

Maſt. What ſay you then to this Queſtion? If 48 Joyners in two days make 200 Light Horſemens ſtaves (ſuppoſing they work but 12 Hours in a Day) and ſuch need requireth that 384 Joyners are ſet to the finiſhing of thoſe 200 ſtaves; in what time, ſay you, will they make them up?

Schol. I ſee here that I muſt turn my 2 Days into hours. And ſo doing, I ſet my Numbers thus:

Joyners.	*Hours.*	*Joyners.*
48.	24.	384.
	48	

Saying, if 48 men require 24 Hours, 384 men will make an end ſooner. For it is grounded upon an old Proverb, *many hands make light work.*

	192	
	96	
384)	1152	(3 hou. Anſw.

I multi-

I multiply 48 into 24, and it produces 1152, which I divide by 384, and my Quotient is 3 hours, which is my desire.

CHAP. XI.

The double Rule of Proportion direct.

The double Rule.

Mast. WEll, since you understand the last Rule, I will shew you another, which is so called, because there is in it double working, in which thing only it differeth from the single Rule of Proportion direct.

Schol. Then by an example I shall understand it well enough, I doubt not.

Question of carriage.

Mast. So you will, and let this be the example: If the carriage of one hundred weight (that is 112 pound) 30 Miles do cost 12 Pence, how much will the carriage of 5 C. weight cost being carried 100 Miles?

Schol. I pray you shew me the working of it.

Mast. You must make two workings of it: the first thus: If 1 C. weight cost 12 Pence, how much will five hundred weight cost? Set your Figures thus:

C.	d.	C.
1.	12 ::	5

And multiply 5 by 12, and the Product is 60, which if you divide by one, the Quotient will be still 60, that is the price of 5 C. weight for 30 Miles.

Then begin the second work, saying: If 30 Miles require 60 Pence, how much will 100 Miles require? Set your Figures thus:

Miles.	*Pence.*	*Mil.*
30.	60 ::	100

Then multiply 100 by 60, which produces 6000, which being divided by 30, will yield 200 pence. Then you must say, that so many pence the carriage of 500 pound weight 100 Miles, will cost after the rate of 12 pence for the 100 carried 30 Miles.

Schol. Now I believe I apprehend it.

Mast. These and such other like Questions of the double Rule of three, are to be answered much sooner at one working by the Rule of Proportion composed of five Numbers, which I will shew you by and by, and then you may use it which way you think good.

Schol.

Schol. Sir, I thank you much for your courteſie. And I long till this Rule be ended, that I may ſee how I ſhall behave my ſelf with that new Rule of five numbers: for that I have ever ſince you taught me hitherto in the Golden Rule, both forward and backward, wrought but with three numbers only.

Maſt. But we will awhile go on with this Rule of Three, therefore anſwer to this Queſtion.

If Thirty Buſhels of Wheat ſowed, yielded in one Year 360, how many will 80 Buſhels yield in 7 Years: ſowing every Year of thoſe ſeven, ſtill fourſcore Buſhels? *Queſtion of ſowing.*

Schol. Firſt I ſay, that if 30 Buſhels will yield 360 in one Year, then 80 Buſhels will yield 960 in one Year. Then for the ſecond work, I ſay, If one Year yield 960, then 7 Years will yield 6720; as the following work doth ſhew.

B. Seed.	*Encreaſe.*	*B. Seed.*	*Enc.*	*Year.*	*Buſh.*	*Year.*	*Buſhel.*
30.	360 ::	80.	960.	and as 1.	960 ::	7.	6720

But now Sir if I deliver 30 Buſhels of Corn to another Man for 7 Years, agreeing ſo that he ſhall ſow every Year the whole increaſe of the Corn, and I at the end of theſe 7 Years to have the half of the whole increaſe: I would know how many Buſhels will amount to my part, ſuppoſing the increaſe to be after the rate of the laſt Queſtion, for 30 Buſhels in one Year to yield 360? *A queſtion of Corn.*

Maſt. In ſuch a Queſtion you muſt have ſo many ſeveral workings as there be Years: As for Example, in the firſt Year Thirty Buſhels yield 360, then to know the yielding of the ſecond Year, I muſt ſay, If thirty yield 360, how many yieldeth 360? Work by your Rule, and you ſhall find 4320 Then ſay for the third year, If thirty yield 360, How many will 4320 yield? you ſhall have 51840, and ſo every Year multiplying the whole increaſe by 360, and dividing it by 30, the increaſe of the next Year will appear, as the following work doth ſhew: where I have ſet 7 Letters for the 7 Years, ſo that the increaſe of the ſeventh Year is 10749 4240 Buſhels: how many Quarters that is, and alſo how many Weyes, you may by Reduction ſoon find.

a=1ſt Year
360 Buſhells

b=2d Year
30 : 360 : : 360. 4320

c=3d Year
30. 360 : : 4320. 51840

d=4th Year
30. 360 : : 51840. 622080

e=5th Year
30. 360 : : 622080. 7464960

f=6th Year
30. 360 : : 7464960. 89579520

g=7th Year's Encreaſe.
30. 360 : : 89579520. 1074954240

Queſtion of Mowing. Now with one Queſtion more I will prove you. If ſix Mowers do mow 45 Acres in 5 days, how many Mowers will mow 300 Acres in 6 days?

Schol. If 45 Acres require 6 Mowers, then 300 Arcres require 40. Now again, if 5 days require 40 Mowers, then 6 days need but 33 Mowers.

Maſt. Why do you not make mention of the 2 that remaineth in the laſt Diviſion? for the laſt part of the Queſtion is wrought by the Backer Rule, where the firſt Number 5 is multiply'd in the Second, that is 40, whereof amounteth 200, which if you divide by the third Number 6, the Quotient will be 33, as you ſaid: but then there will remain 2, which cannot well be divided into 6 parts: however you may underſtand by the 2 ſixth parts, the third part of one man's work, which you muſt put to the 33; or elſe you muſt ſay that 33 Workmen will end all the 300 Acres in 6 days ſave 2 mens work for one day, or two days work for one man. But in ſuch broken Numbers call'd Fractions you ſhall hereafter be inſtructed.

CHAP.

CHAP. XII.

The Rule of Proportion compoſed of five Numbers.

THe Rule of Proportion compoſed, is the ſame with the Double Rule of Three foregoing, only the Numbers given are wrought with one ſtating by this Rule.

The first part of the Rule of Proportion compound direct.

The Rule for ſtating your Queſtion.

The 3d Number from the Left-hand muſt always be of the ſame Denomination with that which is ſought for, thoſe two Numbers on which the 3d depends muſt be the firſt and ſecond Numbers, then place the other 2 Numbers in ſuch Order, that the 4th may be of the ſame Denomination with the firſt, and the 5th of the ſame with the ſecond; which done, for working the Queſtion this is

The Rule. Multiply the firſt Number by the ſecond, and that ſhall be your Diviſor: Then again, multiply the other 3 Numbers, the one by the other, and their Product ſhall be your Dividend. Thirdly, divide the Dividend and the Quotient is your 6th Number ſought for.

And now according to my promiſe, we will firſt work the Queſtion of Weight and Carriage, which I deliver'd you in the double Rule of Three, to be reſolv'd by this Rule, which was this.

If the carriage of 1 *C.* weight 30 Miles coſt 12 pence, what will the carriage of 5 *C.* weight ſtand me in being carried 100 Miles.

C. Weight.	*Miles.*	*Pence.*	*C. Weight.*	*Miles.*
1	30	12	5	100

Now obſerve how theſe five Numbers ſtand: Then multiply the firſt Number by the ſecond; as 30 by 1, which maketh but 30, that Number keep for your Diviſor. Then multiply the other 3 Numbers, one into another, that is, 12 by 5 which maketh 60. Laſtly, 60 by 100, which as you ſee, ariſeth to 6000, which 6000 you muſt divide by the Product of the two firſt Numbers, which here is 30. And you ſee there is found 200 pence, which is the Duty that you ought to pay for the Carriage of 500 Weight 100 Miles, after the rate of 12 pence a Hundred, and agreeth with the Concluſion in the double Rule of Three, Queſtion the Firſt.

Schol. Well now if it pleaſe you, to have patience, I will ſee how I can end the Queſtion next following of 30 Buſhels of Wheat ſowed, which in one Year yieldeth 360, how many then will 80 Buſhels yield in 7 Year following, every Year of thoſe 7 ſtill 80 Buſhels, and according to your Reaſons I ſet my Numbers thus.

Buſh.	*Year.*	*Buſh.*	*Buſh.*	*Year.*
30	1	360	80	7
		80		
		28800		
		7		
		201600		

Then I multiply 30 by 1, and it maketh 30 my Diviſor; then multiplying the other 3 Numbers one into another, as here appeareth they make 201600, which I divide by 30: and my Quotient is 6720 Buſhels, my deſire: for ſo much alſo it came to at two workings by the double Rule of Three.

Maſt. Yet one Queſtion more I will propound unto you, and ſo leave this Rule.

Queſtion of Intereſt. What comes the Intereſt of 258 Pound, for 5 Months to, after the rate of 8 pound, taken in the 100 pound for 12 Months?

Schol. Sir, this is yet within the compaſs of ſome reaſonable uſe: Therefore to do juſtice in this caſe, I will ſee how I can work the ſame, which I ſet down thus, praying you, if I have not done well, to ſhew me mine errour.

li.	*mon.*	li.	li.	*mon.*
100	12	8	258	5.

Maſt. Proceed, you have done very well according to the Rules.

Schol. Then I doubt not but to end it well: I multiply 100 by 12, it produceth 1200. and the three other Numbers multiply'd together produce 10320, which I divide by 1200: and my Quotient is eight pounds. Then according as you have taught me heretofore, I turn the 720 pound that remain, into Shillings: and dividing it by the firſt Number, my Quotient is 12 Shillings. So I anſwer, that the loan of 258 pound for 5 Months, after the rate of 8 pound in the 100 pound for a Year, comes to 8 pound 12 ſhillings.

Maſt. You ſay true, I commend your diligence: now ſee the manner of the ſecond part of this Rule.

CHAP.

CHAP. XIII.

The Reverse Rule, or the Second Part of the Rule of Proportion compound of five Numbers.

Mast. IN this Double Rule of Proportion Reverse the 3 first Numbers are stated as in the last Rule, but the 4th and 5th are changed, as you see in the Example. And then the Rule for working is thus: Multiply the third Number and the fourth together, and that Product shall be your Divisor. Then multiply the fifth by the second, and the Product thereof by the first: and that is the Dividend, and the Quotient is the Answer. For example: I propound this Question for a proof of my last Question of Interest. *Rule.*

A Merchant hath receiv'd 8 pound 12 shillings, for Interest of certain Money for 5 Months term, which he receiv'd after the rate of eight pounds in the 100 for a Year. The Question is now, how much Money was deliver'd to raise this Interest. *The Proof of the last Question.*

Observe the manner, how the Question is stated.	li. *months.*	li. *months.*	li. s.
	100 12	8 5	8 12

Schol. Sir, I perceive it very well: and according to the Doctrine which you prescribed for the working thereof, now it is set down, I think I can perform it.

Mast. You say well, go on if you will, and try your skill in the Question: But this Note take with you by the way, In as much as here is mention made of Shillings: turn all your Money as you work into Shillings *Note.*

Schol. If it please you to mind me a little, I will quickly end it: for I have but my first, my second, and my last Number to be multiply'd together for my Dividend: And my third into my fourth for my Divisor.

l.	*mo.*
100	12
20	
2000 Shill.	
12	
24000	} Mult.
172	
48	
168	
24	
4128000 Product or Dividend.	

l.	*mo.*	*l.*	*s.*
8	5	8	12
20.		20	
Shill. 160		172 Shill.	
5			

Divisor 800

	s.	*l.*
800) 4128000	(5160	or 258.
4000		
12		
8		
48		
48		
00		

Which 4128000 I divide by 800, and my Quotient is 5160 Shillings, which in Pounds is 258 my desire, and proveth the truth of the last Question in the Direct Rule of Proportion Compound.

Mast. I will here for this time in whole Numbers end this Rule, and will instruct you in Vulgar Fractions. You may at your convenient leisure for your exercise, work the last Question by the Rule of Three at twice.

CHAP. XIV.

Notation of Vulgar Fractions.

Schol. SIR, I now earnestly desire of you some brief Uses of Fractions, whereby I may be able perfectly to understand the common works of them, and the vulgar use of those Rules, which without them cannot well be wrought.

Mast. If

Mast. If my Leisure were as great as my Will is good, you should not need to use any Importunity for the attaining of that thing, whereby I may be perswaded that I shall any way profit the Commonwealth, or help the honest Studies of any good Members in the same: wherefore while time will permit me, I am willing to help you as much as I can.

Therefore, first, to begin with the Explication of this name Fraction, what take you it to be?

Schol. Sir, I think a Fraction (as I have heard it often named) to be a broken Number, that is to say, to be no whole Number but part of a Number.

Mast. A Fraction indeed is a broken Number, and so consequently the part of another Number, but that must be understood of such another Number as cannot be divided into any other parts than Fractions: for although I may take the third part of 60, or the 4th part of it, and so of other parts diversly, yet those parts are not properly, nor ought to be called Fractions, because they may be expressed by whole Numbers, for the 1 third part of it is 20, the 4 part is 15, the 12 part is 5, and so forth of other parts, all which are whole Numbers.

Wherefore properly speaking, a Fraction expresseth the parts or part of an Unit only: and therefore it followeth, that no one Fraction alone can be so great, that it shall make 1, as by Examples I will shew, as soon as I have taught you to know the form how a Fraction is expressed or represented in Writing. *What a Fraction is properly.*

CHAP. XV.

Numeration of Vulgar Fractions.

BUT first to begin with the expressing of a Fraction, which is Numeration, you must understand that a Fraction is represented by two Numbers set one over the other, and a Line drawn between them, as thus, $\frac{1}{3}$, $\frac{3}{4}$, $\frac{2}{5}$, $\frac{10}{17}$; which four Fractions you must pronounce thus, $\frac{1}{3}$ one third part, $\frac{3}{4}$ three quarters, $\frac{2}{5}$ two fifth parts, $\frac{10}{17}$ ten seventeen parts, *&c.* *The expressing of Fractions.*

Schol.

Schol. I understand this form of their Expression and Pronunciation, but their Meaning and Valuation seemeth more obscure. Yet I think that by the two first Fractions, I understand the Valuation of the two latter Fractions, and consequently of others.

Mast. Value them then, that I may see you take me right.

Schol. $\frac{2}{5}$ is two fifth parts, that is to say, if One or a Unit be divided into 5 parts, that Fraction doth express two of those 5 parts: $\frac{10}{17}$ doth signifie, that if one be divided into 17 parts, I must take ten of them. And this I gather from the two first Examples: for $\frac{1}{3}$, that is, one third part, doth easily declare, that if one thing be divided into three parts, I must take out one of them: so $\frac{3}{4}$, that is three quarters, doth declare, that One being divided into four Quarters I must take (for this Fraction) three of those Quarters.

If there be no more difficulty in this Numeration, then I pray you go forward to their Addition and Substraction, &c. For I understand that the same kinds of works are in Fractions, that be in whole Numbers.

Mast. There are the same kinds of Works in both, *viz.* Addition, Substraction, &c. but the Order of them is divers, as I will declare by and by, but yet more in Numeration before we leave it. You must understand, that those two Numbers which express a Fraction, have several names, the uppermost which is above the line, is called the Numerator, and that below the line is called the Denominator.

Numerator.
Denominator.

Schol. And what is the reason of their divers Names? For (in my Opinion) both are Numerators, seeing both do express the Numeration of the Fraction.

Mast. You are deceiv'd; for one only (which is the uppermost) doth express the Numeration, and the Denominator doth declare the Number of parts, into which the Unit is divided, as in this Example; when I say, divide a Pound weight of Gold between four Men, so that the first Man may have $\frac{3}{13}$, the second $\frac{1}{13}$, the third $\frac{3}{13}$, and the fourth $\frac{6}{13}$.

Now do you perceive that by the Denominator (which is the same in all four Fractions) it is intended that the Pound weight should be divided into so many parts, I mean 15, and by the 4 several Numerators, is limited the divers proportion that each Man should have, that is, that when the whole is parted into 15, the first Man shall have two of those 15 parts: the second Man three of them: the third Man four: and the fourth Man six. And so may you see the several uses of those two Numbers, I mean of the Numerator and the Denominator.

And

And hereby you perceive that the Numerator of no proper Fraction can be greater than the Denominator, for if it is ſo great the value of the Fraction is a Unit: ſo that it were not proper to ſay, You ſhall have $\frac{12}{12}$ of any thing, ſeeing it were better expreſſed, you ſhall have the whole thing.

Schol. No, for 'tis vain to divide any thing, and then to apply the Diviſion to no uſe. And much leſs reaſonable were it to ſay $\frac{16}{15}$: for if the whole be divided into 15 parts only, it is not poſſible to take 16 of them, that is to ſay, more than all together.

Maſt. This is true touching the Natural and moſt proper Fractions, yet improperly (and after a Vulgar Acceptation for eaſineſs in work) both thoſe forms be called Fractions, becauſe they are written like Fractions although they be none indeed: for $\frac{15}{15}$ and generally in ſuch other, where the Numerator and Denominator be equal, are not Fractions, but the whole thing with all its parts: And ſo $\frac{16}{15}$ is not to be called a Fraction, but a mixt Number, of a whole Number and a Fraction, for it is $1\frac{1}{15}$, that is, one whole and $\frac{1}{15}$ parts; as ſhall be declared in Reduction. Therefore where the Numerator is either equal or greater than the Denominator the Fraction is an improper Fraction. *An improper Fraction or a mixt Number.*

Schol. But is there any neceſſity, why they ſhould write down ſuch kind of Fractions?

Maſt. There is cauſe why they ſhall ſometimes for eaſineſs in work write ſome Numbers after that manner like Fractions: but they needed not to call them Fractions, but (as they be) whole Numbers, or mixt Numbers, (that is, whole Numbers with Fractions) expreſſed like Fractions, or as improper Fractions.

Now you muſt underſtand, that as no Fraction properly can be greater than one, ſo in ſmalneſs under one the nature of Fractions doth extend infinitely, as the nature of whole Numbers is to increaſe above one infinitely, ſo that not only one may be divided into infinite Fractions or parts, but alſo every Fraction may be divided into infinite Fractions or parts, which commonly be called Fractions of Fractions: and they are expreſſed variouſly, as for example, $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$, that is, three quarters of 2 third parts of one half of a Unit. Whereby is ſignified, that if one be divided into two halfs, and the one half into three parts, and two of thoſe three parts be divided into four quarters, this Fraction of Fractions doth repreſent three of thoſe Quarters, or fourth parts. *Fractions of Fractions.*

Schol.

Schol. I pray you let me prove by an example in common Money, whether I do rightly understand you or no. One Crown which I take for an Unit, doth contain 60 Pence; therefore the half of it is 30 Pence, $\frac{2}{3}$ of that half is 20 Pence, whereof $\frac{1}{4}$ is Fifteen pence; so then 15 Pence is $\frac{1}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$ of a Crown: and so is 3 Pence, $\frac{1}{5}$ of $\frac{1}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$ of a Shilling.

Mast. You understand this well enough.

Schol. Sir, I gather then from what you have said that there are 3 kinds of Fractions, *i. e.*

1. A proper Fraction, having the Numerator less than the Denominator, as $\frac{1}{2}$, $\frac{17}{18}$, $\frac{155}{156}$, *&c.*

2. An Improper Fraction, having the Numerator equal to, or greater than the Denominator, as $\frac{12}{12}$, *&c.* both which kinds are Fractions of a Unit. And

3. Fractions of Fractions, which is when the first Fraction toward the left hand is not immediately the Fraction of a Unit, but of that Fraction next it toward the right hand, as $\frac{1}{3}$ of $\frac{2}{3}$ of a Unit, *&c.*

But pray Sir, why do you call all these Fractions Vulgar, is there any other sorts?

Decimals what.

Mast. Yes, a Vulgar Fraction is as much as to say a Common or Natural Fraction, arising from any Division: But there are Decimal Fractions, which have their Denominators 10, or some Number produced by Multiplying 10 in it self, *ad infinitum*, as 10, 100, 1000, 10000, *&c.* which are therefore called Decimals, by which kind of Fractions operations may be easier performed, though the Vulgar do them more exactly in most Cases.

And now since you understand these varieties, I will proceed to the rest of the works: First advertising you, that there is another order to be followed in Fractions, than there was in whole numbers: for in whole numbers this was the order: Numeration, Addition, Substraction, Multiplication, Division, and Reduction: but in Fractions,

The order of Fractions.

(to follow the same Rule in proceeding from the easiest works to the harder) we must use this order, Numeration, Reduction, Addition, Substraction, Multiplication and Division.

Schol. That Addition and Substraction should go together, and Division to follow Multiplication, natural order doth perswade: but why Reduction should be first in order here, next to Numeration, and Addition and Substraction, in the middle, I desire to understand the reason.

Mast.

Mast. As in the Art of whole numbers, Order would reasonably begin with the easiest, and so go forward by degrees to the hardest: so the same reason teacheth in Fraction the like order. But that is not all, for you must consider that Addition or Substraction of Fractions can very seldom be wrought without Reduction: and contrariwise, Reduction may be wrought without this form of Addition or Substraction: therefore was it orderly required, that Reduction should go before Addition and Substraction, and this reason serveth for the placing of Reduction before the other Rules.

Schol. Then, if Reduction be the easiest, I pray you shew me the form of it, first by rule, and then by example.

CHAP. XVI.

Reduction of Vulgar Fractions.

Of Reduction of Fractions, there are five varieties.

I will now shew you the diversities of Reduction of Fractions, which commonly hath five varieties or forms.

First, To reduce sundry Fractions to one Denominator, and also into one Fraction. 1.

Secondly, To reduce Fractions of Fractions, into one Fraction: or a Compound Fraction to a Simple. 2.

Thirdly, To reduce an improper Fraction into a proper, expressing the Unit or Units, and the proper Fraction of it distinctly. And contrary to convert such a mixt Number of Units with Fractions, into the form of a Fraction, that is, into an improper Fraction. 3.

Fourthly, To reduce Fractions written in great numbers, into lesser numbers, *i. e.* into their smallest terms. 4.

Fifthly, To reduce Fractions of one Denomination to another, or to find the value of any Fraction of Coin, Weight or Measure. 5.

Schol. This distinction in Doctrine pleaseth me much, but more with hope than present fruit: for as yet I do scarcely understand the varieties, and much less the practice and use of them.

Mast. Reduction is an orderly alteration of Numbers out of one form into another, which is never done orderly but for some needful

uſe, as in every of the ſaid five ſeveral forms, I will diſtinctly declare.

The Firſt form of Reduction.

Firſt therefore, when two, or more ſeveral Fractions of any Unit be propounded: as for example, $\frac{2}{3}$ and $\frac{1}{6}$, becauſe it is hard to tell what proportion of the intire Number thoſe two Fractions do expreſs, therefore was Reduction deviſed to be a mean whereby theſe ſeveral Fractions might be brought into one Denominator and Fraction.

And in theſe Fractions, this is the Rule for bringing them to one Denominator.

How to reduce Fractions of diverſe Denominators into one Denominator.

Multiply firſt the Denominators together, and the product thereof ſet twice down under two ſeveral lines for two new Denominators, or rather for one common Denominator. Then multiply the Numerator of the firſt Fraction, by the Denominator of the ſecond, and ſet the product thereof for the Numerator over the firſt line. Likewiſe multiply the Numerator of the ſecond Fraction by the Denominator of the firſt, and ſet that Total over the ſecond line for the Numerator of that Fraction: and ſo are theſe two firſt Fractions of ſeveral Denominators brought to one Denominator.

Schol. If I underſtand you, as I think I do, my example ſhall declare the ſame. The Fractions, which you propounded were theſe $\frac{2}{3}$ and $\frac{1}{6}$, whoſe Denominators (being 16 and 6) I multiply together, and there ariſeth 96, which I ſet under two lines, thus: $\frac{\ }{96}$, $\frac{\ }{96}$.

Then I multiply the Numerator of the firſt Fraction by the Denominator of the ſecond, ſaying, 3 into 6 maketh 18, that I ſet over the firſt line for a new Numerator, and it will be thus, $\frac{18}{96}$.

Likewiſe I multiply the Numerator of the ſecond Fraction, by the Denominator of the firſt, ſaying 4 times 16 maketh 64, that I ſet for the ſecond Numerator, and the Fraction will appear thus $\frac{64}{96}$.

So that both Fractions brought to one Denominator, muſt ſtand thus $\frac{18}{96}$ and $\frac{64}{96}$.

Maſt. You have done well.

Note the Reduction of three Fractions or more, into one.

So have you now the Art to bring two ſuch Fractions into one Denomination: And if there be more than two, then muſt you multiply all the Denominators together, and ſet the product thereof ſo many times down as there be Fractions; and then to get for each one a new Numerator, multiply the Numerator of the firſt by the Denominator of the ſecond, and the product thereof multiply by the Denominator of the third, and ſo forth, if there be more; likewiſe multiply the Numerator of the ſecond, by the Denominator of the

firſt,

first, and the total thereof by the Denominator of the third; and in the same method multiply the Numerator of the third, into the Denominator of the first, and the Product thereof into the Denominator of the second, and so forth, if there were more, and the last product is the Numerator to the respective Fraction, whose Numerator you multiplied into the several Denominators; as in these three Fractions, ½, ⅔, ¾ do make by Reduction these other three Fractions of one Denominator, $\frac{36}{72}\ \frac{48}{72}\ \frac{54}{72}$. All which you may bring into one Fraction by adding the Numerators together, and putting the total for the total Numerator, reserving still that same common Denominator. And those three Fractions make one improper Fraction, but more of that in Addition.

Schol. All this I understand.

Mast. When Fractions of Fractions be propounded, you shall multiply the Numerators of each into other, and set the total for the new Numerator, and then multiply all the Denominators one into another likewise, and the product is the new Denominator, and so are they reduced to one Fraction. *The second form of Reduction of Fractions of Fractions into one Fraction.*

Schol. If that be all, then I understand it already, as by this example I will shew. These are the Fractions, ⅔ of ¾ of ⅞ of ⅔ which I would reduce to one proper simple Fraction.

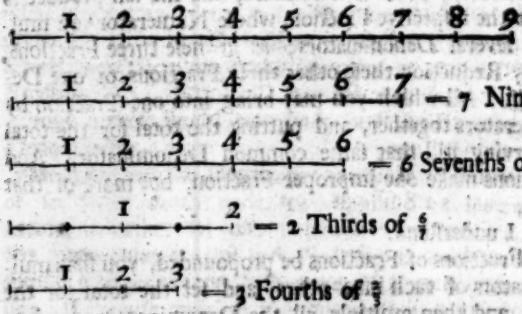
Therefore I begin with the Numerators, and multiply them together, saying, 2 by 3, maketh 6; and 6 by 6, maketh 36, which multiplied by 7, yieldeth 252: that I set over a line for the Numerator thus: $\frac{252}{}$

Then I multiply the Denominator, 4 by 3 maketh 12, and that by 7 maketh 84, which multiplied by 9, yieldeth 756, the new Denominator. And so the whole reduced Fraction is this, $\frac{252}{756}$ which is too hard a Fraction for me to understand yet.

Mast. You think so, and no wonder, but by and by you shall learn to know it easily, for this Fraction is no more indeed than ⅓, although it be in greater terms, and therefore more strange, and more obscure.

And this might suffice for this Reduction, but I will shew you by a Figure of measure the just rate and reason of this kind of Fractions, and also the due understanding of their Reduction.

The Demonſtration of the Compound Fraction, $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{7}{9}$, of Unity or 1.



1. By the Demonſtration above is plainly ſhewed the Nature not only of a Simple Fraction as any one of the former, but alſo of a Compound, as thoſe above taken together.

2. It is plain that $\frac{1}{2}$ of $\frac{2}{3}$ is equal to $\frac{2}{3}$, or $\frac{1}{2}$ of the Unit.

3. Alſo that $\frac{2}{3}$ of $\frac{6}{7}$ is equal to $\frac{4}{7}$ of Unity.

4. That $\frac{6}{7}$ of $\frac{7}{9}$ is equal to $\frac{6}{9}$, or $\frac{2}{3}$ of Unity.

5. The Reaſon of reducing a Compound Fraction to a Simple, or a Simple one into its Loweſt Terms, and many other uſeful things in Vulgar Fractions, is learnt from the Diviſion of the Lines above.

The third form of Reduction of improper Fractions.

But now it is time that we come to the third form of Reduction, which treateth of improper Fractions, that is to ſay, mixt numbers of Units and Fractions, as $\frac{26}{5}$, which doth include 5 Units wholly, and $\frac{1}{5}$ over.

Two ſeveral ways in this Reduction.

There be two ſeveral kinds of Reduction, concerning ſuch Fractions. Sometimes it will be needful to convert theſe Fractions into Units, or Units and a Fraction. And ſometimes, contrariwiſe, it will be convenient to reduce mixt numbers, that is, Units written with Fractions, into the form of one ſimple Fraction, and ſo there be two ways.

Schol. Pray what is the method of the firſt way to turn improper Fractions into Units with their proper Fractions?

Maſt. That is thus; Your Numerator being greater than the Denominator, muſt be divided thereby, and the Quotient thereof expreſſeth

preſſeth the Units; the remainer ſhall be put for the Numerator of the Fraction that reſteth, and the Denominator muſt be the ſame that was before. *Reduction of improper Fractions into Units, with their proper Fractions if any.*

Schol. For example, I take $\frac{17}{5}$ and dividing 17 by 5, the Quotient will be 3, and there will remain 2.

Maſt. That you muſt write thus, $3\frac{2}{5}$, where (you ſee) I have written 3 without any line, as entire Numbers ought to be written, and the 2 that remained, I have ſet over the former Denominator, with a line; as a proper Fraction. And this Number doth ſignifie now three Units, and $\frac{2}{5}$ of one.

Schol. Then if I would by Units here underſtand Crowns, ſo it were 3 Crowns, and $\frac{2}{5}$, that is 2 Shillings.

Maſt. Right: and therefore $\frac{17}{5}$ did ſignifie the ſame: But this happeneth ſometimes that when the Reduction is ſo wrought, there remaineth nothing: And then it is not a mixt number, but a ſimple intire number, repreſented like a Fraction.

Schol. As $\frac{15}{5}$ will make 3 juſt; and $\frac{18}{3}$ will make even 6. This I will remember. But now, what is the ſecond form of Reduction that you ſpeak of for theſe ſorts of Fractions.

Maſt. Whenſoever you have any of theſe two ſorts of Numbers, that is to ſay, whole numbers without Fractions, or whole numbers with Fractions, and you would turn them into the form of a Fraction, you muſt multiply the whole Number by the Denominator, and to the product add the Numerator, and the Summ is the new Numerator, keeping ſtill the former Denominator: As if you have 6 $\frac{3}{4}$ which you would convert into an improper Fraction, you muſt multiply 6 by 4, whereof cometh 24, and thereto add the Numerator, which is 3, and ſo have you 27 for the Numerator, and 4 ſtill for the Denominator. *Reduction of whole numbers either alone, or joyned with Fractions into improper Fractions.*

Schol. Then is $\frac{27}{4}$ equal to $6\frac{3}{4}$.

Maſt. You ſay right, and ſo backward (as appeareth by the former Reduction) $6\frac{3}{4}$ maketh $\frac{27}{4}$. And thus one of theſe Reductions may be the proof of the other. *Note.*

Schol. This I perceive: But now if you would turn whole numbers without Fractions into any Fractions, I ſee not how that can be done, becauſe there is no Denominator to make the multiplication by.

Maſt. That is well obſerved: but this know, that no man intendeth to turn any whole number into a Fraction, but he hath in his mind that Denominator by which the multiplication muſt be made: for

for the proof whereof I ſet down 7, which is a whole number. As if you will have this number converted into any certain Fraction, you pleaſe.

Schol. I pray you reduce 7 into a Fraction.

Maſt. Then you care not what the Fraction be, ſo it be ſome Fraction.

Schol. No.

Maſt. Then how can you think that you require me to do any thing certain, when you leave me to do as I liſt?

I will limit my ſelf (ſeeing you will not) to turn it into quarters. And therefore I multiply 7 by 4, and there ariſeth 28 to be ſet for the Numerator, and the 4 muſt be ſet for the Denominator, and the Fraction will be thus, $\frac{28}{4}$.

Schol. Indeed I perceive this to be reaſonable, for without much trial I underſtand that $\frac{28}{4}$ of any thing doth make 7. And ſo then if I would turn 8 into 5 parts, it will make $\frac{40}{5}$ which is all one with 8: for 8 Crowns turned into 5 parts (that is into Shillings) will make 40 Shillings, that is $\frac{40}{5}$ of a Crown.

The Fourth form of Reduction.

Maſt. Seeing you underſtand now theſe three kinds of Reduction, I will declare unto you the fourth kind, that is, when Fractions be written in greater terms, how they may be brought to leſſer terms, and the Fraction retain the ſame value.

As for example: In the firſt kind of Reduction this was your own example, $\frac{3}{11}$ and $\frac{5}{7}$, which when you would reduce, you were fain to turn them firſt into one Denominator, and ſo appeared they thus, $\frac{21}{77}$ and $\frac{55}{77}$.

Terms of Fractions.

Schol. Sir, I underſtand now, not only that the Fractions may better be underſtood as Fractions, when they are in leſſer terms, but alſo I underſtand what you mean by greater terms and leſſer terms, whereof I was before in doubt: for I ſee you call the Numerator and Denominator, the terms of the Fraction.

Rule.

Maſt. I am glad you underſtand it ſo well: now then when you would value any Fractions, becauſe they may beſt be done when the terms are ſmalleſt, you ſhall reduce them to the ſmalleſt that you can, which you may do thus: Divide the greateſt of any ſuch two terms by the leſſer, and if any thing remain, by that remainer divide the laſt Diviſor: and if any thing remain now, by that divide the firſt Diviſor (which was before the remainder of the laſt Diviſion) and ſo continue ſtill, till nothing remain in the Diviſion: and then obſerve

serve your last Divisor, for that is the number that will easily reduce your Fractions, if you divide both the Numerator and the Denominator by it, and put for the Numerator the Quotient of its Division, and for the Denominator also the Quotient, that riseth by its Division.

Schol. I take for example $\frac{18}{96}$, and because 96 is the greatest number, I divide it by 18, and the Quotient is 5, and there resteth 6, what shall I do with this Quotient?

Mast. Nothing in this work, but now seeing there remaineth somewhat, by that remainer you must divide the last Divisor.

Schol. If I shall divide 18 (which was the last Divisor) by 6, that was the remainer, the Quotient is 3, and nothing remaining.

Mast. As for the Quotient, I omit it, but because there doth remain nothing, therefore is 6 (which was your last Divisor) that number by which you may reduce the Fraction propounded into its lowest terms.

Schol. Then, as you taught me, I must divide the Numerator 18 by 6, and the Quotient is 3, which I must put for the Numerator over a line, as in the Margent: And then by the said 6 must I divide also the Denominator 96, and the Quotient will be 16, which I must take for the Denominator, and so is the Fraction $\frac{3}{16}$. And I think this Rule doth prove the work of the first Reduction.

$\frac{3}{16}$

Mast. But note this, that if you find no such Divisor, to reduce the Fraction till you come to 1, because one doth make no Division, therefore that Fraction is already in its least Terms, as by $\frac{71}{100}$ you may prove, and so $\frac{85}{96}$, and many others of the like nature.

A second way of Reducing a Fraction to its lowest Terms.

For Example: I would reduce $\frac{288}{576}$ into its least term or value, and therefore I set $\frac{288}{576}$ with a long line drawn from it, thus,

288	144	72	36	18	9	3	1
576	288	144	72	36	18	6	2

And because both the Numerator and Denominator end in even Numbers, I see this may be abbreviated by 2, or 4, or 6, *&c.* Therefore, I first take the half of the Numerator, saying the half of 2 is 1, the half of 8 is 4: and again the half of 8 is 4: which 144 is now a new Numerator, and therefore I part it with a dash as you see.

Then

Then I also take the half of 576, saying, the half of 5 is 2, the half of 17 is 8, and the half of 16 is 8, and so I have 288 for a new Denominator.

Then I say, the half of 144 is 72, and the half of 288 is 144: Thus continuing the Division by 2, until I come to the last work, as appeareth here in the Example, where the same is reduced to $\frac{1}{2}$ which is equal to $\frac{288}{576}$.

Yet one other way of easie Reduction in this kind there is: when your Fraction hath any Cyphers in the first places of both Terms, then may you, by casting away the Cyphers, make a brief Reduction, as thus, $\frac{300}{600}$. Here take away the Cyphers, and it will be $\frac{3}{6}$, which is the same in value with $\frac{300}{600}$.

Schol. Sir, I thank you, this is very brief and easie.

Mast. Then I may go to the fifth or last kind of Reduction, which teacheth how to turn any Fraction proposed into any other Denomination, or into any part of common Coins, Weights, or Measures, *&c.*

The fifth kind of Reduction.

First then you must mind whether your Fraction be a simple Fraction, or a Fraction of sundry parts, I mean of more terms than two. And if your Fraction be a Fraction of Fractions, or otherwise compound, you may reduce it to one simple Fraction: And then consider the Denomination of that other Fraction, into which you would turn this: for by that Denominator you must multiply the Numerator of your first Fraction, and the Product thereof divide by the Denominator of your first Fraction, and that Quotient will be the Numerator to the Denominator proposed: as for Example; I have this Fraction $\frac{3}{5}$, which I would turn into ten parts: therefore I multiply this 10 by 3, the Numerator of my Fraction, and there ariseth 30, which I divide by 5, the Denominator and the Quotient is 6, which must be the Numerator to 10, and so $\frac{3}{5}$ will be $\frac{6}{10}$.

To reduce Fractions to a Denomination appointed.

Schol. This is easie enough to do.

Mast. Then you shall see another example of the same Fraction that is not so easie: as if I would turn $\frac{3}{5}$ into 8 parts, work that.

Schol. I must multiply 8 by 3, and there ariseth 24, which I divide by 5, and the Quotient is 4, then is the new Fraction $\frac{4}{8}$.

Mast. And see you nothing doubtful in this work?

Schol. I see that when 24 was divided by 5, there remained 4, which I did not mind, because you speak nothing of any remainer, but only of the Quotient.

Mast.

Mast. I hope you remember what I said to you in Division of whole Numbers, that you should not mind the remainer there, but only note it as a Summ that could not be divided without knowledge of Fractions. Wherefore now observe this, that in all divisions of whole Numbers, when there is any remainer, you must set it over a Line as a Numerator, and put the Divisor for the Denominator, and that Fraction doth make the Division compleat, and is part of the Quotient: As if I would divide 48 by 5, the Quotient will be $9\frac{3}{5}$: so in your former Work when 34 was divided by 5, the Quotient should be $4\frac{1}{5}$, and so the new Fraction should be thus $\frac{1}{5}$ and $\frac{1}{5}$, of $\frac{1}{4}$, that is $\frac{1}{5}$ of the entire Number, and $\frac{1}{5}$ of $\frac{1}{4}$ part of any thing, which you may prove by Example of some Coin.

Schol. Then I take a Crown, whose $\frac{1}{4}$ is 3 *s.* Now I would prove whether the 3 *s.* be $\frac{1}{4}$ and $\frac{1}{5}$ of $\frac{1}{4}$, I shall have a troublesom piece of work to do.

Mast. Indeed for whole Pence your Example is a little troublesome: yet turning the Crown into Half-pence, it is easie enough.

Schol. I wil try that.

Mast. The best way however to work that Question is thus: $\frac{1}{5}$ being abbreviated as before I taught, is $\frac{1}{20}$. Now half a Crown is 2 Shillings 6 pence. Now $\frac{1}{5}$ of $\frac{1}{4}$ is a Fraction of Fractions, which if you do reduce into one entire Fraction, as before you have learned, in saying 5 times 8 is 40, for a new Denominator, and once 4 is 4, for a new Numerator: it maketh $\frac{4}{40}$, which abbreviated also makes $\frac{1}{10}$; now the tenth part of a Crown is 6 Pence, which put to 2 Shillings 6 Pence make also 3 Shillings your desire.

But now one Example more for this Rule, and then we shall end it. If I have $\frac{7}{15}$ of a Soveraign (accounting the Soveraign 20 Shillings) how many Shillings is that $\frac{7}{15}$?

Schol. I must multiply 7 by 20 and that maketh 140, which I divide by 15, and the Quotient is $9\frac{5}{15}$, or in lesser terms $\frac{1}{3}$.

Mast. That is 9 Shillings, and one third part of a Shilling, that is 4 Pence, as by the same Rule you may prove.

Let me by the same Rules see you find the value of $\frac{3}{15}$ of a Pound Troy-weight.

Schol. To do that I must multiply the Numerator 2 by 12 the Ounces in a Pound, and divide the Product by 15 the Denominator, and the Quotient is 1 Ounce and 9 remains, which multiply by 20 the Penny-weights in an Ounce, and divide the Product by the Denominator 15 and the Quotient is 12 Penny-weight and nothing remains; so the value of $\frac{2}{15}$ of a Pound Troy I find to be 1 $\frac{2}{15}$: 12 Penny-weight.

Mast. You have done very well, and since you are so expert in Reduction we will pass forward to Addition of Fractions.

2
12 Ounces
15) 24 (1 Ounce
15
9 Ounces rem.
20 Penny-weight.
15) 180 (12 pen. wt.
15
30
30
0 rem.

CHAP. XVII.

Addition of Vulgar Fractions.

Addition of Fractions that have a common Denominator.

WHenever you have any Fractions to be added, you must consider whether they be of one Denominator or not, and if they be of one Denominator, then add the Numerators together, and set that Summ for the Numerator over the common Denominator, and so have you done: But if the Fractions be not of one Denominator, or any of them be mixt of whole Numbers and Fractions, then must you first reduce them to one Denominator, and after add them.

Schol. This seemeth easie enough, now I have learned to reduce, without which I could never have wrought this: And therefore now I see good reason why you did place Reduction before Addition.

Mast. It is well considered, but yet refuse not to expresse your understanding of it by an Example.

Schol. Then I will add first $\frac{2}{15}$ to $\frac{5}{15}$, and because the Denominators are like (and so needeth no Reduction) I add 2 to 5, which maketh

maketh 12, and then is my Summ $\frac{11}{12}$, that is in ſmaller numbers, being abbreviated $\frac{3}{4}$.

And if many Numbers be to be added, as here $\frac{1}{2}$, $\frac{4}{5}$, $\frac{3}{10}$, firſt I muſt reduce them (becauſe they have divers Denominators) into one Denominator, and then they will be thus, $\frac{10}{20}$ $\frac{16}{20}$ $\frac{6}{20}$, or in leſſer terms, $\frac{11}{20}$ $\frac{12}{20}$ $\frac{16}{20}$, which by Addition do make $\frac{34}{20}$, that is $2\frac{14}{20}$. *To add Fractions of divers Denominations.*

Maſt. Well, let me ſee you add $14\frac{1}{2}$ to $\frac{1}{2}$ and $\frac{2}{3}$ of $\frac{3}{4}$.

Schol. The Compound Fraction $\frac{2}{3}$ of $\frac{3}{4}$ in a Simple is $\frac{6}{12}$ or $\frac{1}{2}$, which added to the other $\frac{1}{2}$, the Summ is 1, which added to $14\frac{1}{2}$ is $15\frac{1}{2}$ the Anſwer.

Maſt. I am pleaſed to ſee you do it not only right but the beſt way, in working firſt the Fractions and adding their Summ to the whole Number given, which in this caſe needs not be reduced to a Fraction as ſome do.

Now may we go on to Subſtraction.

CHAP. XVIII.

Subſtraction of Vulgar Fractions:

SUbſtraction hath the ſame Precepts that Addition had; as to the Denominators, which if alike, then muſt you ſubſtract the one Numerator from the other, and the reſt is to be ſet over the common Denominator, and ſo your Subſtraction is ended: but if you have many Fractions to be ſubſtracted out of many, then muſt you reduce them to one Denomination, and into two ſeveral Fractions, that is, all that muſt be ſubſtracted into one Fraction, and the reſidue into another Fraction, and then work as I ſaid before. *Subſtraction of Fractions.*

Schol. For the firſt Example I take $\frac{1}{12}$ to be ſubſtracted out of $\frac{11}{12}$, and the reſt will be $\frac{10}{12}$ or $\frac{5}{6}$.

For another Example I take $\frac{3}{7}$ to be ſubſtracted out of $\frac{4}{5}$, which I muſt reduce, and it will be thus, $\frac{15}{35}$ and $\frac{21}{35}$.

Then I ſubſtract 21 out of 28, and there reſteth 4, which I ſet over the common Denominator for a Remainer, thus, $\frac{4}{35}$: that is I.

Now for the third Example, I take $\frac{3}{4}$ and $\frac{4}{5}$ to be ſubſtracted from $\frac{7}{8}$ and $\frac{9}{10}$: and becauſe their Denominators be different, I reduce them into one Denominator as thus, $\frac{140}{160}$, $\frac{160}{160}$, $\frac{168}{160}$, $\frac{172}{160}$.

Then I add the two firſt, and they make $\frac{300}{160}$. Alſo I add the two laſt, and they make $\frac{340}{160}$. Then I ſubſtract 3040 out of 3408, and there reſteth 368, ſo is the remainer $\frac{368}{160}$, that is in ſmaller terms $\frac{23}{10}$. And thus have I done with Subſtraction, except you have any more to teach me.

Maſt. Let me ſee one Example more of Fractions of divers Denominators.

Schol. I take $\frac{7}{8}$ to be ſubſtracted from $\frac{9}{24}$ which being reduced will ſtand thus $\frac{168}{192}$ and $\frac{72}{192}$: Now I ſhould ſubſtract 168 out of 72, but I cannot.

168	72
$\frac{7}{8}$	$\frac{9}{24}$
192	

The greateſt of two Fractions.

Maſt. Then may you perceive that you miſtook the Fractions: for you can never ſubſtract the greater out of the leſſer, although you may add, multiply or divide the greater with the leſſer. And although $\frac{7}{8}$ hath both its terms leſſer than $\frac{9}{24}$ yet is $\frac{9}{24}$ the leſſer Fraction: for generally if you multiply the Numerator and the Denominators of two Fractions croſſways, that Fraction is the greateſt of whoſe Numerator cometh the greateſt Summ, as in this Example, 7 multiplied by 24 maketh 168, and 9 being multiplied by 8 yieldeth but 72, therefore is the firſt Fraction the greateſt of theſe two.

But if you would ſubſtract a Fraction out of a whole Number, what would you do?

Schol. I would reduce the whole Number into a Fraction of the ſame Denomination that my Fraction is, and then work by Subſtraction.

Maſt. So you may, but it is much eaſier, if your Fraction be a proper Fraction, that is to ſay, leſs than an Unit, to take an Unit from the whole Number, and then turn it into an improper Fraction, and ſo work your Subſtraction. As if I would ſubſtract $3\frac{2}{3}$ from 4, I may take 1 from 4 and turn it into $\frac{3}{3}$, from which $3\frac{2}{3}$ I abate $3\frac{2}{3}$, there will remain $\frac{1}{3}$. And if the firſt be an improper Fraction, then may I take ſo many Units from the whole Number, that they may make an improper Fraction, greater than that firſt, and then work by Subſtraction. As if there be propoſed $\frac{10}{3}$ to be ſubſtracted from 6, becauſe $\frac{10}{3}$ is more than 3, and not ſo much as 4, I muſt take 4 from 6, and turn them into thirds thus $\frac{12}{3}$, then abate $\frac{10}{3}$ from $\frac{12}{3}$, there reſteth $\frac{2}{3}$; ſo the whole remainder is $2\frac{2}{3}$. Or elſe you may at your

pleaſure.

pleasure take $\frac{3}{4}$, which is $\frac{12}{16}$ from 6: then set 1 under 6, as thus $\frac{6}{1}$. And reduce those two Fractions into one Denominator, as here appeareth $\frac{12}{16}$ from $\frac{6}{1}$: then $\frac{12}{16}$, from $\frac{16}{16}$ resteth $\frac{4}{16}$, which maketh $2\frac{1}{4}$ your desire. And thus will I make an end of the Work of Substraction of Fractions, and proceed to Multiplication.

$$\begin{array}{r} 8 \\ \frac{12}{16} - \frac{4}{16} \\ 3 \end{array}$$

CHAP. XIX.

Multiplication of Vulgar Fractions.

Multiplication of Fractions.

WHen any two Fractions are proposed to be multiplied together, the Numerator of the one must be multiplied by the Numerator of the other: and the Product thereof must be set for a new Numerator: likewise the Denominator of the one must be multiplied by the Denominator of the other, and that Product shall be set for the Denominator, and this new Fraction expresseth the Product of the Multiplication of the two first Fractions proposed, whereof take this Example; $\frac{3}{5}$ multiplied by $\frac{5}{12}$ doth make $\frac{15}{60}$.

$$\begin{array}{r} 15 \\ \frac{3}{5} - \frac{5}{12} \\ 60 \end{array}$$

Schol. I perceive then that 3 being the Numerator of the first Fraction is multiplied by 5 being the Numerator of the second Fraction, whereof cometh 15, the Numerator of the third Fraction. And so likewise 5 being the Denominator of the first Fraction, is multiplied by 12 the Denominator of the second Fraction, which produceth 60 the new Denominator, so that though I perceive how the Work is done, I do not see how $\frac{15}{60}$ is greater than $\frac{3}{5}$, for if I shall use my former manner of Examination by the parts of some Coin, I see that $\frac{3}{5}$ of a Crown is 36 Pence, and $\frac{5}{12}$ of a Crown is 25 Pence, whereof the one multiplied by the other, doth make 900 Pence, which is 15 Crowns, but by your Multiplication there amounteth $\frac{15}{60}$, which is but 15 Pence, and that is much less than either of the first Fractions.

Mast. That difference is between Multiplication in whole Numbers and Multiplication in broken Numbers, that in whole Numbers, the Summ produced is greater than either of the other where-

of

of it came: but in Fractions it is contrariwise: for the Product is leſſer than either of the other two Fractions whereof it is produced.

Schol. I very much deſire to underſtand the reaſon thereof.

Maſt. Although I purpoſed to reſerve the Reaſons of works Arithmetical for another time, yet I will ſhew you this, becauſe of the ſtrangeneſs of the work.

You ſee in whole Numbers, that of two Numbers being multiplied together is made the third Number, which third Number doth bear the ſame Proportion to the Number multiplied, that the multiplier doth bear to an Unit. And ſo in Fractions the third Number or Product beareth the ſame Proportion to each of the two firſt Fractions that the other of thoſe two Fractions doth bear to an Unit.

Schol. Sir, I underſtand your words thus: when 40 is multiplied by 12, there is produced 480, which 480 doth contain 40 ſo many times in it as 12 doth contain Units. But now I ſee not how the third Number in this Example of Fractions can contain any of the two former (as it happened in whole Numbers) ſeeing it is leſſer than either of them.

Maſt. No wonder if you cannot ſee that which is not poſſible to be ſeen, how the third number in multiplication of Fractions ſhould be greater than any of the two former Fractions: but yet this may you ſee (which I ſaid) that the third number in Fractions ſo multiplied doth bear the ſame proportion to any of the two former Fractions, that the other of thoſe two Fractions doth bear to an Unit, as in your Example, $\frac{3}{4}$ being multiplied by $\frac{4}{11}$ doth make $\frac{12}{44}$. Now I ſay that $\frac{12}{44}$ doth bear the ſame proportion to $\frac{3}{4}$ that $\frac{4}{11}$ doth bear to an Unit, as you may in your own form of Examination by Coin try it: for in an old Angel (which in times paſt was current for 7 Shillings 6 Pence) are 180 Half-pence, which I ſet for the entire Unit, whoſe parts (according to the Fractions aforeſaid) are theſe, for $\frac{12}{44}$ ſet 45 Half-pence, for $\frac{3}{4}$ take 108 Half-pence, and for $\frac{4}{11}$ put 75 Half-pence. Now doth 45 bear the ſame proportion to 108 that 75 doth bear to 180, for 45 is $\frac{4}{11}$ of 108, and ſo is 75 alſo $\frac{4}{11}$ of 180.

But theſe reaſons may be better reſerved till another time, when the knowledge of Proportions in due order ſhall be taught: yet in the mean time I will ſhew you how it cometh to paſs, that in Fractions the third Summ muſt needs be leſſer than any of the other two.

Conſider

Conſider this, that when a Fraction is propoſed, as in the former Example, ½ if it be multiplied by more than 1, it will make more than one entire number. As if I multiply ½ by 5, that is to ſay, if I take it 5 times, it will make three entire Units. Example: in a Crown ½ of it maketh 5 Shillings, which if I take five times, it will amount to 15 Shillings, that is, three entire Crowns; ſo if I take the ſame ½ but twice, it will yield 6 Shillings, that is one entire Crown, and ½. Now if I take it but once, it cannot be more than it was before, that is 5 Shillings. And if I take it leſs than once, it cannot be ſo much as it was before. Then ſeeing that a Fraction is leſs than one, if I multiply a Fraction by another Fraction, it followeth that I take the firſt Fraction leſs than once, and therefore the product muſt needs be leſs than the firſt Fraction.

Schol. Sir, I thank you much for this reaſon. And I think I underſtand it, as by Example of this Fraction ½ I will expreſs. If I take ½ of a Crown once, that is to ſay, if I multiply ½ by 1, it will be as it was before, but 5 Shillings: ſo if I do multiply it by ½, that is, if I take but half one time, then it will be but half ſo much: likewiſe if I multiply it by ⅓, that is, if I take but the third part of one, it will yield but 12 Pence, that is, the third part of the firſt Fraction.

And ſo to make an end: if I take but the twelfth part of one, that is, if I multiply it by 1/12 it will yield but the twelfth part of the firſt Fraction, which is but 3 Pence. And it followeth, that if 1/12 make 3 Pence, then 5/12 muſt needs make five times ſo much, that is, 15 Pence.

Maſt. Then I perceive you have ſufficient underſtanding in this ſort of Multiplication for this time, wherefore I will proceed to the reſt.

To multiply a whole Number into a Fraction.

In Multiplication it happeneth ſometimes, that there be whole Numbers to be multiplied with Fractions, and may be in two ſorts: for either the whole Number is ſeveral from the Fraction, or elſe the whole Number is joyned with one, or both of the Fractions, and ſo maketh a mixt number. If it be in the firſt ſort, then needeth there no Reduction, but only multiply the Numerator of the Fraction by that whole Number, and the Product ſet for the new Numerator.

Schol. I underſtand you thus. If I have 6/13 to be multiplied by 16, then muſt I multiply that 16 with 6, which is the Numerator, whereof cometh 96, and that I muſt ſet for the Numerator: keeping

ing

ing ſtill 23 for the Denominator, and ſo the Fraction will be $\frac{92}{23}$ that is $4\frac{0}{23}$.

Maſt. Or if it happen the Denominator to be ſuch a Number as may evenly be divided by the ſaid whole number propoſed, then divide it thereby, and ſet the Quotient of that Diviſion for the Denominator, but reſerve ſtill the Numerator, and ſo is the Multiplication ended.

Schol. Then ſuppoſe this Example $\frac{7}{20}$ to be multiplied by 5, and becauſe 5 will juſtly divide 20, therefore I take the Quotient of that Diviſion, which is 4, and ſet it inſtead of 20, and ſo the Fraction will be $\frac{7}{4}$ that is $1\frac{3}{4}$.

Maſt. Which is all one with $\frac{35}{20}$ that would have followed of the other way of working.

Schol. I underſtand it well.

How to multiply mixt numbers.

Maſt. Now then for the other kind, where the number is mixt, take this Rule: firſt reduce the ſaid whole number and Fraction into one improper Fraction (as I ſhewed you in Reduction) and then multiply them together, as if they were proper Fractions.

Schol. $13\frac{3}{5}$ being to be multiplied by $\frac{5}{8}$, firſt I muſt reduce the mixt number, as in this Example appeareth, by multiplying 13 by 5, and that maketh 65, whereto I muſt add the Numerator 3, and ſo the Fraction will be $\frac{68}{5}$ which two Fractions now I multiply after the uſual way, and the product will be $\frac{340}{40}$, or $\frac{34}{4}$, or $8\frac{1}{2}$.

Maſt. You have done well: and ſo you ſee, that although moſt part of the forms of Multiplication may be wrought without Reduction, yet ſome cannot, and now I ſhall not need to tell you that Multiplication is proved by Diviſion, and Diviſion likewiſe by Multiplication: but the like work that I ſhewed you in Multiplication, I will ſhew you in Diviſion.

CHAP.

CHAP. XX.

Division of Vulgar Fractions.

Mast. WHensoever two Fractions are proposed, the one to be divided by the other, I must set down first the Fraction that shall be divided (which is called the dividend) and then after it the other which is the Divisor: Then I multiply the Numerator of the dividend by the Denominator of the Divisor, and the Product I must put for a new Numerator of the Quotient. Again I multiply the Denominator of the dividend by the Numerator of the Divisor, and the Product thereof I put for the Denominator of the Quotient. And this third Fraction is the Quotient of the said Division. *Division of Fractions.*

Schol. This seemeth easie, as for Example: If I would divide $\frac{5}{8}$ by $\frac{2}{6}$, first I multiply 5, (being the Numerator of the dividend) by 6, which is the Denominator of the Divisor, and thereof riseth 30: then I multiply 8 (being the Denominator of the dividend) by 2 being the Numerator in the Divisor: and the product is 16, the Denominator of the Quotient thus: $\frac{30}{16}$.

Mast. You are quicker in understanding now, than you were when I taught you the Art of whole Numbers, but that is no wonder: for the more knowledge that a man getteth the readier he finds his wit: but yet of two things I will advise you, which you might have observed here for the ease in the work.

Whensoever you divide one Fraction by another, either they are both equal, or else the one is greater than the other: if they be equal, their Quotient shall be such that the Numerator and the Denominator of it shall be equal also. And if the two first Fractions be unequal, their Quotient shall shew the same by the inequality of the Numerator and Denominator, as in these Examples following will appear.

First, if equal Fractions as $\frac{4}{9}$, and $\frac{12}{27}$ are the one to be divided by the other, the Quotient will be $\frac{108}{108}$, as you may perceive by the Rule aforesaid.

Now in unequal Fractions, as $\frac{4}{9}$ by $\frac{12}{18}$ the Quotient will be $\frac{72}{108}$, where the Numerator is greater than the Denominator.

Schol. I see it is so: but I see not the reason why it should be so.

Note how to know the proportion between two Numbers.

Mast. The reason is this; when any Fraction is divided by another, the Quotient declareth what Proportion the dividend beareth to the Divisor. So $\frac{1}{2}$ divided by $\frac{1}{4}$ maketh 2, declaring that $\frac{1}{4}$ is contained twice in $\frac{1}{2}$.

And note this, that the Numerator in the Quotient representeth the Dividend, and the Denominator representeth the Divisor. And this is always true, whether the greater Fraction be divided by the lesser, or the lesser by the greater. But this proportion will not be exactly known, till you have learned the Art of Proportions: I have shewed something of it in the Rule of Reduction. But now for the easie remembrance of the Quotient in Division, as soon as you have set down your two Fractions the one against the other, then make a straight line for the Quotient: and as soon as you have multiplied the Numerator of the Dividend, by the Denominator of the Divisor, set the Number produced over the said line, and then multiply the other two Numbers, and set their product under the same line.

Schol. I perceive you would not have me trust to memory till I were more experienc'd, lest I happen to forget. For Example,

If I would divide $\frac{2}{3}$ by $\frac{3}{4}$, I must set the numbers one against the other (as here doth appear) and then make another line for the Quotient as you see, where I may set the Numbers of the Quotient, as soon as any of them is multiplied. So then as soon as I have multiplied 2 by 4 which maketh 8, I set that 8 over the line in the Quotient thus: And then multiply 3 by 3, which makes 9: and that 9 must be set under the line, and then will the whole Quotient appear thus $\frac{8}{9}$: whereby it appeareth (as I remember your words) that $\frac{2}{3}$ is in proportion to $\frac{3}{4}$ as 8 is to 9, but how may I know that?

$\frac{3}{4})\frac{2}{3}($
8
9

Mast. Although you might better perceive it by the Rule of Reduction, yet this Example may be declared in common Coins, as in a common Shilling of 12 Pence, of which $\frac{2}{3}$ maketh 8 Pence, and $\frac{3}{4}$ doth make 9 Pence, and so you may easily see that their Proportions do agree.

Schol. If there be whole Numbers to be divided by a Fraction, how shall I perform it?

To divide a whole Number by a Fraction.

Mast. When any whole Number shall be divided by a Fraction, you must multiply the said whole Number with the Denominator of the Fraction, and set the total thereof for the new Numerator, and for the Denominator set the Numerator of the Fraction.

Schol.

Schol. Then 20 divided by $\frac{1}{4}$ will make $\frac{80}{3}$, as here appeareth $\frac{1}{4}) \frac{20}{1} (\frac{80}{3}$

To divide the Fraction by the whole number.

Mast. Right: but if you would divide the Fraction by the whole Number, then multiply the Denominator by the whole Number and set the Product for the Denominator, without changing the Numerator.

Schol. Then to divide $\frac{20}{23}$ by 4, it will be $\frac{20}{92}$, as here appeareth. $\frac{4}{1}) \frac{20}{23} (\frac{20}{92}$

Another brief way.

Mast. You say well. And by the same Example you give me cause to remember another brief way to do the same: for if you had divided the said Numerator by 4, and set the Quotient for the Numerator, keeping still the old Denominator, it would have been not only as well done, but also in a Fraction of lesser terms.

Schol. I guess it to be in the method you taught in Multiplication: And for proof thereof $\frac{20}{23}$ being the Dividend, and 4 the Divisor, I divide the Numerator 20 by 4, and the Quotient is 5, which I set over 23, thus $\frac{5}{23}$. And I see that it is all one with $\frac{20}{92}$, as by dividing or abbreviating both these terms by 4, I may easily prove.

Mast. And if there be mixt Numbers, (either one or both) you must first reduce that mixt Number into an improper Fraction, and then work as you have learned.

Schol. That was sufficiently taught in Multiplication. Therefore I pray you go forward to some other thing.

CHAP. XXI.

The Golden Rule, or single Rule of Three Direct, in Fractions.

The Rule of proportion in Fractions.

Mast. FOR the placing of the three Numbers proposed in the Question whereby to find the fourth, and for the form of their work, with other like Notes, I refer you to that which you have already learned.

Note this for a general Rule.

But this easie method of working Fractions note, that if your three Numbers be Fractions, multiply the Numerator of the first

Number by the Denominator of the second: And that Product multiply by the Denominator of the third Number, and the Product thereof shall be the Denominator of the Answer. Then multiply the Denominator of the first Number by the Numerator of the second, and the whole thereof by the Numerator of the third, and the Product thereof shall be the Numerator of the Answer.

EXAMPLE.

A Question of Velvet. If $\frac{3}{4}$ of a Yard of Velvet cost $\frac{2}{3}$ of a Soveraign, esteemed at 20 Shillings, what shall $\frac{5}{6}$ cost?

Schol. If it please you to let me make the Answer, I would first place these three Numbers as I learned in whole Numbers, thus:

Yd.	*Sov.*	*Yd.*
$\frac{3}{4}$.	$\frac{2}{3}$: :	$\frac{5}{6}$.

And then according to your Rule, I must multiply 3, being the Numerator in the first Number, by 3, the Denominator of the second: and thereof cometh 9, which I multiply again by 6, the Denominator of the third Number, and so have 54, which is the Denominator of the Quotient. Then I multiply 4, the Denominator of the first, by 2 the Numerator of the second, and there ariseth 8, which again I multiply by 5, the Numerator of the third, and it maketh 40. Then I put 40 over 54, and it will be $\frac{40}{54}$ that is $\frac{20}{27}$ in lesser terms, and then the work will stand thus:

$\frac{3}{4}$.	$\frac{2}{3}$: :	$\frac{5}{6}$.	$\frac{40}{54}$.	

But what that is in Money I cannot tell, except I shall work it by Reduction, as you taught me.

Mast. You may reduce it when you will, but it were disorder here to mingle divers works together, where we do not seek the value of the thing in common Money, but a fourth Proportion a Fraction, which you have well done.

A Question of Silver. *Schol.* For a second Example hereof, I put this Question. If $\frac{11}{12}$ of 1 Pound weight of Silver, be worth $\frac{12}{14}$ of a Soveraign, what is, $\frac{1}{2}$ of 1 pound weight worth?

Answer $\frac{144}{88}$, Soveraign; see the work.

$\frac{11}{12}$.	$\frac{12}{14}$: :	$\frac{1}{2}$.	$\frac{144}{88}$	or $1\frac{7}{11}$

The proof of the Golden Rule. *Mast.* This Rule you understand well enough, and as for any other at this time I will not repeat, only for the proof of this Rule by Fractions, as well as whole Numbers. Multiply the first number by the fourth, and note the Product; then multiply the second by the third,

third, and mind that Product alſo. Now if thoſe two Products be equal, then is your work well done, elſe you have erred.

CHAP. XXII.

The Rule of Three Reverſe, in Fractions.

Maſt. IN the Reverſe Rule, the Queſtion is ſtated as in whole Numbers, and for the performing the work, multiply the Numerator of the firſt by the Numerator of the ſecond, and the Product thereof by the Denominator of the third, and that Product ſhall be the Numerator of the Anſwer. Then multiply the Denominator of the firſt by the Denominator of the ſecond, and that by the Numerator of the third, and that Product ſhall be the Denominator. Example. *Note this alſo for a general Rule.*

I lent my friend ¾ of a Hundred pound ſeven Months, upon promiſe that he ſhould do as much for me again; and when I would have borrowed of him he could lend me but ⅕ of a Hundred pound: now I demand how long time I muſt keep his Money in, juſt recompence of my Loan, accounting 13 Months in the Year? *A Queſtion of a Loan.*

Schol. The firſt Number muſt be the firſt Money borrowed, that is ¾ of the Hundred pounds: the ſecond Number the 7 Months, that is 7/13 of a Year: and the third Number, the Money that was lent in recompence, that is ⅕ of a Hundred: then I ſet the Numbers thus: $\frac{3}{4} \cdot \frac{7}{13} :: \frac{5}{12}$

Then (as you taught me) I multiply 3 (being Numerator of the firſt Number) by 7, the Numerator of the ſecond Number, and it maketh 21, which I multiply by 12 the Denominator of the third, and ſo have I 252 for the Numerator of the Anſwer: then I multiply 4, the Denominator of the firſt, by 13 the Denominator of the ſecond, and it produceth 52, which I multiply again by 5, the Numerator of the third, and it will make 260, that is the Denominator of the Anſwer. Then I muſt put 252 over 260, ſo will the Anſwer be $\frac{252}{260}$, or $\frac{63}{65}$.

Maſt.

Mast. Another Question I will propose, to the intent you may see thereby the reason of the Statute of Assize of Bread and Ale, which in all Statute Books, in Latin, French, and English, has been formerly much corrupted for want of knowledge in this Art; for the right understanding whereof I propose this Question.

If when the price of a Bushel of Wheat is 5 *s.* 3 *d.* the Penny White-loaf weigh 7 Ounces 1 Penny weight: What shall the Penny Loaf weigh when the price of a Bushel of Wheat is 6 *s.* 6 *d.*

The Question is stated thus:

S.	℥	S.
$5\frac{3}{12}$.	$7\frac{1}{20}$.	$6\frac{6}{12}$

And the Numbers in Fractions stand thus:

$$\frac{63}{12}. \quad \frac{141}{20}. \quad \frac{78}{12}$$

And being done according to the Rule above, the Answer is $\frac{606196}{18720}$ Ounce, or being divided as directed, in the last Case of Reduction, the value of that Fraction is, ℥ 5 : 13 *dw*: 21 *gr.*

And by this method the Calculation of the Assize of Bread mentioned in the Bills of Mortality ought to be made.

CHAP. XXIII.

The Rule of Fellowship.

Definition. *Mast.* THE Rule of Fellowship is that, by which several Partners trading in Company with different shares in a joint Stock, know how to proportion the Loss or Gain to each Partners share in the whole Stock. Whence it follows, that it is a Rule usefull in making Averages in case of Loss by Sea for the Master of Ships, whereby each Freighter knows what he is to allow for Goods thrown over-

over-board, &c. in proportion to the value of his Goods in the Veſſel. And whereby likewiſe Inſurers of Ships, Merchandize, or Houſes know how much they muſt pay to make good Loſſes by Sea or Fire, in proportion to the Summs they have inſured, &c.

This Rule is ſometimes without difference of Time, and ſometimes there is in it difference of Time. Firſt, I will ſpeak of that without difference of Time, of which let this be an example. *The Rule of Fellowſhip without time.*

Four Merchants of one Company made a Bank of Money; the firſt laid in 30 Pound, the ſecond 50 Pound, the third 60 Pound, and the fourth 100 Pound, which Stock they trade with ſo long, till they had gained 3000 Pound. Now I demand of you, what ſhould each receive at the parting of this Money. *A queſtion of company.*

Schol. I perceive that this Rule is like the other, but yet there is a difference which I underſtand not.

Maſt. Then I will ſhew it you: firſt by Addition, you muſt bring all the particular Summs of the Merchants into one Summ, which ſhall be the firſt Summ in your working by the Golden Rule, and the whole Summ of the gains by that Stock ſhall be the ſecond Summ. Now for the third Summ, you muſt ſet each mans ſhare one after another, and then work by the Golden Rule, and the fourth Summ will ſhew you each mans gain: as in example.

	30
	50
	60
	100
	240

The Stock of the four Merchants makes in one Summ 240 Pounds: ſet that in the firſt place, the gains in the ſecond, and the firſt mans portion of Stock in the third place, thus:

240. 3000 :: 30. 375

Now multiply the ſecond by the third, and it will be 90000, which divide by 240, and there will appear 375 Pounds; and that is the ſhare of the firſt man.

Now for the ſecond man ſet the 50 Pound that he brought, in the third place, and work as before: and his part will be 625 Pound.

240. 3000 :: 50. 625

Likewiſe for the third man, ſet his Money, which was 60 Pounds, and his part will be 750 Pounds, as here appeareth.

240. 3000 :: 60. 750

And ſo for the fourth man; if you ſet his Summ, which is 100 Pound, his gain will be 1250 Pound, as by the Work.

240. 3000 :: 100. 1250

Schol.

Schol. This I understand, but is there any way to examin whether I have done right or no?

Note this common proof.

Mast. For the tryal hereof, add together all their four shares of the gain, and if their Summ make the total of their gains, then is the work well done.

Schol. The four shares of the gain are as follows, which added together, make 3000, which is the just Summ of Money that they gained, whereby I know the work is well done.

	375
	625
	750
	1250
The Summ-	3000 Proof.

Mast. Now I will put you another example, of Loss; for one reason serveth for both.

A question of loss.

If three Merchants in one Ship, and in Company, had bought Merchandize, so that the first had laid out 200 Pound, the second 300 Pound, the third 500, and it chanced by tempest, that they did cast over-board into the Sea Merchandize to the value of 100 Pound; how much should each man bear in this Loss for an Average?

Schol. If I shall do in this as you did in the other Question, then I must add their three shares together, 200, 300, 500, which maketh 1000. Then I say, if 1000 lose 100, then shall 200 lose 20, and 300 shall lose 30, and 500 shall lose 50, as it doth appear plain; for,

1000. 100 :: 200. 20
1000. 100 :: 300. 30
1000. 100 :: 500. 50

Summ-100 Proof.

Mast. Well, since you have done this, I will propound a Question of more importance, which shall make you not only the abler to understand this Rule, but will also assist you in the next Rule of Fellowship with Time, if your Money be of divers Denominations.

For this must not be forgotten in all such Questions: if the number be of divers kinds, you must by Reduction bring it into one kind, that is to say, to the least value that is named in the Question. And likewise if the time be of divers kinds, as some Years, some Months, Weeks, and Days, you must make all Months, Weeks, or Days, according as the least name of time in the Question is, as for example.

A question of Sheep.

First in diversity of Money. Three Partners bought 2000 Sheep, and paid for them 241 Pound 13 Shillings 4 Pence, of which Summ one paid 101 Pound 10 Shillings. The second 82 Pound

Pound 17 Shillings 10 Pence. And the third paid 57 Pound 5 Shillings 6 Pence: How many Sheep muſt each of them have? Anſwer, The firſt ſhall have 840, the ſecond 686, and the third 474, and that you muſt work thus.

Solution. Firſt, conſidering that your Money is of divers Denominations, you muſt (by Reduction) bring it all into the ſmalleſt Denomination, that is to ſay, pence; and ſo will the total Summ be 58000 pence.

Now if you turn each mans Money into pence alſo, the firſt mans Summ will be 24360 pence: the ſecond mans Money will be 19894 pence: and the third mans Money will be 13746 pence.

Now to know how many Sheep every man ſhall have, let the whole Summ of Money, that is, 58000 pence be ſet in the firſt place, and in the ſecond place ſet the number of Sheep, and in the third place ſet each mans Money, and then by the Golden Rule there will appear the number of Sheep that each man ought to have.

	l.	*Sheep.*	*l.*	*Sheep.*
As firſt,	58000.	2000::	24360.	840
	58000.	2000::	19894.	686
	58000.	2000::	13746.	474

Summ-2000 for proof.

Here you ſee, that if 58000 pence buy 2000 Sheep, 24360 pence will buy 840 Sheep, and ſo of the reſt.

But in Averages it is moſt uſual to calculate the loſs *per Cent.* or ſo much in the 100 pound, and by that they multiply each mans ſhare, and know conſequently each mans loſs. For inſtance; *A*, *B* and *C* inſure 700 *l.* on the bottom of a Ship, and ſhe was ſo ſhatter'd by bad Weather, that it coſt 52 *l.* 10 *s.* to refit her. I demand, firſt, how much *per Cent.* will be the Average; and ſecondly, what each Inſurer is to pay of that loſs in proportion to his Summ written in the Policy, which was 300 by *A*; 250 by *B*; and 150 by *C*.

The Summ to be paid *per Cent.* is found thus.

l. *s.*

700. 52 : 10 :: 100
20

1050 Shill. } multiply
100. } 5.

700) 1050 00. (150 Answer, or 7 *l.* 10 *s.* *per Cent.*
100

700

350

00 rem.

	l.	*s.*	*d.*
So that *A* for 300 *l.* must pay 3 times 7 *l.* 10 *s.* or	22	10	00
B for 250 *l.* must pay 2 times and half 7 *l.* 10 *s.*	18	15	00
C for 150 *l.* pays once 7 *l.* 10 *s.* and ½ of it, or	11	05	00
Total-700	Total for proof--	52	10	00

A question of a Testament.

Another question shall be this.

A man divised over unto his three Sons 7851 Crowns, to be divided so, that the first Son should have ½, the second Son ⅓, and the third Son ¼, which is not possible; for ½ and ⅓ and ¼ do make $\frac{26}{24}$, or $1\frac{1}{12}$, that is, $1\frac{1}{12}$, so it is more than the whole, but reduce these Fractions into one Denomination, the least that they will come to, and they will be $\frac{6}{12}$, $\frac{4}{12}$, $\frac{3}{12}$, and so you may part the Goods into such proportion, as these three Numerators bear together, that is, the first to have 6 for every 4 to the second, and the second to have 4 as often as the third hath 3. Thus, if 13, the Total of the Numerators require 7851 Crowns, what shall 6 (the first) require, and so of the rest; it will appear their Portions will be for the first, $3623\frac{7}{13}$, for the second $2415\frac{9}{13}$, and for the third $1811\frac{10}{13}$: and these three shares added together, will make the total Summ of the whole Goods, as you may easily see in this example.

Note, That the total of the Fractions is 2, as by Addition.

	Crowns.
	$3623\frac{7}{13}$
	$2415\frac{9}{13}$
	$1811\frac{10}{13}$
Total-7851 proof.	

Another Question is this.

A

A man lying upon his Death-bed, bequeathed his Goods (which were worth 3600 Crowns) in this manner: Becauſe his Wife was great with Child, and he yet uncertain whether the Child was Male or Female, he made his Will conditionally, that if the Wife bare a Daughter, then ſhould the Wife have half his Goods, and the Daughter $\frac{1}{3}$, but if ſhe were delivered of a Son then that Son ſhould have one half of his Goods, and his Wife but $\frac{1}{3}$. Now it chanced that ſhe brought forth both a Son and a Daughter; the Queſtion is, How ſhall they part the Goods agreeable to the Teſtator's Will?

Schol. If ſome cunning Lawyers had this matter to manage, they would determine this Teſtament to be quite void, and ſo the man to die inteſtate, becauſe the Teſtament was made inſufficient, ſince this condition was not expreſſed in it, and alſo it might have chanced that ſhe ſhould have brought forth neither Son nor Daughter, as often hath been ſeen: ſo is the Will inſufficient in that point alſo.

Maſt. Such Lawyers would ſeem too cunning, and more cruel: for the mind of the Teſtator is to be taken favourably for the Legatees, when there ariſeth ſuch doubt. But let us try this work, not by force of Law, but by Proportion Geometrical, ſeeing the Teſtator did intend to provide for each of them.

Schol. If the Son ſhall have $\frac{1}{2}$ by virtue of the Teſtament, ſo muſt the Mother have $\frac{1}{3}$. Again, becauſe ſhe hath a Daughter alſo, therefore ought ſhe to have one half, and the Daughter $\frac{1}{3}$, that is both ways $\frac{11}{23}$, and $\frac{11}{32}$, which cometh to the whole Goods, and $\frac{3}{5}$ more. Wherefore it ſeemeth impoſſible.

Maſt. In this matter the mind of the Teſtator is to be underſtood, that ſuch Proportion ſhould be between the Portion of the Wife and the Son, as is between $\frac{1}{2}$ and $\frac{1}{3}$, that is, the Son muſt have $\frac{3}{5}$, for $\frac{2}{5}$ to his Mother, ſo ſhall he have 3 to 2, that is as much as his Mother, and half as much more; and the Mother muſt have the like rate in compariſon to her Daughter. Then I muſt find out three numbers in ſuch proportion, that the firſt may have as much as the ſecond, and half as much more; and the ſecond as much, and half as much as the third: ſuch numbers be 9, 6, 4.

Schol. I pray you Sir, how ſhall I find out theſe numbers?

Maſt. You have been taught that in Progreſſion, which you muſt recollect.

Schol. Then in this Example, to find the proportion of $\frac{1}{2}$ to $\frac{1}{3}$, I muſt divide (as you taught me in Diviſion of Fractions $\frac{1}{2}$ by $\frac{1}{3}$, and the Quotient will be $\frac{3}{2}$, that is $1\frac{1}{2}$; whereby I perceive that the proportion in this

To find the proportion between two numbers.

this queſtion is as 3 to 2. Therefore as you taught me in Progreſſion, I multiply 3 by 2, and the Product is 6, which muſt be the middle number: then I multiply the middle number 6 by 2, which is the leaſt term, and the Product is 12, that I divide by 3, being the greater term, and the Quotient is 4; ſo is 4 the leaſt number of the three. Then I multiply 6 by 3, whereof cometh 18, and that I divide by 2, and ſo I have 9, which is the greateſt number of the three.

Hereby I perceive then that the Son muſt not have $\frac{1}{2}$ of the Goods, neither the Mother $\frac{1}{3}$, nor the Daughter $\frac{1}{3}$, but the Goods muſt be divided into ſuch proportion, that the Son may have 9 Crowns for 6 to his Mother, and the Mother 6 Crowns for every 4 to her Daughter. Then I apply it to the Golden Rule in three examples, as followeth, and as in the laſt queſtion.

The firſt number is the Summ of thoſe three numbers 9, 6, 4: and the third is one of them ſeverally:

the ſecond is the Total of the Goods in the Will mentioned or deviſed over: and then by the work of the Golden Rule, I find out the fourth number in every work: that is, for the Son 1705 $\frac{5}{19}$, for the Mother 1136 $\frac{16}{19}$, and the Daughter 757 $\frac{17}{19}$; which numbers added together, make the Summ of the whole Goods, as may be ſeen by this example.

19. 3600 :: 9. 1705 $\frac{5}{19}$
19. 3600 :: 6. 1136 $\frac{16}{19}$
19. 3600 :: 4. 757 $\frac{17}{19}$

	1705 $\frac{5}{19}$
	1136 $\frac{16}{19}$
	757 $\frac{17}{19}$
	3600

§. 2. *Fellowſhip with Time.*

TO the intent you may as well underſtand the ſame Rule with diverſity of time, I propoſe this example.

Four Merchants made a common ſtock, which at the years end was increaſed to 35145 pound. Now to know what ſhall be each mans portion of gain, you muſt know each mans ſtock, and time of continuance.

Queſtion of a Bank. The firſt man of theſe four put in 669 *l.* which he took from the Stock again at the end of 10 Months. The ſecond man put in 810 pound, for 8 Months. The third man put in 900 pound for 7 Months. And the fourth put in 1040 for 12 Months.

Note. Aeneral Rule. Multiply each mans Stock by his time, and proceed with the three Products, as in Fellowſhip without Time, as in Queſtion the firſt, *&c.* foregoing, thus:

The

The firſt mans Summ is 669 pound, which I multiply by 10, his time, and it maketh 6690. The ſecond mans Stock 810 pound, multiplied by 8 (which was his time) maketh 6480. The third mans Summ 900 pound, multiplied by 7 (for that was his time) yieldeth 6300. The fourth mans Summ was 1040 pound, and his time 12: multiply the one by the other, and it will be 12480.

The four Summs thus multiplied by their time, muſt be ſet orderly in the third place, in the Rule of Three, and in the firſt place muſt be ſet the whole Summ of all four Products, which is 31950; and the gain muſt be in the ſecond place, which is 35145. Now to end the queſtion, I ſay firſt, if 31950 did gain 35145, what did 6690 gain? Anſwer 7359 pounds, &c. as by this work appears.

31950. 35145 :: 6690. 7359.
31950. 35145 :: 6480. 7128
31950. 35145 :: 6300. 6930
31950. 35145 :: 12480. 13728

A queſtion of loſs.

Three Partners made a joint Stock of 650 *l.* whereof the firſt put into the common Stock 200 pound for ten Months. The ſecond put in 350 pound, and the third 100 pound, but for how long the two latter is unknown: But breaking off their Partnerſhip, the firſt found himſelf a loſer 80 pound, the ſecond 56 pound, and the third 24 pound. The queſtion is, for how long time was the Money of the two latter in company.

For the Solution hereof, and of ſuch other like queſtions, you muſt multiply the firſt mans 200 pounds, that he put into the Stock by his time of continuance, which was ten Months, and it maketh 2000: wherefore now I affirm, if his Money that loſt 80 pound multiplied by his time make 2000: what ſhall his Money make that loſt 56 pound, and his that loſt 24 pound, which two numbers I commit to the trial of the Rule of Three, at two workings, thus.

If 80 give 2000, what giveth 56? And again, if 80 give 2000, what giveth 24?

80. 2000 :: 56. 1400
80. 2000 :: 24. 600

To conclude, if you now divide 1400, the ſecond mans portion, by 350, which was his Stock that he put into Company, you ſhall find in your Quotient 4 Months, and for ſo long time did the ſecond man put his Money into the common Stock.

Laſtly, if you divide 600 by 100 the third mans Stock that he put into the Company, the Quotient ſheweth his time of continuance,

ance, which was six Months. And thus is the question resolved.

Schol. Sir, I have attentively observ'd your working, and the more we do herein, the more, methinks, I am in love with this excellent Art.

A question of Canons.

Mast. Then what say you to this Question?

There is in a Cathedral Church 20 Canons, and 30 Vicars, those spend by the year 2600 pound but every Canon must have to his part 5 times so much as every Vicar hath; how much is every mans Expence say you?

Schol. I pray you make the answer your self, so shall I perceive the way to answer such like Questions.

Mast. In this Question, you must do as in those before, that have diversity of time, for here is diversity of Portions. Therefore shall you multiply the number of the persons by their difference of Portions; (as you did in the other by time.) Then must you multiply the 20, (which is the number of Canons) by 5, so will it be 100. Then 30, (that is the number of Vicars) by 1, and it will be 30: put these two Summs together, and they make 130. Then say thus; If 130 spend 2600 pounds, what does 100 spend? the Rule sheweth 2000 pounds.

Again for Vicars, if 130 spend 2600 pound, what does 30 spend? Answer 600 pound, as these Examples shew.

130. 2600 :: 100. 2000

130. 2600 :: 30. 600

And by this rule you see what the 20 Canons may spend; which Summ if you divide by 20, you shall see each Canons proportion 100 *l.* and so of the Vicars, if you divide their Summs by 30, the Quotient will declare every Vicars portion 20 *l.* and 5 times the Vicars is equal to the Canons.

CHAP.

CHAP. XXIV.

Rules of Practice.

THE Rule of Practice is that by which the value of any Quantity of any Commodity at any Rate the Pound, Ell, Yard, &c. is briefly cast up by considering what aliquot or even part or parts of a Shilling or Pound the said rate or price is. *Definition.*

In which Rule the Multiplications and Divisions are so performed mentally (the Divisors and Multipliers being always small) that the Quotients or Products are only put down.

2. Hence it follows, that you must first learn what such aliquot parts of a Shilling and Pound are; and secondly, how to divide any number mentally thereby; which done, I shall, thirdly, give you Rules to work any thing that can fall out in Practice; and fourthly, an example to each Rule.

I. *The even parts of a Shilling, and of a Pound, which may be taken mentally, without putting down the formal work of Division,* viz.

The even parts of a Shilling.

d.	*Divisors.*	
1 —	1 Twelfth	of a Shilling.
1½ —	1 Eighth	
2 —	1 Sixth	
3 —	1 Fourth	
4 —	1 Third	
6 —	1 Half	

The

The even parts of a Pound.

l.	*d.*	*Divisors.*	
0	2	one	120
0	3	one	80
0	4	one	60
0	6	one	40
0	8	one	30
1	0	one	20
1	8	one	12
2	0	one	10
2	6	one	8
3	4	one	6
4	0	one	5
5	0	one	4
6	8	one	3
10	0	one	half

} of a Pound Sterling.

But before you can divide by all these Divisors, as aforesaid, you must learn readily to multiply the 9 Digits by 12, which must be therefore got by heart, *viz.*

12 times

1	is	12
2		24
3		36
4		48
5		60
6		72
7		84
8		96
9		108

II. *To know how to divide any Summ by any of these Divisors in the even parts of a Pound or Shilling before-mentioned.*

Example 1. To take a 20th of any Summ, as suppose 6764.

Rule. Cut off the 4 in Units place of the number given, and half the 676 remaining, is the answer to your question, as in the Margent, and the 4 cut off is of the same Denomination with the number given, as if 6764 is Shillings, the 4 is so.

676 | 4

$\frac{1}{20}$ — 338*l.* 4*s.*

Schol.

Schol. Sir, I thank you for these Preparatory Rules, I believe I perfectly understand you, and will shew it by supposing the foregoing 6764 to be two-pences, which by your Table is a 120th part of a Pound: therefore I cut off 4 in the Units place, for the Cypher in 120, as you taught me, and then I say 12 in 67 is 5 and 7 over, 12 in 76 is 6, six times 12 is 72, from 76 rests 4, which being in tens place makes the 4, cut off 44 two-pences, or 7 *s.* and 4 *d.* And I know this to be true, because 56 *l.* 7: 4, is a 6th part of the *l.* 338: 4: 0, in your Example foregoing, as 2 *d.* here, is a sixth of 1 *s.* in that Example: And by the same Rule and more Easie, I can take an 80th, a 60th, *&c.* of any Number whatsoever.

$\frac{1}{120}$ of 6764

is = *l.* 56 : 7 : 4

Mast. Well then, since you understand these fundamental Rules, I will proceed to give you such particular Rules for the working any Question, as will make you an Excellent proficient in the Rules of Practice.

Number of Rules.	The price or value of any thing.	*Rules for brief Working to give the Answer in Pounds.*
	s.	
1	1	Take $\frac{1}{20}$ of the given Number.
2	2	Take $\frac{1}{10}$ of the given Number.
3	3	Add $\frac{1}{10}$ to $\frac{1}{20}$ of the given Number.
4	4	Take $\frac{1}{5}$ of the given Number.
5	5	Take $\frac{1}{4}$ of the given Number.
6	10	Take $\frac{1}{2}$ the given Number.
7	6, 8, 12, 14	For any even Number of Shillings, Multiply what the Number given would come to at two Shillings, by the Number of two Shillings contained in the price given.
8	7, 9, 11, 13	For any odd Number of Shillings (not before mentioned) work first for the Even Number, next Less than the odd one given, as taught, and to that Quotient add $\frac{1}{20}$ of the given Numb. Except for 15 *s.* for which see Rule 84.

Number of Rules.	The value or price of any thing.	*Rules for brief working to give the Answer in Pounds.*
	d.	
9	2	Take $\frac{1}{120}$ of the given Number.
10	3	Take $\frac{1}{80}$ of the given Number.
11	4	Take $\frac{1}{60}$ of the given Number.
12	6	Take $\frac{1}{40}$ of the given Number.
13	8	Take $\frac{1}{30}$ of the given Number.
14	1	Take $\frac{1}{20}$ of $\frac{1}{12}$ of the given Number.
15	5	Take $\frac{1}{8}$ of $\frac{1}{6}$ of the given Number.
16	7	Add $\frac{1}{60}$ to $\frac{1}{80}$ of the given Number.
17	9	To $\frac{1}{40}$ of the given Number add $\frac{1}{2}$ the 40th.
18	10	Take $\frac{1}{8}$ of $\frac{1}{3}$ of the given Number.
19	11	Add $\frac{1}{80}$ to $\frac{1}{30}$ of the given Number.
	fa.	
20	1	Take $\frac{1}{80}$ of $\frac{1}{12}$ of the given Number.
21	2	Take $\frac{1}{80}$ of $\frac{1}{6}$ of the given Number.
22	3	Take $\frac{1}{80}$ of $\frac{1}{4}$ of the given Number.

Having thus shewed you the briefest way of finding the value of any Quantity of Merchandize, when the price of the Unit is any Number of Shillings, Pence or Farthings; I proceed to shew you how to work when the price of a Unit is,

1. Shillings and Pence that are the Even parts of a Pound, and from thence,

2. Shillings, and Pence that are not an Even part of a Pound.

3. Pence and Farthings.

4. Pounds, Shillings, Pence and Farthings.

	s.	d.	
23	1	8	Take a Tweltfth of the given Number.
24	2	6	Take $\frac{1}{8}$ of the given Number.
25	3	4	Take $\frac{1}{6}$ of the given Number.
26	6	8	Take $\frac{1}{3}$ of the given Number.

Shillings

Rules for brief working to give the Answer in Pounds.

Shillings and Pence that are no even part of Pound, viz.

Numb. of Rules.	The price or value of any thing. s.	d.	
27	1	1	To $\frac{1}{20}$ of the given Number, add $\frac{1}{12}$ of that Twentieth.
28	1	2	To $\frac{1}{20}$ of the given Number, add $\frac{1}{6}$ of that 20th.
29	1	3	Take $\frac{1}{4}$ of $\frac{1}{5}$ of the given Number.
30	1	4	Take $\frac{1}{3}$ of $\frac{1}{5}$ of the given Number.
31	1	5	From $\frac{1}{12}$ take $\frac{1}{80}$ of the given Number.
32	1	6	To $\frac{1}{30}$ of the given Number, add $\frac{1}{5}$ that 20th.
33	1	7	From $\frac{1}{12}$ of the given Num. take $\frac{1}{80}$ of that 12th.
34	1	9	From $\frac{1}{15}$ of the given Number take $\frac{1}{80}$ thereof.
35	1	10	From $\frac{1}{15}$ of the given Number take $\frac{1}{120}$ thereof.
36	1	11	Add $\frac{1}{80}$ to $\frac{1}{12}$ of the given Number.
37	2	1	To $\frac{1}{12}$ of the given Number, add $\frac{1}{8}$ of that 12th.
38	2	2	To $\frac{1}{10}$ of the given Number, add $\frac{1}{120}$ thereof.
39	2	3	To $\frac{1}{10}$ of the given Number, add $\frac{1}{8}$ of that 10th.
40	2	4	To $\frac{1}{10}$ of the given Number, add $\frac{1}{5}$ of that 10th.
41	2	5	From $\frac{1}{8}$ of the given Number, take $\frac{1}{80}$ of that 8th.
42	2	7	To $\frac{1}{8}$ of the given Number, add $\frac{1}{80}$ of that 8th.
43	2	8	To $\frac{1}{10}$ add $\frac{1}{80}$ of the given Number.
44	2	9	To $\frac{1}{8}$ add $\frac{1}{80}$ of the given Number.
45	2	10	To $\frac{1}{8}$ add $\frac{1}{10}$ of the given Number.
46	2	11	To $\frac{1}{8}$ of the given Number, add $\frac{1}{8}$ of that 8th.

Note, That where the price of the Unit is thus valuable, as half a Crown, or three Shillings; less than half a third or fourth part of a Shilling, are seldom or never taken notice of in Bargains: I shall therefore go no farther with 1 *d.* advance.

47	3	3	From $\frac{1}{3}$ of the given Number take $\frac{1}{20}$ of that 6th.
	3	4	See Rule the 25th.
48	3	6	To $\frac{1}{3}$ of the given Number, add $\frac{1}{30}$ of that 6th.
49	3	8	To $\frac{1}{10}$ of the given Number, add $\frac{1}{12}$ thereof.
50	3	9	From $\frac{1}{3}$ of the given Number take $\frac{1}{80}$ thereof.
51	4	3	To $\frac{1}{3}$ add $\frac{1}{16}$ of the given Number.

Rules for brief working to give the Answer in Pounds.

Numb. of Rules.	The price or value of any thing. s.	d.	
52	4	4	To $\frac{1}{3}$ add $\frac{1}{20}$ of the given Number.
53	4	6	To $\frac{1}{10}$ add $\frac{1}{8}$ of the given Number.
54	4	8	To $\frac{1}{30}$ add $\frac{1}{3}$ of the given Number.
55	4	9	From $\frac{1}{4}$ take $\frac{1}{16}$ of the given Number.
56	5	3	To $\frac{1}{4}$ of the given Number, add $\frac{1}{16}$ of that 4th.
57	5	4	To $\frac{1}{10}$ add $\frac{1}{6}$ of the given Number.
58	5	6	To $\frac{1}{4}$ add $\frac{1}{40}$ of the given Number.
59	5	8	To $\frac{1}{4}$ add $\frac{1}{30}$ of the given Number.
60	5	9	From $\frac{3}{10}$ (as for 6 *s. per* Rule 7th) take $\frac{1}{80}$ of the N. given.
61	6	3	To $\frac{1}{4}$ of the given Number, add $\frac{1}{4}$ of that fourth.
62	6	4	From $\frac{1}{3}$ take $\frac{1}{60}$ of the given Number.
63	6	6	Add $\frac{1}{8}$ to $\frac{1}{3}$ of the given Number.
	6	8	See Rule 26th.
64	6	9	To $\frac{3}{10}$ (as *per* Rule, 7) add $\frac{1}{8}$ of those tenths.
65	7	3	To $\frac{1}{16}$ add $\frac{1}{4}$ and $\frac{1}{80}$ of the given Number.
66	7	4	To $\frac{1}{3}$ add $\frac{1}{8}$ of the given Number.
67	7	6	To $\frac{1}{4}$ add $\frac{1}{8}$ of the given Number.
68	7	8	To $\frac{1}{30}$ add $\frac{1}{8}$ of the given Number.
69	7	9	From $\frac{4}{10}$ (as *per* Rule 7) take $\frac{1}{80}$ of the given Number.
70	8	3	To $\frac{4}{10}$ (as *per* Rule 7) add $\frac{1}{80}$ of the given Number.
71	8	4	To $\frac{1}{4}$ add $\frac{1}{8}$ of the given Number.
72	8	6	To $\frac{6}{10}$ (as *per* Rule 7) add $\frac{1}{8}$ of the given Number.
73	8	9	From $\frac{1}{3}$ the given Number take $\frac{1}{8}$ of that half.
74	9	3	To $\frac{1}{10}$ put twice down; add $\frac{1}{4}$ and $\frac{1}{80}$ of the given Numb.
75	9	4	To $\frac{3}{10}$ (as *per* Rule 7) add $\frac{1}{8}$ of the given Number.
76	9	6	From $\frac{1}{3}$ the given Number take $\frac{1}{10}$ of that half.
77	9	8	From $\frac{1}{3}$ the given Number take $\frac{1}{30}$ of that half.
78	9	9	From $\frac{1}{3}$ the given Number, take $\frac{1}{80}$ thereof.

Note, That notice being seldom or never taken of less than Six-pence in bargains where the price of the Unit is 10 *s.* or upward: I shall therefore now shew how to work for any number of Shillings and 6 *d.*

that

Rules for brief working to give the Answer in Pounds.

that are above 10*s*. I having already shewed in Rule 7 and 8, how to work for any number of Shillings.

Numb. of Rules.	The price or value of any thing. *s.*	*d.*	
79	10	6	To ½ the given Number, add $\frac{1}{20}$ of that half.
80	11	6	From $\frac{6}{10}$ (as *per* Rule 7) take $\frac{1}{10}$ of the given Number.
81	12	6	To ½ the given Number, add ¼ of that half.
82	13	6	From $\frac{1}{10}$ of the given Numb. (as *per* Rule 7) take $\frac{1}{20}$ thereof.
83	14	6	To $\frac{6}{10}$ (as *per* Rule 7) add $\frac{1}{5}$ of the given Number.
84	15	00	From the given Number take ¼ thereof.
85	15	6	From $\frac{1}{10}$ (as *per* Rule 7) take $\frac{1}{20}$ of the given Number.
86	16	6	To $\frac{1}{10}$ (as *per* Rule 7) add $\frac{1}{20}$ of the given Number.
87	17	6	From the given Number take ⅛ thereof.
88	18	6	To $\frac{2}{10}$ of the given Number, add $\frac{1}{20}$ thereof.
89	19	6	From the given Number take $\frac{1}{20}$ thereof.

How to work for any Number of Pence and Farthings (that may probably be the value of a Unit of any Commodity) follows.

	d.	*qr.*	
90	1	1	Add $\frac{1}{80}$ of the given Numb. to $\frac{1}{12}$ of that 80th.
91	1	2	Take $\frac{1}{20}$ of ⅛ of the given Number.
92	1	3	From $\frac{1}{120}$ of the given Numb. take ⅛ of that 120th.
93	2	1	To $\frac{1}{120}$ of the given Numb. add ⅛ of that 120th.
94	2	2	Take ⅛ of $\frac{1}{12}$ of the given Number.
95	2	3	From $\frac{1}{80}$ of the given Number take $\frac{1}{12}$ of that 80th.
96	3	1	To $\frac{1}{80}$ of the given Number add $\frac{1}{12}$ of that 80th.
97	3	2	To $\frac{1}{80}$ of the given Number add ⅛ of that 80th.
98	3	3	To $\frac{1}{80}$ of the given Number add ¼ of that 80th.
99	4	1	To $\frac{1}{120}$ of the given N°. twice put down, add ⅛ of a 120th.
100	4	2	To $\frac{1}{60}$ of the given Numb. add ⅛ of that 60th.
101	4	3	To $\frac{1}{10}$ of the given Numb add ½ that 80th. and ¼ that ½.
102	5	1	From $\frac{1}{20}$ of the given Number take ⅛ of that 40th.
103	5	2	From $\frac{1}{40}$ of the given Number take $\frac{1}{12}$ of that 40th.
104	5	3	To $\frac{1}{80}$ of the given N° add $\frac{1}{120}$ thereof and ¼ of the 80th.
105	6	1	To $\frac{1}{80}$ of the given N° (twice put down) add $\frac{1}{12}$ of an 80th.
106	6	2	To $\frac{1}{40}$ of the given Numb. add $\frac{1}{12}$ of that 40th.

To

Numb. of Rules	The price or value of any thing		*Rules for brief working to give the Answer in Pounds.*
	d.	*q*	
107	6	3	To $\frac{1}{20}$ of the given N°. add $\frac{1}{8}$ of that 40th.
108	7	1	To $\frac{1}{20}$ of the given N°. add $\frac{1}{8}$ of that 40th and $\frac{1}{4}$ of that 6th.
109	7	2	To $\frac{1}{20}$ of the given N°. add $\frac{1}{4}$ of that 40th.
110	7	3	To $\frac{1}{20}$ add $\frac{1}{80}$ of the given N°. and $\frac{1}{4}$ of that 80th.
111	8	1	To $\frac{1}{20}$ add $\frac{1}{120}$ of the given N°. and $\frac{1}{8}$ of that 120th.
112	8	2	To $\frac{1}{20}$ of the given N°. twice put down add $\frac{1}{8}$ of a 60th.
113	8	3	To $\frac{1}{20}$ add $\frac{1}{120}$ of the given N°. and $\frac{1}{8}$ of that 40th.
114	9	1	To $\frac{1}{20}$ of the given N°. add $\frac{1}{2}$ the 40th. and $\frac{1}{12}$ of the half.
115	9	2	To $\frac{1}{20}$ of the given N°. add $\frac{1}{8}$ of the 40th and $\frac{1}{6}$ of that half.
116	9	3	To $\frac{1}{40}$ of the given N° add $\frac{1}{2}$ the 40th. and $\frac{1}{4}$ of that half.
117	10	1	To $\frac{1}{20}$ of the given N°. add $\frac{1}{4}$ of the 20th and $\frac{1}{8}$ of that 4th
118	10	2	From $\frac{1}{20}$ of the given N°. take $\frac{1}{8}$ of that 20th.
119	10	3	To $\frac{1}{40}$ add $\frac{1}{80}$ of the given N°. and $\frac{1}{8}$ of the 40th.
120	11	1	To $\frac{1}{20}$ add $\frac{1}{80}$ of the given N°. and $\frac{1}{12}$ of that 80th.
121	11	2	To $\frac{1}{20}$ of the given N°. add $\frac{1}{80}$, and $\frac{1}{4}$ of that 80th.
122	12	1	To $\frac{1}{12}$ of $\frac{1}{20}$ of the given N°. add $\frac{1}{20}$ thereof.
123	12	2	To $\frac{1}{40}$ of the given N°. (twice put down) add $\frac{1}{12}$ of a 40th.
124	12	3	To $\frac{1}{40}$ of the given N°. (twice put down) add $\frac{1}{8}$ of a 40th.
125	13	1	To $\frac{1}{20}$ of the given N°. add $\frac{1}{12}$ of that 20th.
126	13	2	To $\frac{1}{20}$ of the given N°. add $\frac{1}{8}$ of that 20th.
127	13	3	To $\frac{1}{20}$ of the given N°. add $\frac{1}{8}$ of that 20th and $\frac{1}{8}$ of that 8th.
128	14	1	To $\frac{1}{20}$ of the given N°. add $\frac{1}{8}$ of that 20th and $\frac{1}{8}$ of the 6th.
129	14	2	To $\frac{1}{20}$ of the given N°. add $\frac{1}{8}$ of that 20th and $\frac{1}{4}$ of that 6th.
130	14	3	From $\frac{1}{20}$ of the given N°. & $\frac{1}{4}$ of that 20, take $\frac{1}{12}$ of that 4th.
131	15	1	To $\frac{1}{20}$ of the given N°. add $\frac{1}{4}$ of that 20, and $\frac{1}{12}$ of that 4th.
132	15	2	To $\frac{1}{20}$ of the given N°. add $\frac{1}{4}$ of that 20, and $\frac{1}{8}$ of that 4th.
133	15	3	To $\frac{1}{20}$ of the given N°. add $\frac{1}{4}$ of that 20, and $\frac{1}{4}$ of that 4th.
134	16	1	To $\frac{1}{8}$ of $\frac{1}{2}$ the given N°. add $\frac{1}{12}$ of that 8th.
135	16	2	To $\frac{1}{8}$ of $\frac{1}{2}$ the given N°. add $\frac{1}{16}$ of that 8th.
136	16	3	From $\frac{1}{12}$ take $\frac{1}{80}$ of the given N° and $\frac{1}{12}$ of that 80th.
137	17	1	From $\frac{1}{20}$ of the given N°. and $\frac{1}{8}$ of that 20th (twice put down) take $\frac{1}{4}$ of one of those 4ths.
138	17	2	From $\frac{1}{20}$ of the given N°. and $\frac{1}{8}$ that 20, take $\frac{1}{12}$ of that half.
139	17	3	From $\frac{1}{20}$ of the given N°. (and $\frac{1}{4}$ of that 20 twice put down) take $\frac{1}{12}$ of one of those fourths. Note,

Note, That after the price of any thing exceeds this Rate, there is seldom or never any notice taken of Farthings: so that I have now done with my Rules, only observe,

That when the price of a Unit of any thing is Pounds and Shillings: work for the Shillings, as taught before, and Multiply the given Number by the Pounds, adding the Product to the Quotient; as 3764 Yards of Broad Cloath at 26 *s.* *per* Yard.

Add { 3764 Yards or Pounds
l. 1129 : 4 *s.* at 16 *s.*

Answer *l.* 4893 : 4 : 00

Second Example.

147 hundred at 5 *l.* 5 *s.* *per* Hundred.

147 } Multiply
5 }

l. 735
l. 36 : 15

Answ. *l.* 771 : 15

Note, That if there had been 2 Quarters of a hundred more, you must have added $\frac{1}{2}$ of 5 *l.* 5 *s.* &c. but this is so easie that I need give you no more Examples.

Scho. But suppose the price of any thing is Pounds, Shillings, Pence and Farthings, how shall I know the value of many of those things?

Mast. This Question, and such like, I know are sometimes proposed by some Masters in their Rules of Practice, but there is never any occasion for them in the way of Trade; for where the value of a Unit (as I said before) is above twenty Shillings, they seldom take notice of less than a Shilling in the Advance; sometimes indeed of 6 pence, but never of less in things of that value, and you having learnt how to work by the Rules foregoing for any number of Pounds and Shillings, and Shillings and 6 pences (and even for your Example above any number of Pounds, Shillings, Pence and Farthings

things, by comparing the Rules foregoing together.) I ſhall therefore (having given more particular Rules for all prizes, than any I know extant) proceed to hear *1ſt*, your Inferences from, and *2dly*, Your Objections againſt the foregoing Rules, and endeavour to ſatisfie you therein before I conclude this Rule of Practice.

Schol. The Inferences I make from the foregoing Rules, and from what you have ſince declared is;

1. That there can be no price of any Commodity but it will fall within one of your Rules, which muſt needs be very uſefull.

2. That you have ſhewed how to work for any even Number of Shillings under 20, or for 2, 3, 4, 6, and 8 Pence, or Shillings and Pence that are even parts of a Pound at one working (or Diviſion:) Alſo for any odd Number of Shillings, or Shillings and Pence; or for any Number of Farthings under 4, or for any number of Shillings and Pence; or Pence and Farthings under $4\frac{3}{4}d.$ at two Diviſions.

3. That for any other Number of Pence and Farthings which can be ſuppoſed to be the value of any Commodity you have ſhewed to perform the work by three Diviſions, and ſometimes by two.

3. I obſerve, that all your Diviſions are ſo ſmall that they may be eaſily divided by in the Mind, as Digits, without putting down any Work, or greatly charging the Memory. But perhaps ſome may make theſe

OBJECTIONS.

Obj. 1. That the Rules are ſo numerous that they cannot be carried well in the Mind without much trouble and practice.

Obj. 2. That your Fractions, and Fractions of Fractions, are difficult to be underſtood, eſpecially to know what parts of Coin they repreſent, which unleſs well known it will be impoſſible to perform the work.

Obj. 3. That the Remainers are not only very great where the Diviſors are ſo, and conſequently hard to be caſt up, but in caſe of Fractions of Fractions they are not eaſily diſcovered, as to their Denominations.

GENERAL ANSWER.

Maſt. As to what Inferences you have made, they are true enough accounting the taking $\frac{1}{8}$ of any thing to be no Diviſion, as indeed, it cannot

cannot be ſaid to be any Work, becauſe it is performed by a Daſh, as 3740 divided by 10 is 374, 2974 divided by 10 quotes 297 $\frac{4}{10}$ as has been taught before.

2. And as for the Objections you have mentioned, they are no more than what are made againſt the Rules of Practice in general, and chiefly by ſuch as do not rightly underſtand them; in behalf of which Rules I ſhall therefore give the Objections diſtinct Anſwers.

Anſw 1. That the Rules are numerous is true, but I take it to be much better to give a Learner a Rule for every particular price that can fall in his way: than to tell him in general he ſhall work ſo and ſo for Shillings and Pence, and ſo for Pence and Farthings, *&c.* which is a moſt difficult task for him to apply to the beſt advantage in all caſes, and next to impoſſible without ſuch particular Rules: And as to bearing them all in mind it is not required for two Reaſons; 1. Becauſe no one Tradeſman deals in Commodities of all prizes, eſpecially in a whole-ſale-way, for which theſe Rules are intended, and therefore needs only to mind a few of the Rules which relate to the price of thoſe Goods which he deals in. 2. That even of thoſe few it is not neceſſary he ſhould get them by heart, for he may lay the Book before him, it being portable enough either for Buſineſs at home or abroad.

Anſw. 2. The Fractions are very eaſie, all or moſt having only a Unit for the Numerator, eſpecially conſidering they are placed after the Teaching the whole Art of Broken Numbers: And as to knowing what parts of Coin they repreſent that is not difficult, if it be conſidered,

1. That the deſign of every working is to give the Anſwer in Pounds, and therefore whereſoever it is directed to take ſuch a part of the given Number. If 20 Shillings be divided by the Denominator of that Fraction, the Quotient will ſhew what parts of Coin your Fraction repreſented: as in Rule 54, it is ſaid, Take $\frac{1}{30}$ and $\frac{1}{5}$ of the given Number. Now 20*s.* divided by 30 gives 8 Pence, and 20*s.* divided by 5 gives 4 Shillings, which ſhews that you firſt wrought for 8*d.* and then for 4*s.* and that for proof is the price to Rule the 54th.

2. The value of the Fraction of a Pound being once known, the value of any Fraction of that Fraction is found by the ſame method as before; *for inſtance*,

In Rule 48 it is ſaid, To $\frac{1}{6}$ of the given Number add $\frac{1}{20}$ of that 6th; now to know what parts of Coin I work for, $\frac{1}{6}$ of 20*s.* is 3*s.*

and 4*d.* as by the Table of even parts of a Pound, which being 40*d.* a 20th thereof is 2*d.* which shews I work'd first for 3*s.* 4*d.* and then for 2*d.* which together makes 3*s.* 6*d.* as appears in the price to the said Rule; all which is easily done without either Pen or Ink by any one that pretends to learn Practice; and so much for the Rules to know what you work for, which is not difficult.

Answ. 3. Lastly, as this Objection has two parts I shall answer them distinctly; as,

1. That the Remainers are great where the Divisors are so: I answer, They are never above 120, for that is the greatest Divisor, than which the Remainers can never be greater (as you have learnt in Division) and how easie is it to know (tho' a Man had never learnt in Division) how many Shillings and Pence are in any number of 2 Pences, 3 Pences, Groats, 6 Pences, or 8 Pences under 20*s.* I need not to mention: for Example, 3765 Pounds of any thing at 2*d.* amounts to 31*l.* 7*s.* 6*d.* for $\frac{1}{120}$ part of 3765 is 31*l.* and 45 Two Pences remaining, to know how many Shillings are in which 45, I divide it in my mind by 6, the 2 Pences in a Shilling, and the Quotient is 7*s.* and 3 two pences or 6 pence remains. All this is done with ease; or if the remainer is near 120 deduct the 2 pences it wants and the remainer is the Answer in Shillings and Pence, as if the Remainer were 113 two Pences, that wants but 7 or 14*d.* of 20 Shillings, therefore 14*d.* from 20 Shillings leaves 18*s.* 10*d.* and that is the value of 113 two pences.

2. As to the discovering the Denomination of the Remainers, whether they are Pounds or even parts of a Pound, *i. e.* Whether you are to take the Fraction immediately of a Pound or the Fraction of the Fraction of a Pound; you have nothing to observe but the general Rule given in Division: *That the Remainers are always of the same name with the Dividend.* For instance (for an Example or two will make the whole Answer to this Objection more obvious.) In Rule 138 for any thing at 17$\frac{1}{4}$*d.* per Pound, Ell, Yard, *&c.* it is directed from $\frac{1}{20}$ of the given Number, and $\frac{1}{4}$ of that 20th twice put down) to take $\frac{1}{4}$ of one of those fourths: As to know the value of 3765 pounds of Cot-

3765 at 17$\frac{1}{4}$*d.*

$\frac{1}{20}$ is—188*l.* 5 : 0 at 1*s.*
$\frac{1}{4}$ of that 20th is—47 : 1 : 3 at 3*d.*
again = 47 : 1 : 3 at 3

Summ—282 : 7 : 6 at 18*d.*
$\frac{1}{4}$ of $\frac{1}{4}$ deduct—11 : 15 : 3$\frac{3}{4}$ at 3$\frac{3}{4}$*q.*

Answer—270 : 12 : 2$\frac{1}{4}$

ton

ton at 17¼ *d.* I take in this Caſe for 1 *s.* a 20th part of the given Number, which is 188 *l.* and 5 remains (which is 5 *s.* becauſe I work for 12 *d.* ſo that the Dividend is ſuppoſed to be Shillings.

Then I take ¼ of that 20th, that is, I work for 3 *d.* for that is ¼ of a Shilling; as ¼ of 18 is 4 and 2 remains, ¼ of 28 is 7, ¼ of 5 *s.* is 1 *s.* and 1 *s.* remains, ¼ of which is 3 *d.* which is 47 *l.* 1 *s.* 3 *d.*

That put down again and added to 188 *l.* 5 *s.* the Summ is 282 *l.* 7 *s.* 6 *d.* which 3795 would come to at 18 *d.* but our Example being 17¼ *d.* I am to deduct 3765 three Farthings,

And 3 Farthings being ¼ of 3 *d.* I take therefore ¼ of 47 *l.* 1 *s.* 3 *d.* as 4 in 4 is 1, 4 in 7 once and 3 remains (which by the Rule is 3 *l.* becauſe the 47 is ſo) 3 *l.* is 60 *s.* and the 1 is 61 *s.* ¼ of which is 15 *s.* and 1 *s.* remains or 12 *d.* and the 3 *d.* is 15 *d.* ¼ of which is 3 *d.* and 3 *d.* remains, or 12 *qrs.* ¼ of which is 3 Farthings; which 11 *l.* 15 *s.* 3¾ deducted from 282 *l.* 7 *s.* 6 *d.* the remainer is 270 *l.* 12 *s.* 2¼ the Anſwer as by the Work in the Margent foregoing.

And to make you fully underſtand the manner of working by the foregoing Rules, I will work one other Example which ſhall likewiſe be one of the moſt difficult to perform. As,

What coſt 3765 Yards of Cloath at 13¼ *per* Yard.

In this Example according to Rule the 126, $\frac{1}{20}$ of 3765 is 188 *l.* 5 *s.* $\frac{1}{12}$ of which is 15 *l.* and 8 *l.* remains or 160 *s.* (multiplying the 8 by 20) to which 160 adding the 5 *s.* makes 165, $\frac{1}{12}$ of which 16 is 1 and 4 remains, $\frac{1}{12}$ of 45 is 3 (or 13 *s.*) and there reſts 9 *s.* or 9 *d.* for if you multiply it by 12 and divide it again by 12 it will come to the ſame;

376 5 at 13¼ *d.*

l.	*s.*	*d.*	
188	05	0	at 12 *d.*
15	13	9	at 1 *d.*
3	18	5¼	at 1 *qr.*
207	17	2¼	at 13¼

And ¼ of the 15 *l.* 13 *s.* 9 *d.* is 3 *l.* 18 *s.* 5¼ for ¼ of 15 *l.* is 3 *l.* and 3 *l.* remains or 60 *s.* and the 13 makes 73 *s.* ¼ of which is 18 and 1 *s.* remains and the 9 *d.* is 1 *s.* 9 *d.* or 21 *d.* 4 in 21 is 5 *d.* and 1 *d.* or 4 Farthings remains, ¼ of which is 1 Farthing, which 3 Numbers added together, as in the Example before, the Summ is 207 *l.* 17 *s.* 2¼ for Anſwer.

Schol. Now I think I underſtand the Rule of Practice as you ſhall ſee by this Example

	l.	d.
What coſt 8769 at	4	$\frac{3}{4}$ *per* lb.

$\frac{1}{80}$ of the given Number is—109 : 12 : 3 at 3 *d.*
$\frac{1}{3}$ of that 80th is—54 : 16 : 1 $\frac{1}{2}$ at 1 $\frac{1}{2}$.
$\frac{1}{8}$ of that half is—9 : 02 : 8 $\frac{1}{4}$ at 0 $\frac{3}{4}$.

The Sum of all is the Anſwer—173 : 11 : 00 $\frac{3}{4}$ at 4*d.* $\frac{3}{4}$.

Maſt. I ſee you underſtand theſe Rules, and I will therefore ſhew you farther how to work Queſtions of Simple Intereſt by Practice at any Rate *per Cent.*

Rules for the ſpeedy working Queſtions of Intereſt, Commiſſion to Factors, Cuſtom, or the like, for which ſo much is paid per Cent.

Per Cent or 100	
1	Take $\frac{1}{100}$ of the given Number. *Vid.* The Examples following.
2	Take $\frac{1}{50}$ of the given Number.
2 $\frac{1}{2}$	Take $\frac{1}{40}$ of the given Number.
3	Take the Summ of $\frac{1}{33}$ of the given Number and $\frac{1}{3}$ that 50th.
4	Take $\frac{1}{25}$ of the given Number and put it down twice, and the Summ is the Anſwer.
5	Take $\frac{1}{20}$ of the given Number.
6	Take $\frac{1}{20}$ of the given Number and $\frac{1}{3}$ that 20th.
7	Add $\frac{1}{25}$ to $\frac{1}{50}$ of the given Number.
8	Multiply $\frac{1}{25}$ of the given Number by 4.

Here follows IX *Examples to the* 9 *Rules foregoing.*

Example 1.

At 1 *per Cent.* what is 14 | 73 : 04 : 9 $\frac{1}{4}$

$\frac{1}{100}$ of the given Number is—14 : 14 : 7 $\frac{3}{4}$.

Ex.

Example 2.

	l.	*s.*	*d.*
At 2 *per Cent.* what is 147	3	04	9¼
$\frac{1}{50}$ of the given Number is—	29	09	3½

Example 3.

	l.	*s.*	*d.*
At 2½ *per Cent.* what is 147	3	04	9¼
$\frac{1}{40}$ of the given Number is—	36	16	7¼

Example 4.

	l.	*s.*	*d.*
At 3 *per Cent.* what is 147	3	04	9¼
$\frac{1}{50}$ of the given Number—	29	09	3½
½ of that 50th is—	14	14	7¾
Summ, or Answer	44	03	11¼

Example 5.

	l.	*s.*	*d.*
At 4 *per Cent.* what 147	3	04	9¼
$\frac{1}{50}$ of the given Number—	29	09	3½
that again, add	29	09	3½
Summ, or Answer—	58	18	7

Example 6.

	l.	*s.*	*d.*
At 5 *per Cent.* what is 147	3	04	09¼
$\frac{1}{20}$ of the given Number is—	73	13	02¼

Exa-

Example 7.

	l.	s.	d.
At 6 *per Cent.* what is 147	3	04	09 ¼
$\frac{1}{20}$ of the given Number is—	73	13	02 ¾
$\frac{1}{5}$ of that 20th is—	14	14	07 ¾
Summ, or Anſwer—	88	07	10 ½

Example 8.

	l.	s.	d.
At 7 *per Cent.* what is 147	3	04	09 ¼
$\frac{1}{30}$ of the given Number is—	29	09	03 ½
$\frac{1}{35}$ of the given Number is—	73	13	02 ¾
Summ, or Anſwer—	103	02	06 ¼

Example 9.

	l.	s.	d.
At 8 *per Cent.* what is 147	3	04	09 ¼
$\frac{1}{30}$ of the given Number is—	29	09	03 ½
Multiply by—			04
and the Product or Anſwer is—	117	17	02

Schol. I thank you, Sir, for theſe excellent Rules and Examples, whereby I perceive you have done that in one Line which would require near 20 to do it by the Common way of Multiplication and Diviſion.

Maſt. This way is much ſhorter than any other, except by Decimals, which I ſhall not treat on at preſent, but proceed to ſhew you ſome other Rules relating to Merchants Buſineſs and then conclude the Book with Alligation and the Rules of Falſe Poſition.

CHAP.

CHAP. XXV.

Exchange of Coin.

EXchange of Coin is when one Merchant pays a Summ of Money to another for his Bill drawn on his Correſpondent, to pay the like Value to the firſt Merchant's Order, or Correſpondent in another Country; as if *A.* at *London* has 100 *l.* Sterling to pay at *Rotterdam*, he pays it to *B.* at *London*, for his Bill drawn on *C.* (*B*'s Correſpondent) who is to pay the like value to *D.* who is *A*'s. Correſpondent at *Rotterdam*. *Definition.*

2. Now becauſe the value of Money differs in the Courſe of Exchange, as influenced by the Courſe of Trade and other Circumſtances: as ſometimes the *Engliſh* Pound in the foregoing inſtance, would be valued at 34 *l. Flemiſh* and upward, ſometimes at 32 or leſs; therefore is it prudence in both parties Exchanging to make their Terms as well as they can, the Receiver for his Credit, and both for their Advantages.

3. And becauſe it cannot be known by any Merchant whether he gains or loſes by Exchange, unleſs he firſt knows the Parr (or true value) of the Money he Exchanges for: therefore is a Table of ſuch values very neceſſary, and which take as follows:

A Table ſhewing the value in Engliſh *Coin of all ſuch Foreign Money, as in the Courſe of Exchange is given for Pounds, Shillings or Pence Sterling.*

	Sterling. *l.* *s.* *d.*
1 Pound *Flemiſh* or Twenty Shillings, each ſix Stivers.	00: 12: 00
33 *s.* 4 *d.* Flemiſh is	1: 00: 00
1 Crown *French*, which contains 3 Livers (or *French* Pounds) each Livre 20 Soulze, *&c.* and each Soulze, *&c.* (or Shillings) 12 Deniers.	00: 04: 06

1 Piece

1 Piece of 8, *Mexico*	00:	04:	06:	$1\frac{1}{4}$
Sevil	00:	04:	06:	$1\frac{3}{16}$
Piller	00:	04:	06:	$3\frac{1}{16}$
Peru	00:	04:	03:	$1\frac{3}{16}$
1 *Millree* containing 1000 Rees is	00:	06:	08:	2
1 Ducat *De Banko*	00:	04:	04:	0

Schol. Upon this Table, and Diſcourſe preceeding, I have ſeveral Queries to make which I am unſatisfied in. As

1. What do you mean by Sterling?

Maſt. I mean Engliſh Money of 11 ℥, 2 *dw.* fine, and it is called Sterling, becauſe it was firſt Coin'd in *England* of that Fineneſs by *Eſterlings*, *i. e.* Perſons ſent for from the *Eaſterly* Parts of *Germany* by King *Edward* III. in the 20th. Year of his Reign for that purpoſe, as moſt ſay; but Mr. *Lounds* in his *Report* to the Lords of the Treaſury, *containing an Eſſay for the amendment of the Silver Coin of* England, ſays the word *Sterling* is only a Synonimous Term; (page 17) of his ſaid Book.

For you muſt know that the value of all Coins, are diſtinguiſhed by the Fineneſs, which Fineneſs is more or leſs according as there is more or leſs Allay in a certain quantity of fine Silver or Gold: as *Engliſh* Silver Coin is 11 ℥ 2 *dw.* fine, *i. e.* has 18 Penny weight of Allay in 12 ℥ of Silver, *Holland* Coin 9 ℥, *France* 10 ℥ fine, *&c.* and our Crown Gold has 2 Carats of Allay in 24 Carats, or 12 ℥ of fine Gold.

Schol. I thank you for this Explanation, and deſire you,

2. To let me know what Countries uſe the ſeveral ſorts of Coin in the Tables foregoing.

Maſt. I underſtand you, therefore obſerve, That

England Exchanges Pounds, Shillings and Pence Sterling. for Pounds, Shillings and Pence *Flemiſh* with *Amſterdam*, and *Rotterdam* in *Holland*, *Antwerp* in *Brabant*, and *Hamburgh* in *Germany*;

Pence Sterling for Crowns with *France*.

Pence for Pieces of Eight *Mexico*, with *Legborn* and *Genoa* in *Italy*, and *Cadiz* and *Madrid* in *Spain*.

Shillings and Pence for Millrees with *Lisbon*, and *Oporto* in *Portugal*.

Pence Sterling for Ducats with the City of *Venice*.

Schol.

Schol. And pray Sir, how ſhall I know how many pieces of the aforeſaid *Engliſh* Coin I muſt give in Exchange for any Number of pieces of the Foreign Coin foregoing?

Maſt. This depends on the courſe of Exchange, as I have ſhewed you before, and in Reduction you have the Arithmetical Work, which I hope you have not forgot.

Schol. I remember it very well. But pray how ſhall I know when I gain, and when I loſe by Exchange?

Maſt. Admit then you had 500 Pound Sterling to pay at *Antwerp*: You pay it here at *London* to a *Dutch* Merchant, agreeing with him at the ſame time how many *Flemiſh* Shillings and Pence ſhall be paid to your Correſpondent for every 20 Shillings Sterling of your 500 Pound; now if you agree to have paid to your Correſpondent leſs than 33 Shillings 4 Pence for 20 Shillings Sterling, it is evident you loſe by the Exchange: If he is to pay your Correſpondent more than 33 *s.* 4 *d.* for every 20 Shillings Sterling paid by you, then do you gain by the Exchange; If there be paid juſt 33 *s.* 4 *d.* for every 20 Shillings paid by you, then do you neither gain nor loſe by the Exchange; and all this is very undeniable, for by the foregoing Table, 33 *s.* 4 *d.* is in value 20 Shillings Sterling; and the ſame uſe may be made of the reſt of the Table in the Exchange to other places to know whether you gain or loſe thereby.

Schol. I will work your laſt Example, and alſo ſee what the Merchant that paid the 500 *l.* here, will gain ſuppoſing the Bill of Exchange runs to pay 34 *s.* 6 *d.* *Flemiſh*, for 1 Pound Sterling; and to do that, I ſay if 20 Shillings at *London* require 34 *s.* 6 *d.* at *Antwerp*, What ſhall 500 *l.* at *London* require at *Antwerp*?

s.	*s.*	*d.*	*l.*
20.	34:	6::	500
	12		20
	414 *d.*		10000 Shillings.
			414
			20) 4140000
			12) 207000 *d.*
			20) 17250 Shillings.

Answer 862 *l.* 10 *s. Flem.*

Mast. Very well, and how much does the Merchant that paid the 500 *l.* gain by this Exchange?

Schol. 34 *s.* and 6 *d.* is 1 *s.* 2 *d.* above 33 *s.* 4 *d.* the Parr; therefore I say,

If 1 *l.* Sterling gain 1 *s.* 2 *d. Flemish*, what will 500 *l.* Sterling gain?

l.	*s.*	*d.*
500 at	1:	2
l. 25 at	1 *s.*	
l. 4:	3 *s.*	4 *d.*

Flem. l. 29: 3: 4 Answer by practice.

Mast. So might you also have done the Question of Exchange by Practice much sooner than by the Golden Rule as you did it. For instance,

l. 500

l.		*s.*	*d.*
500	at	34 :	6

500 *l.* at 20 *s.*
350 *l.* at 14 *s.*
12 *l.* 10 *s.* at 6 *d.*

l. 862 10 *s.* Answer

Which is the Summ in *Flemish* Pounds to be paid upon the Bill.

And now you say the Merchant that paid the 500 *l.* got 29 *l.* 3 *s.* 4 *d.* *Flemish*, how many Pounds Sterling is that?

Schol. That I can find by this Rule.

If 33 *s.* 4 *d.* *Flemish* require 20 *s.* Sterling, what will 29 *l.* 3 *s.* 4 *d.* *Flemish* require?

Mast. It is true, you may do it that way; but you may do it much sooner by considering that 5 Nobles *Flemish* is 3 Nobles Sterling; therefore Multiply by 3, and Divide by 5 thus:

Multiply { 29 : 3 : 4 *Flemish.*
3

Product *l.* 87 : 10 : 0
Of which Product $\frac{3}{5}$ is — 17 : 06 : 0 Sterl. Answer.

Schol. Sir I thank you, I find the Rules of Practice are of admirable use, not only in casting up the value of Merchandize, and Interest of Money, but also in casting up Bills of Exchange, and Reducing of Foreign Coin to *English* and the contrary.

CHAP. XXVI.

Concerning Barter, or Exchange of one ſort of Merchandize for another.

Definition.

Maſt. THE Rule of Barter ſhews how much of any one commodity at a certain price may be given for another commodity at another price.

Schol Pray give me a Rule and an Example or two to explain it, which I doubt not but will enable me to underſtand this.

Queſtion I.

Two Merchants, willing to Exchange Merchandize, the one hath 24 broad Cloaths at 10 *l.* 10 *s.* the Piece, the other hath Mace at 12 Shillings the Pound. The queſtion is, how many pounds of Mace he ought to give for the others Cloath, to ſave himſelf harmleſs and be no loſer.

Rule, Seek firſt by the Rule of Three, or Practice, what the 24 Cloaths coſt at 10 *l.* 10 *s.* the Piece, and you ſhall find 252 pound: Then to find the quantity of Mace, ſay by the Rule of Three, if 12 *s.* buyone pound, what ſhall 252 pounds buy me? Work, and you ſhall find 420 pound of Mace: And ſo many pound ought he to give for the other Merchants Cloaths.

The Proof, or Queſtion II.

Two Barter. The one hath 420 pounds of Mace, at 12 *s.* the pound, to barter or exchange for broad Cloath, at 10 Pounds 10 Shillings the Piece. The Queſtion is, how many broad Cloaths he ought to have for all his Mace.

Anſwer, Firſt find the value of 420 at 12 Shillings, which is 5040 Shillings. Then ſay again, if 10½ Pounds give one Cloath, what ſhall 5040 Shillings give? Work, and you ſhall find 24 Cloaths, your deſire.

Queſtion

Question III.

Two Exchange Merchandize. The one hath Pepper at two Shillings four Pence the Pound, to sell for ready Money; but in barter he will have no less than three Shillings the pound. And the other hath Holland at five Shillings six pence the Ell ready Money. The question is, at what price he ought to deliver the Ell in the barter to save himself harmless?

Rule, Say by the Rule of Three direct, if 2⅓ ready Money give 3 Shillings in barter, what shall 5 ¼ give in barter? You shall find 7 ¼ Shillings, and at that price ought the second Merchant to sell his Holland in barter, that the first may not over-reach him.

The Proof, or Question IV.

Two barter. The one hath Holland at 5 *s.* 6 Pence the Ell, to sell for ready Money; and in barter he will have 7 ¼ Shillings. The other hath Pepper at 2 Shillings 4 Pence the pound, to sell for ready Money. The question is now, how he ought to sell in barter, to make his gain proportionable?

Rule, Say by the Rule of Three direct, if 5 ¼ ready Money give 7 ¼ Shillings in barter, what ought 2⅓ to take in barter? Multiply and divide, and you shall find 3 Shillings, your desire.

Question V.

Two barter. The one hath Cloath of Arras at 30 *s.* the Ell ready Money, but in barter he will have 35⅓ *s.* And the other hath white Wines which he delivered in barter for 16 pounds the Tun. The question is now, what his Wines cost the Tun in ready Money.

Answer, Say by the Rule of Three direct, If 35⅓ Shillings in barter, give but 30 Shillings ready Money, what does 16 Pound in barter require ready Money? Work, and you shall find 13 Pound 10 Shillings $\frac{20}{71}$ *s.* And so much cost his Wines *per* Tun ready Money.

The Proof, or Question VI.

Two barter Merchandize. The one hath white Wines at 13 Pounds 10 *s.* $\frac{20}{71}$ *s.* the Tun to sell for ready Money; but in barter he delivered it for 16 *l.* The other, to make his bargain as good, delivereth Arras at 35 ⅓ *s.* the Ell. The question is now, what an Ell of his Arras cost in ready Money.

Answer.

Answer, Say by the Rule of Three direct, if 16 Pounds in barter give but 13 Pounds 10 $\frac{2}{7}$ Shillings in ready Money, what ſhall 3 $\frac{1}{3}$ Shillings yield in barter? Work, and you ſhall find 30 Shillings, your deſire.

Queſtion VII.

Two barter. The one hath Kerſeys at 14 Pounds the Piece ready Money; but in barter he will have 18 Pounds: and yet he will have the $\frac{1}{3}$ part of his over-price in ready Money. And the other hath Ginger at eight Groats the pound to ſell for ready Money. The queſtion is, how he ought to deliver the Ginger by the pound in barter to ſave himſelf harmleſs, and make the barter equal.

Rule, For the working of this queſtion, and ſuch like, you muſt underſtand, if the party over-ſelling his wares, require to have alſo ſome Portion in ready Money, as $\frac{1}{3}$, $\frac{1}{4}$, *&c.* then ſhall you firſt rebate the demanded part, whatſoever it be, from the over-price, and alſo from the juſt price. And thoſe two numbers that ſhall remain after the Subſtraction is made, ſhall be the two firſt numbers in the Rule of Three. And the juſt price of the ſame Merchandize ſhall be the third number, which by the operation of the Rule of Three direct, will give you a true ſolution how, and at what price you ſhall over-rate that your Merchandize, to ſave your ſelf harmleſs, and make the barter equal.

Example.

Take the $\frac{1}{3}$ (of eighteen) which is the over-price of his Cloath, which $\frac{1}{3}$ of eighteen is ſix, which you muſt ſubſtract from 18, there reſt 12. And alſo abate it from 14, which is the juſt price of the Cloath, and there remaineth 8, which 8 and 12 are the two firſt numbers in the Rule of Three. Then take eight Groats, or 2 $\frac{2}{3}$ *s.* for the third number. Then ſay by the Rule of Three direct, if eight pounds give 12 pounds, what ſhall 2 $\frac{2}{3}$ *s.* give? Multiply and divide, and you ſhall find 4 Shillings. And for ſo much ſhall the ſecond Merchant ſell his Ginger, or his Commodity in barter, to ballance the others demands.

The Proof, or Queſtion VIII.

Two barter. The one hath fine Kerſeys, at 14 Pounds the Piece ready Money; but in barter he will have 18 Pounds: and yet he will have the $\frac{1}{3}$ part of his over-price in ready Money. And the other hath Ginger, which he having diſcretion enough to make the barter equal, delivered

delivered in barter for 4 Shillings the pound. The question is now, what his Ginger cost him in ready Money.

Rule, After you have made the Subtraction, abating 6 the ⅓ part of 18, both from 18 and 14 (as before was taught you) then will there remain 8, and 12 for your two first numbers in the Rule of Three. Then say, if 12 give 8, what shall come of 4, the over price of the pound of Ginger? Multiply and Divide, and you shall find 2 *s.* 8 pence, your desire.

Question IX.

Two Merchants barter. The one hath the following Commodities at the following Rates ready Money. *viz.*

C.	*S.*	
171 Sugar at	30.	*per* Hundred, and
l.	*d.*	
9764 Cotton, at	15	*per* pound Nett

but in barter he will have 5 *per Cent.* advance. These he is willing to barter with another Merchant for the following Commodities,

	l.	*s.*	*d.*	
Beef at	25:	0		*per* Barrel,
and Cheese at	0:	3		*per* Pound

as valued ready Money:

The question is how much Provisions the second Merchant may give the first, advancing proportionable in barter, contracting to deliver 200 *l.* weight of Cheese for every Barrel of Beef.

	l.	*s.*	*d.*
Rule, The value of the Sugar is	256:	10:	00
and 9764 *l.* Cotton at 15 *d.* is —	610:	5:	00
The Summ is —	866:	15:	00

Then for the 5 *per Cent.* advance, say, if 100. 105:: what 866 *l.* 15 *s.* 00. Multiply and Divide, and you will find the value of the Sugar and Cotton, at 5 *per Cent.* advance to be *l.* 910 $\frac{375}{1000}$.

Then

Then for the price of a Barrel of Beef at 5 *per Cent.* advance is *l.* $1\frac{3125}{100000}$, for

100. 105 :: $1\frac{65}{1000}$ $1\frac{3125}{100000}$

And the advance of 5 *per Cent.* upon a pound of Cheese makes the $3\frac{13125}{100000}$ *d.* for *l.* *l.* *l.* *l.*

100. 105 :: $\frac{125}{10000}$ $\frac{13125}{1000000}$

which $\frac{13125}{10000000}$ Multipyly'd by 200, gives the price of 200 pound of Cheese. *viz.* *l.* $2\frac{625}{1000}$

which added to the *l.* $1\frac{3125}{100000}$ the Summ is *l.* $3\frac{3375}{10000}$; by which dividing the abovesaid price of the Cotton and Sugar and 5 *per Cent.* advance. *viz.* $910\frac{875}{10000}$, the Quotient is $231\frac{13333}{100000}$.

Which shews that the second Merchant must give the first $231\frac{13333}{100000}$ Barrrel of Beef. And as many 200 pounds, or $46226\frac{666}{1000}$ pounds of Cheese for his Cotton and Sugar.

And for proof of this

	l.
$231\frac{13333}{100000}$ Barrels of Beef at $1\frac{3125}{10000}$ *l. per* Barrel	$= 303\frac{2468}{10000}$
And $46226\frac{666}{1000}$ Pounds of Cheese at $\frac{13125}{100000}$ *l. per* pound is	$= 606\frac{8407}{10000}$
Which together make the price of the Cotton and Sugar, and 5 *per Cent.* advance. *viz.*	$= 910\frac{875}{10000}$

This you may perform by Vulgar Fractions, as I have formerly taught you; though I have here done it by Decimals, to save the trouble of Dividing.

CHAP.

CHAP. XXVII.

Containeth a Declaration of the diverſity of the Weights and Meaſures of moſt places of Chriſtendom for Traffick. At the end of which Diſcourſe are two Tables, the one for Weight, and the other for Meaſure, proportionate, and reduced to an equality of our Engliſh *Meaſure and Weight; by the aid whereof, the Ingenious may eaſily by the Rule of Three, convert the one into the other at pleaſure, &c.*

AT *London*, and ſo all over *England*, are uſed 2 kinds of Weights and Meaſures, as the *Troy* weight, and the *Avoirdupoiſe*. From the *Troy* weight is derived the proportion and quantity of all kind of dry and liquid Meaſures, as Pecks, Buſhels, Quarters, *&c.* wherewith is bought and ſold all kind of Grain and other Commodities meaſured by the Buſhel; and in liquids, Ale, Beer, Wine, Oyl, Butter, Honey, *&c.* Upon theſe grounds and Statutes is Bread made and ſold by the *Troy* weight: and ſo is Gold, Silver, Pearl, precious Stones, and Jewels. The leaſt quantity of this *Troy* weight is a grain: twenty four of theſe grains make a penny weight, twenty penny weight's an Ounce, and twelve Ounces a Pound; two Pounds or Pints of this weight make a Quart. And ſo aſcending into bigger quantities, is produced the Meaſures whereby are ſold our other natural ſuſtenance, *viz.* Ale or Beer, with all other neceſſary Commodities, as Butter, Honey, Herrings, Eels, Sope, *&c.* All which laſt before mentioned, though their Meaſures (wherein they are contained) be framed and derived from the *Troy* weight, yet are they in traffick with divers Commodities, as Lead, Tinn, Flax, Wax, with all other Commodities, both of this Realm, and of other Foreign Countries whatſoever, bought and ſold by the *Avoirdupoiſe* weight, of ſixteen Ounces to the Pound, and 112 Pound for the Hundred weight. And to every Hundred is allowed but 12 Pound weight at

See further of theſe Weights and Meaſures in Reduction.

the Common-beam. From hence is also derived the weight of *Suffolk* Cheese, which containeth thirty two Cloves, 8 pound to a Clove, and weigheth in all 256 pounds. And also the Barrel of *Suffolk* Butter is, or should be of like weight with the weight of Cheese, *viz.* 256 pounds. 14 of these pounds make a Stone, and 26 Stone containeth a Sack of *English* Wool: Forreign Wool, to wit, *French*, *Spanish*, and *Effrick*, is also sold by the Pound, or *C.* weight, but sometimes by the Rove; 25 pound to a Rove. Other Commodities of Tale are bought and sold by the *C.* Five-score to the *C.* except headed ware, to wit, Cattel, Nails, and Fish, which are sold after Sixscore to the *C.* There are also two other sorts of measures, to wit, the Ell and the Yard. By the Ell is usually measured Linnen Cloath, as Canvas, *&c.* And by the Yard, Silks, Woollen-Cloaths, Muslins, *&c.*

Antwerp.

At *Antwerp* are also two sorts of weights, their Gold and Silver weight, and their common weight. Gold and Silver is weighed by the mark, the mark is 8 Ounces, the Ounce 20 Esterlings, and the Esterling 32, as our Grains. The Goldsmiths divide that into smaller, but not the Merchants. The proof of Gold is made by Carats, whereof 24 make a Mark of Fine Gold, the Carat is 24 Grains: the proof of the Money is made by Deniers: 12 Deniers is 1 *l.* fine, that is, a Mark of fine Silver: the Denier also is divided into 24 Grains, and the Grain into four quarters.

The Mark of Gold or Silver at *Antwerp*, *Troy* weight, which is 8 Ounces, maketh 7 $\frac{1}{3}$ Ounces common weight, with which all other Merchandize is weighed. So that the Troy weight is greater than the common weight by 6 $\frac{1}{4}$ in the *C.* By this weight of *Troy* they also weigh Musk, Amber, Pearl, *&c.*

All Silks are bought at *Antwerp* by the *Bruges Ell*, which is greater than the common measure, by which they retail by 2 in the hundred. Their common Ell is $\frac{3}{4}$ of our yard, and $\frac{3}{5}$ of our Ell.

Lions.

At *Lions* is used 3 sorts of weight. whereof the first is the common Town weight, with which they weigh all kind of Spicery, and divers other Merchandise. The second is called *Geneva* weight, which is 8

in

in the Hundred greater than the Common weight, with which they weigh Silks, &c. The third is *French* weight, called commonly the Mark weight, and 100 pounds thereof maketh 106 ½ pound *Geneva*, and 314½ of their common weight: with which *French* weight are weighed all things that pay Custom or Toll.

At *Lions* is also used two sorts of Ells or Aulnes. The one wherewith they measure gross Cloaths, as Canvass, and such like. The other is called the *French* Ell or Aulne, with which they measure all other kind of Merchandise, whereof 7 common Town Ells make 11 ordinary *French* Ells.

Roan.

At *Roan* 6½ Muides of Salt, being the measure of the place, make a Hundred at *Armuiden* in *Zeland*, and the Hundred of *Bronage* measure of *Armuiden* maketh at *Roan* 11 Muides. 30 Muides make a Last of Corn, and 16 a Last of Oats. 100 pound weight there, maketh at *London* 114½, and 190¼ at *Antwerp*. And 200 Ells make at *London* 115½.

Noremburg.

A 100 pound weight at *Noremburg* maketh at *London* 111 ⅔; at *Antwerp* 107 ½, and 100 Ells at *Noremburg* make at *London* 75⅓, at *Antwerp* 95 ⅓, &c.

Lisbon.

The Hundred weight at *Lisbon* maketh 4 Roves, every Rove 32 pounds, so that their Hundred weight is 128 pounds, and their pound containeth 14 Ounces, and 100 pounds of their weight maketh at *London* 113 ⅓.

Their Silk, Cloth of Gold, and Woollen, is measured with a measure which they call a Cubit, containing about ¾ of a Varre of *Castile*. But their common measure is called a Varre, which maketh five Palms, and containeth 1¼ Varre of *Castile*: our Ell of *London* is equal with the Varre of *Lisbon*.

Note, That all kind of Merchandise coming to *Lisbon* by Land, payeth less in Custom than that that cometh by Water.

Sevill.

The Rove of *Sevil* is Thirty pound, four Roves make their Hundred weight, which is 120 pound. The 100 pounds of *Sevil* maketh at *London* 102 pounds. Their other common measure is a Varre, whereof one Hundred maketh at *London* 74 Ells, and at *Rome* 40 Canes, *&c.*

Venice.

At *Venice* be two sorts of weight, the one called *la Grosse*, the other *la Suttle*; with the Grosse is weighed all kind of great wares, and with the small all kind of Spicery, and such like.

Their own common Measure are Braces, whereof 100 make at *London* 55 ⅓ Ells, at *Antwerp* 92 ⅓, *&c.*

Florence.

At *Florence* the 100 pound weight maketh at *Aquila*, for Saffron, 110, and 145 pounds of *Florence* make at *Roan* but 100 pounds; the weight of *Florence* and that of *Lucque* is all one.

Their other measures are Braces, whereof 100 maketh at *Antwerp*, *Bruges* measure, 81 ⅓ Ells; 100 Braces there make at *London* 49 Ells, *&c.*

Lucque.

The *Lucque* Sattens commonly sold at *Lions* by weight, and 133 ⅓ pound, make at *Lions* 100 pound, so that 1 pound ⅓ maketh at *Lions* but one pound.

Their other measures are Braces, whereof 100 of them make at *London* 50 Ells, at *Antwerp* 83 ⅓ Ells, *&c.*

Aquila.

At *Aquila* their 100 pounds make at *London* 71 ½, their 136 ⅔ pounds of Saffron make at *Geneva* but 100, and 11 *l.* of *Geneva* maketh 15 *l.* at *Aquila.*

Valentia.

Valentia.

At *Valentia* be two ſorts of weights, a great and a ſmall. The Hundred weight, or great weight containeth four Roves, the Rove 36 pound, ſo the Hundred great weight is 144 pound, and the Hundred weight ſmall containeth but 120 pounds, and is alſo parted into 4 Roves, which is 30 pounds to a Rove. By the ſmall is ſold the Scarlet Grain, with all other kind of Spicery, and by the great is ſold Wool, with all ſuch like groſs wares. The 1¼ pounds of Silk at *Valentia* maketh at *Lions* one pound *Geneva* weight. The charge of great Merchandize at *Valentia* containeth 432 pounds, and in ſmall Wares 360 pounds.

The weight here and at *Barſellona* is all one.

Their 100 pound weight maketh at *London* 78 pound, at *Antwerp* 75.

Dantzick.

At *Dantzick* the Rule is, that whoſoever buyeth any Merchandiſe there, buyeth it by the Ship-pound, which is 320 li. 20 Liſpounds make a Ship-pound, and the Liſpound containeth 16 pound, which Ship-pound of *Dantzick* maketh at *Antwerp* 266⅔ li. Their 100 li. weight maketh at *London* 86, &c.

Their other common meaſures are Ells, whereof 100 maketh at *London* 72 ½, and at *Antwerp* 120½ Ells.

Tholouſe.

At *Tholouſe* 6 Cabes of Woad make a Charge, two Ciſterns of Corn-meaſure and all kinds of Grain make a Charge, the Ciſtern weigheth 160 li. weight of that place. Their 100 in weight maketh at *London* but 91 ¼ pound.

Genoa.

At *Genoa*, 100 li. of their weight maketh at *London* 71½, and at *Antwerp* 68¾, at *Venice*, to wit, Suttle, 106 li.

Their other common Meaſures are *Palms*, whereof 100 make at *London* 23⅓ Ells, and at *Antwerp* 34⅓.

The reſt are ſupplied in two Tables, which hereafter follow.

The-

The Table of the Agreement of the weights of divers Countries, being reduced to an equality, are as followeth.

112 pounds weight at *London*, make at			112 pounds weight at *London*, make at	
Antwerp	107 1/3		*Venice* groſs weight	107 1/2
Frankeford	099		*Venice* ſuttle weight	166 2/3
Colen and *Ausbergh*	102 2/3		*Aquina*	157 1/3
Noremburg	100 1/2		*Vienna*	089 1/2
Roan	098		*Preſlaw*	134 5/6
Paris	102 1/2		*Leypſick*	101 1/2
Lions	118 5/6		*Dantzick*	129 1/2
Diep	100 1/4		*Lubeck*	097 1/2
Geneva	090 1/3		*Barcellona*	144 1/2
Toloufe	122 1/4		*Lisbonne*	099
Rochell	124 1/2		*Geans*	157 1/2
Marſelles	124 1/2			
Sevil, &c.	109 1/2			

The next Table is the Agreement of Meaſures of divers Countries reduced unto an equality, by the aid whereof you may with the uſe of the Rule of Three convert either more or leſs of any one Meaſure unto the other

60 Ells, or 75 Yards at *London* make at

Antwerp	100	
Noremburg	104 1/2	*Ells.*
Frankford	125	*Ells.*
Leypſick	125	*Ells.*
Preſlaw	125	*Ells.*
Dantzick	183	
Vienna in *Auſtria*	87	
Lions in *France*	60 3/4	*Aulnes.*
Paris in *France*	57	*Aulnes.*
Roan in *Normandy*	52	*Aulnes.*
Lisbon	60	
Sevil in *Spain*	81	*Varres.*
Caſtile in *Spain*	81	*Varres.*
Madera Iſles	62	
Venice	108	
Lucques	120	*Braces.*
Florence	122 1/2	*Braces.*
Milain	138	*Braces.*
Rome	90	—*Canes.*
Geans	288 6/13	—*Palms.*

CHAP.

CHAP. XXVIII.

Of Loss and Gain in Merchandizing.

Mast. IF one Yard cost 6*s.* 8*d.* and the same is sold again for 8*s.* 6*d.* the Question is what is gained in one hundred pounds laying out on such Commodities.

Answer, By the Rule of Three say, If 6⅔ give 8½, what giveth 100? Multiply and divide, and look, what your Quotient bringeth forth, above your laying out, is the nett gains and solution to your Question. If you follow the work, your Solution will bring forth 127*l.* 10*s.* which is 27*l.* 10*s.* more than your Principal, and so much is gained in 100 pounds laying out.

Item, to work it the other way, (which I take to be the neareſt) ſeek the difference betwixt the juſt price and the other price, which is one ſhilling ten pence, then ſay by the *Rule of Three*,

If 6⅔*s.* gain 1½*s.* what will 100 pound gain?

Multiply and divide, and you ſhall find 27*l.* 10*s.* and ſo much is gained in 100*l.* laying out.

You may uſe which of theſe two ways you think good.

The Proof.

If a Yard of Cloth be delivered for 8*s.* 6*d.* whereupon was gained after the rate of 27*l.* 10*s.* in 100 pounds laying out: the queſtion is, what the Yard coſt at the firſt hand?

Anſwer. Put your gain 27*l.* 10*s.* to 100 pounds, which maketh 127*l.* 10*s.* Then ſay, If 127*l.* 10*s.* give but 100 pounds, what will 8½*s* give? Work, and you ſhall find 6*s.* 8*d.* the true Solution to your Queſtion.

Schol. Another Example or Proof upon the firſt Queſtion I will put thus;

If one Yard coſt 6*s.* 8*d.* the Queſtion is, at what price the ſame is to be ſold again, to gain 27*l* 10*s.* in a 100 pound laying out?

I ſay by the Rule of Three, If a 100*l.* be advanced to 127*l.* 10*s.* what 6⅔*s*? I multiply and divide, and find 8*s.* 6*d.* the ſolution.

Mast.

Maſt. If one Ell coſt 7 *s.* 8 *d.* to be ſold again for 8 *s.* 6 *d.* the Queſtion is, What is gained in 20 pounds laying out in ſuch Commodities?

Schol. I ſeek the difference betwixt the juſt price, and the other price which is 10 pence, and then apply the Rule of Three, as before is taught, ſaying, If 7 $\frac{1}{2}$ *s.* give $\frac{5}{6}$ ſhillings, what giveth 20 *l.*? To do that I multiply and divide, and I find 2 *l.* 3 $\frac{11}{15}$ *s.* and ſo much is gained in 20 *l.* laying out.

Maſt. See alſo the proof by an Example of Loſs.

A Merchant hath bought *Holland* Cloth at 8 *s.* 6 *d.* the Ell, which proveth not to his expectation; whereupon he is content to loſe 2 *l.* 3 $\frac{11}{15}$ in 20 pounds laying out. The Queſtion is, what price ought to be made of the Cloth, abating this loſs?

Anſwer. Do as before in Gains hath been taught, putting 2 *l.* 3 $\frac{11}{15}$ *s.* to your 20 pound, all together maketh 22 *l.* 3 $\frac{11}{15}$. Then ſay by the Rule of Three, If 22 *l.* 3 $\frac{11}{15}$ *s.* give but 20 *l.* what ſhall come of 8 $\frac{1}{2}$ *s.*? work, and you ſhall find 7 *s.* 8 *d.* the juſt price that the Ell ought to be ſold for, after the rate of this loſs.

Again, if 20 $\frac{1}{4}$ Yards coſt 36 *l.* 10 *s.* how ſhall I ſell the ſame again to make of 3, 4?

Schol. To do this I ſay by the Rule of Three, If 3 do give 4, what will 36 $\frac{1}{2}$ give? I multiply and divide, and find 48 $\frac{2}{3}$ pounds: then I ſay again, If 20 $\frac{1}{4}$ yards do give 48 $\frac{2}{3}$ pounds, principal and gain, what will one yard be worth at that price? I multiply and divide, and find 2 *l.* $\frac{28}{243}$.

Maſt. If 12 $\frac{1}{2}$ Yards coſt me 11 pounds 5 ſhillings, and I ſell the Yard again for 16 ſhillings, the Queſtion is, Whether I do gain or loſe, and how much in the pound Sterling?

Schol. I look what the 12 $\frac{1}{2}$ Yards come to at 16 *s.* the Yard, and find it ten pounds. But they coſt 11 pound 5 *s.* So there is loſt upon the whole 1 pound 5 ſhillings. Then to know how much is loſt in the pound, I ſay by the Rule of Three, If 11 $\frac{1}{4}$ pound do loſe 1 $\frac{1}{4}$ pound, what will 1 pound loſe? Multiply and divide, and you ſhall find 2 *s.* 2 *d.* $\frac{2}{3}$, and ſo much is loſt in the pound Sterling.

Maſt. If I ſell 100 weight of any Commodity for 4 pound, whereupon I do loſe after 10 pound in the 100 pound, I demand how much I ſhall loſe or gain in the 100 pound, in caſe I had ſold the ſame for 4 pound ten ſhillings.

Schol. I ſay, if 90 pound yield 100, how much will 4 give? I multiply and divide, and find 4 $\frac{4}{9}$. Then I ſay again, if 4 $\frac{4}{9}$ give me 4 $\frac{1}{2}$ what

4½, what will 100 come to? I multiply and divide, and find 101 pound ½, which is more than 100 pound by 1*l.* 5*s.* and ſo much is gained in the 100 pound.

Maſt. A Merchant hath ſold Currans for the ſumm of 430 pound, and he hath gained after the Rate of 10 pound in the 100 pound, the Queſtion is, to know how much he gained in all?

Schol. I ſay by the Rule of Three, If 100 pound do gain 10 pound, what will 430 pound gain? I multiply and divide, and find 43, and ſo much hath he gained in all.

Maſt. If one Yard be worth 28*l.* 6*d.* for how much ſhall 10 Yards be ſold to gain after 8 pound 6 ſhillings 8 pence in the 100 pound?

Schol. I firſt add 8*l.* 6*s.* 8*d.* to 100. Then I ſay, if 100*l.* do give 108½*s.* for principal and gain, what will 28½*s.* principal yield? I multiply and divide, and find 30½*s.* Then I ſay again by the Rule of Three, if 1 Yard do give 30½*s.* what ſhall ten Yards give? I multiply and divide, and find 15*l.* 8*s.* 9*d.* And for that price ſhall the ten Yards be ſold to gain after the rate of 8*l.* 6*s.* 8*d.* upon the 100.

Maſt. Very well, I will now ſhew you a Branch or Proof out of this Queſtion.

A Merchant hath ſold Clothes for 15*l.* 8*s.* 9*d.* and he hath gained in the whole the ſumm of 1*l.* 3*s.* 9*d.* The Queſtion is to know how much he hath gained in the 100 pound.

Schol. To know this, I firſt deduct the gains from the price, and there will remain 14*l.* 5*s.* 0*d.* Then ſay by the Rule of Three direct, If 14*l.* ¼ give me 1*l.* 3¾, what will 100*l.* give? Multiply and divide, and you ſhall find 8*l.* 6*s.* 8*d.* the effect deſired. The proof is apparent in the Queſtion before.

Maſt. If one Yard coſt 36*s.* how much ſhall 12 Yards be ſold for, to gain after the rate of 10*l.* in the 100?

Schol. Firſt ſay, If 100 give 110*l.* principal and gain, what will 36*s.* give? I multiply and divide, and find 39¾*s.* Then I ſay again by the Rule of Three, If one Yard yield 39¾ ſhillings, what ſhall 12 Yards gain? I multiply and divide, and find 23*l.* 15¾*s.* which ¾*s.* is 2¼*d.* And for the ſame price ſhall the 12 Yards be ſold, to gain after the rate of 10 in the 100.

The Proof.

Mast. If 12 Yards be sold for 23 *l.* 15 *s.* 2½ *d.* whereupon is gained after 10 *l.* in the 100, the Question is, what the Yard cost the first peny?

Answer, First say, If 12 give 23 *l.* 15 *s.* 2½ *d.* what one Yard? Multiply and divide, and you shall find 39½ *s.* Then say again by the Rule of Three, If 110 pounds give but 100, what shall 39½ *s.* give? Work, and you shall find 36 *s.* the just price of the yard at the first hand.

Again, When one Merchant selleth Wares to another, and he giveth to the Buyer 1 *l.* 6 *s.* 8 *d.* upon the score, or 20 *l.* the Question is, how much shall the Buyer gain upon the 100 *l.* after that rate?

Schol. First, I add 1 *l.* 6 *s.* 8 *d.* unto 20 *l.* and they are 21⅓; I then say, If 20 pound give 21⅓, what shall 100 give? I multiply and divide, and find 106⅔. So the buyer getteth after the rate of 6⅔ *l.* upon the 100 *l.*

Mast. If one Yard cost me 2 *s.* 8 *d.* ready Money, and after I sell the same again for 2 *s.* 10 *d.* to be paid for it at the end of 3 Months: the Question is, what I gain upon the 100 *l.* in 12 Months.

Loss and Gain upon Time.

To answer this, first say, If 2½ gain ⅝, what shall 100 *l.* gain? Multiply and divide, and you shall find 6¼ pound. Then say again, by the Rule of Three, If three Months gain 6¼ pound, what shall 12 Months gain? Work, and you shall find 25 *l.* and so much shall I gain in 12 Months after that rate.

You may also work it all at one working by the first part of the Rule of Three composed, saying, If 2½ *d.* in three Months do gain ⅝ of a Shilling (which is 2 *d.*) what will 100 *l.* gain in 12 months?

Schol. I propose this Question, If a Yard be delivered for 2 *s.* 10 *d.* to be payed at 3 months, whereupon was gained after the rate of 25 *l.* in the 100 for 12 months, the Question is now, what the Yard cost at the first hand?

Answer. First I say, if 12 months gain 25 *l.* what shall 3 months gain? I work, and find 6¼ *l.* Then I say again, If 106¼ *l.* give but 100, what shall 2½ *s.* give? I work, and find 2 *s.* 8 *d.* which is the just price that the Yard cost at the first hand.

Mast. Very well, but if one Yard of Cloath cost me 2 *s.* 8 *d.* ready money, for what term shall I sell the same again for 2 *s.* 10 *d.* so that I might gain after the rate of 25 pound upon the 100 pound in 12 months?

Schol.

Schol. First, if 2 ½ gain ½, what ſhall 100*l.* gain? I multiply and divide, and find 6 ¼*l.* Then I ſay again, If 25 pound come of 12 months Truſt, what Truſt ſhall come of 6 ¼? I work, and find 3 months, the juſt term that the Cloath ought to be delivered at 2*s.* 10*d.* to gain 25*l.* upon the 100*l.* in 12 months.

Maſt. If one Yard coſt me 2*s.* 8*d.* ready money, for what price ſhall I ſell the ſame again to be paid at the end of 3 months, ſo that I may gain after the rate of 25 pound in the 100 pound for 12 months?

Schol. I firſt ſay, If 12 gain 25*l.* what ſhall 3 months gain? I multiply and divide, and find 6 ¼*l.* Then I ſay, if 100*l.* give 106 ¼, what giveth 2 ⅔*s*? I work, and find 2*s.* 10*d.* and for that price muſt the Yard be ſold to gain after 25 pound in the 100 pound for 12 months.

CHAP. XXIX.

Concerning Allowance for Tare, Trett, &c.

Maſt. IF ever you be concerned in Merchandizing you will hear of Allowance for Tare and Trett, as alſo of Groſs-weight, Suttle-weight, and Nett-weight, which I ſhall explain to you firſt, and then ſhew you how to compute the uſual allowances for any Quantity of a Commodity the briefeſt way by ſundry Examples.

Schol. Sir, I thank you for your kind offer, for truely at preſent the Names you have mentioned appear to be more difficult than the Allowances in themſelves neceſſary.

Maſt. You will beſt judge of that when you underſtand the Nature thereof; therefore know, that **Tare** is an Allowance made for the weight of the Bagg, Cask, Cheſt, *&c.* wherein any kind of Merchandiſe is put: for if a Merchant buys (for inſtance) a Bag of Cotton, Cheſt of Sugar, *&c.* he cannot take the Commodity out to weigh it alone, and therefore ſince neither the weight of the Bag, or Cheſt, or that of the Commodity can be known without much loſs, damage of the Goods, and trouble to the Owners, it became

neceſſary therefore that they ſhould both be weighed together, and that the Seller ſhould allow the Buyer ſo much *per* hundred weight according to the Commodity; as the Allowance for

	l.	*l.*
Allum is	12	*per* 112.
Almonds	14	
Brimſtone	8	
Cotton-Wool	4	
Cotton-Yarn	5	*per* 100
Copperas	8	*per* 112
Currans	16	
Copper	8	
Feathers	4	
Hopps	4	
Iron-wire	6	
Lattin-wire	6	
Lambs-wool	4	
Poliſh Wool	4	
Steel Hemp	14	
Salt-peter	12	
Tallow	12	

But there are for ſome Commodities allowed more or leſs Tare according to the Quantity made up together; as,

For Raw-Silk,

The Bale being 3 *C.* weight and upward they allow 16 lb. for Tare.
From 3 *C.* weight down to 2 *C.* 14 lb. Tare.
And from 2 *C.* weight downward 12 lb. Tare.

Alſo for Virginia *Tobacco,*

All Hogſheads under 3 *C.* weight allow 70 lb. Tare.
From 3 *C.* to 4 *C.* 80 lb. Tare.
From 4 *C.* to 5 *C.* 90 lb. Tare.
And from 5 *C.* upward, 100 lb. Tare.

Trett is allowed the Buyer, being 4 lb. at every hundred and four, and was formerly allowed only to Freemen of *London*, but is now claimed and given to moſt Buyers within the City of *London*.

This 4 *l.* at 104 is allowed for the Impurity that may be mixed with a Commodity, *i. e.* in Cinnamon, Cloves, Mace, Tobacco, Cotton-yarn, Cotton-wool, and other things that have waſte.

Groſs-weight is the weight of the Commodity and thing wherein it is contained.

Suttle.

Suttle-weight is the weight of the Commodity when the Allowance for Tare is deducted.

Nett-weight is the weight after all Allowances are made, being the same with Suttle in such Commodities as Trett is not allowed for.

Schol. I heartily thank you for these Explanations, and now if you please, I will try to make Allowance for Tare in this

Example;

In 476 *C.* 3 *q.* 14 *l.* Gross, I would know how much Nett-weight, allowing 4 lb at 112 Tare, which I do thus;

l. *l.* *C.* *q.* *l.*
112. 4 : : 476 : 3 : 14
4
1907 Quarters,
28
15260
3815
53410 Pounds
4
lb. *C.* *q.* *l.*
112) 213640 (1907 ¼ Tare, or 17 : 00 : 02 ½.

Most. But this Question may be much sooner answered by multiplying the given Hundred, by 4 the Tare to be allowed *per* hundred, and adding to the Product 3 Quarters of 4 for the 3 Quarters of a Hundred, and for the 14 lb, ½ of 4 thus;

C. q. l.
476 : 3 : 14 Groſs.
4

1904 lb. for 476 C.
3 lb. for the 3 quarters.
0½ lb. for the 14 lb.

Anſwer—1907½ lb. Tare.
or 17 : 00 : 03½ Tare.

Deducted from the Groſs the Remainer is——— 459 : 03 : 10½ Nett.

Schol. I will do this Example by the ſame method: To know the Nett-weight of 37 C. 1 q. 19 l. Groſs at 14 l. *per* 112 Tare.

C. q. l.
37 : 1 : 19
14

148
37

518 lb. Tare of 37 C.
3½ lb. Tare of the 1 quarter.
1½ lb. Tare of the 19 lb.

Total 523½ lb. Tare in all, which deducted from the Groſs—4191

Reſteth—3667½ lb. Nett.

Meſt. You have done that very well, and Note, That where the Commodity is valued by the Pound you may reduce the Hundreds, *&c.* Groſs into Pounds, and ſo deduct the Tare; but if the thing is valued by the Hundred, then reduce the Tare into Hundreds, ſo will the Nett-weight be in Hundreds, except in ſmall parcells.

I will

I will shew you one Example of Tare and Trett.

If I bought the 4 Bags of Cotton following: what does the same amount to at 14*d. per lb.* Nett, allowing 4 *lb.* at 112 for Tare and 4 at 104 for Trett.

	C.	q.	lb.
N°. 1 Cont.	2	1	22 Groſs.
2 ——	3	0	07
3 ——	1	3	20
4 ——	2	2	14
4 ——	10	0	07 Total Groſs.
	0	1	12 ¼ Tare deducted.
Remains —	9	3	22 ¾ Suttle.

9
98 ¾
97

1086 ¾ lb. Suttle.
⅛ thereof is — 41 ¾ Trett, deduct.

remains — 104 5 lb. Nett, at 14*d. per lb.*

52 : 05 for 1*s.*
8 : 14 : 2*d.* for 2*d.*

l. 60 : 19 : 2 Anſwer.

CHAP. XXX.

Queſtions of Lengths and Breadths.

IF a piece of Arras be 7 Ells and ¼ long, and 5 Ells and ½ broad, how many Ells ſquare doth the ſame piece contain?

Anſwer. Multiply the length by breadth, that is to ſay, 7 ¼ by 5 ½, and thereof will come 41 ¹¹⁄₁₆ Ells: and ſo many Ells ſquare doth the ſame piece contain.

2. Again,

2. Again, A piece of Arras doth contain 22 Ells ſquare, and if the ſame were in length $3\frac{1}{3}$ Ells, I demand how many Ells in breadth the ſame piece doth contain.

Anſwer. Divide 22 Ells by $3\frac{1}{3}$, and thereof cometh $6\frac{3}{5}$. And ſo many Ells doth the ſame contain in breadth.

3. A Merchant hath $3\frac{1}{4}$ Ells of Arras, at $1\frac{3}{4}$ Ells broad, which he will change with another man for a piece of Arras that is $\frac{7}{8}$ Ells ſquare. The queſtion is how many Ells of that ſquareneſs ought the firſt Merchant to have?

Anſwer. Multiply the firſt Merchant's piece his length by the breadth, and you ſhall find it containeth $4\frac{15}{56}$ Ells, which divide by $\frac{7}{8}$ and you ſhall find $5\frac{1}{7}$ Ells, and ſo many Ells of that ſquareneſs ought the latter Merchant to give the firſt.

4. A Student hath bought $3\frac{1}{3}$ Yards of broad Cloath at 7 quarters broad, to make a Gown, and will line the ſame throughout with Lamb at a foot ſquare each skin, the queſtion is now, how many skins he ought to have.

Anſwer. Seek firſt the number of Yards ſquare that his Cloath containeth, which to do, multiply $3\frac{1}{3}$ his length, by $1\frac{3}{4}$ his breadth, and you ſhall find $6\frac{1}{3}$ Yards ſquare: then ſay by the Rule of Three, If one yard ſquare give 9 foot, what ſhall $6\frac{1}{3}$? Work, and you ſhall find $55\frac{1}{8}$ skins.

5. A Lawyer hath a rich piece of Ceiling which is 24 foot and 3 Inches long, and 7 foot and $2\frac{1}{2}$ Inches broad: the Plaiſterer is to be paid by the yard ſquare: the queſtion is how many yards this containeth?

Anſwer. Multiply the length by the breadth, that is, $24\frac{1}{4}$ foot by $7\frac{1}{4}$ foot, and you ſhall find $174\frac{77}{88}$ foot ſquare, which 174 you ſhall divide by 9 (for ſo many foot make a yard ſquare) and you ſhall find 19 yards 3 foot and $\frac{77}{88}$ of a foot, and ſo many yards do this Ceiling contain.

6. I have bought 342 Ells *Flemmiſh* of Arras work, at two Ells broad *Flemmiſh*, and I would line the ſame with Ell-broad Canvas of *Engliſh* meaſure. The queſtion is, how many Ells *Engliſh* will ſerve my turn?

Anſwer. For as much as three Ells *Engliſh* are worth five Ells *Flemmiſh*, therefore put three Ells *Flemmiſh* into his ſquare, in multiplying three by it ſelf, which maketh nine. Likewiſe multiply the *Engliſh* Ell, which is five quarters every way into it ſelf, and you ſhall find 25. Then multiply 342, which is the length of the piece,

by

by 2, which is the breadth, and thereof cometh 684, then ſay by the Rule of Three, as before, if 25 Ells ſquare of *Flemmiſh* meaſure, be worth nine Ells ſquare of *Engliſh* meaſure, what are 684 of *Flemmiſh* meaſure? Multiply and divide, and you ſhall find 246 $\frac{6}{25}$ Ells *Engliſh*.

The ſame is alſo wrought by the Reverſe Rule of Three, in ſeeking the ſquare Quarters contained in two *Flemmiſh* Ells (which are 18) and alſo in ſeeking the ſquares contained in the *Engliſh* Ell (which are 25:) then ſay by the Rule of Three backward, If 18 quarters require 342 Ells, what ſhall 25 quarters give? Multiply and divide by the Rule of Three Reverſe, and you ſhall find as before 246 $\frac{6}{25}$ Ells *Engliſh*.

7. At three Shillings four Pence the *Flemmiſh* Ell, what is the *Engliſh* Ell worth after that rate?

Anſwer. Say, if three quarters give 3 $\frac{1}{3}$ *s.* what giveth five quarters? Multiply and divide, and you ſhall find 5 *s.* and 6 $\frac{2}{3}$ *d.*

8. *Maſt.* At 8 *s.* 4 *d.* the *Flemmiſh* Ell ſquare, what is the *Engliſh* Ell worth after that rate?

Schol. According to the reaſon of the laſt Queſtion, I conſider that a *Flemmiſh* Ell ſquare is equal to nine quarters of a Yard *Engliſh*, and an *Engliſh* Ell ſquare is equal to 25 quarters of a Yard. Therefore I ſay by the Rule of Three, if 9 quarters give 8 $\frac{1}{3}$ *s.* what 25 quarters? I find 23 $\frac{1}{9}$ *s.* And ſo is the *Engliſh* Ell worth.

9. At 6 *s.* 8 *d.* the Ell ſquare, what ſhall a piece of Cloath coſt that is 7 $\frac{1}{2}$ Ells long, and 3 $\frac{1}{4}$ Ells broad?

Anſwer. Multiply the breadth by the length, and you ſhall find the Product 23 $\frac{7}{8}$ Ells ſquare; which multiply by $\frac{1}{3}$ and you ſhall find 8 *l.* 2 *s.* 6 *d.* and ſo much the piece of Cloath coſt.

10. *Maſt.* A Mercer ſold 3 pieces of Silk, to wit, 24 $\frac{1}{2}$ 13 $\frac{3}{4}$ and 25 yards, at 6 $\frac{3}{4}$ *s.* the yard, and was glad to receive in part of payment again a Cloath containing 34 $\frac{1}{2}$ yards at 7 $\frac{1}{2}$ Shillings the yard. The Queſtion is now, what the Debtor oweth the Creditor? Work and you ſhall find he oweth the Mercer 8 *l.* 1 $\frac{201}{431}$ *s.*

CHAP. XXXI.

Equation of Payment.

Queſt. A Merchant doth owe a Summ of Money, whereof the $\frac{1}{3}$ is to be paid at ſix Months, and the $\frac{1}{4}$ at eight Months, and the reſt at a Year. If he would pay all at one payment, the queſtion is, what time ought to be given him?

Anſw. I have omitted the quantity of the Summ, for you ſhall underſtand the Rule is General, and giveth a true ſolution to what Summ ſoever ſhall be propoſed: But now for order ſake in teaching, I do imagin the Summ to be 60 pounds, whereupon the manner of this work is to multiply the proportionate part of the Money by the time, as in company. Then 20 being the firſt payment, and the $\frac{1}{3}$ of 60, which $\frac{1}{3}$ multiplied in broken numbers by 6 its time of payment, maketh $\frac{6}{3}$, which in whole numbers, as appeareth by the Example in the Operation, maketh two Months: next 30, which is the $\frac{1}{2}$ multiplied by its term 8, yields 4, Months, then the reſt, which is 10 *l.* muſt needs be abbreviated into the proportion it beareth to 60, which is $\frac{1}{6}$, which $\frac{1}{6}$ multiplied by his time 12 Months, produceth $\frac{12}{6}$, and maketh two Months. All which added together, as appeareth in the Operation, maketh 8 Months, which is the juſt time that all thoſe payments ought to be paid at once.

$\frac{1}{3}$ by $\frac{6}{1}$	$\frac{6}{3}$	2 Months.
$\frac{1}{2}$ by $\frac{8}{1}$	$\frac{8}{2}$	4 Months.
$\frac{1}{6}$ by $\frac{12}{1}$	$\frac{12}{6}$	2 Months.

But Queſtions of this nature may alſo be done by multiplying the ſeveral Summs by the time they ought to be paid at, and Dividing the Summ of the Products by the Total Debt, and the Quotient is the Anſwer to your Queſtion.

Example. A Merchant oweth 800 *l.* which he is to pay thus:

$\frac{1}{2}$ or *l.* 133 : 6 : 8 ready Money
$\frac{1}{3}$ or *l.* 200 : at 2 Months
$\frac{1}{4}$ or *l.* 400 at 4 Months, and the reſt
which is *l.* 66 : 13 : 4 at 12 Months:

The Queſtion is, when one payment muſt be made to ſatisfie the whole Debt?

See

See the work according to the Rule above.

C.		Months.
200	Multplied by	2—is 400
400		4—is 1600
66:13:4		12—is 800

800) 28 00 Summ
Mon.
(3½ Answer.

Schol. I very much admire the excellency of this Rule that shews so exactly how payments may be made according to proportion, without Loss either to the Debtor or Creditor. I will now, to see if I understand what you have said, propose this Question:

A Merchant is to pay 1200 *l.* at 3 several times, *viz.*

400 *l.* at 2 Weeks
600 *l.* at 4 Months, and
200 *l.* at 5 Months: The Question is when the whole may be paid at once.

I Multiply (for Answer)

l.
400 by 2 Month gives — 200
600 by 4 Months —— 2400
200 by 5 Months is — 1000

1200) 36 00 Summ
(3 Quotient, or Answer.

And having performed the work, I find the whole Summ ought to be paid at the end of 3 Months.

Most. I am glad to see you understand this Rule so well: We will now proceed to Examples of a different nature.

I lent my Friend 326 pounds for 5½ Months without any Interest, upon condition to have the like courtesie again when I needed. But when I came to borrow, he could spare me but 149 *l.* 8 *s.* 4 *d.* The

Queſtion is, how long time I ought to have the Uſe thereof, to countervail my Friendſhip ſhewed him.

Anſwer. Say by the Reverſe Rule of Three, if 326 pounds give 5$\frac{1}{2}$ months, what time will 149$\frac{5}{12}$ pounds give? Multiply and Divide, and you ſhall find twelve Months, and ſo long time ought I to uſe his Money.

The Proof.

I lent my Friend 149 *l.* 8 *s.* 4 *d.* for Twelve Months. The Queſtion is, how much Money he ought to lend me again for 5$\frac{1}{2}$ Months to recompence my Friendſhip ſhewed him?

Anſwer. Say by the Reverſe Rule of Three, if Twelve Months give 149$\frac{5}{12}$, what ſhall 5$\frac{1}{2}$ Months give? Work, and you ſhall find 326 pounds, and ſo much ought he to lend me.

A Third Variety.

I lent my Friend 149 *l.* 8 *s.* 4 *d.* for 12 Months, to have the like Friendſhip again when I need. And coming to borrow of him, he very courteouſly lent me 326 pounds (for that he could then ſpare the ſame.) The Queſtion is now, how long I ought to have the uſe of it?

Anſwer. Say by the Rule of Three Reverſe, if 149$\frac{5}{12}$ pounds give 12 Months, what ſhall 326 pounds give? Multiply and Divide, and you ſhall find that at 5$\frac{1}{2}$ Months term I ought to reſtore it again.

Proof.

I Lent my Friend 326 pounds for 5$\frac{1}{2}$ Months. The Queſtion is, how many pounds he ought to lend me for 12 Months to recompence this kindneſs.

Schol. I Work by the Rule of Three Reverſe, as you have done, and you will find *l.* 149 : 08 : 04.

Again, I lent my Friend 430 pounds, to receive for the Intereſt thereof after the rate of 8 pounds in the 100 for 12 Months. The Queſtion is, how long time my Friend ought to have the Uſe thereof, that it may be returned with 8 pounds 12 Shillings gain?

You

You may work it, if you please, by the Rule of Three direct at twice, in saying, if 100 pounds yield 8 pounds, what yieldeth 430 pound? Multiply and Divide, and find 34 pound and $\frac{4}{5}$.

Then again for the second work say, if 34 $\frac{4}{5}$ pounds give twelve Months, what giveth 8 $\frac{3}{4}$ pounds? Multiply and Divide, and you shall find three Months, and so long time ought my friend to use it to return with 8*l.* 12*s.* gain.

Otherwise at one working by the Reverse Rule of Proportion, in saying, if 100*l.* in 12 Months do gain 8*l.* how much time shall 430*l.* be a gaining of 8*l.* 12*s.*? Multiply the first and the second into the last for your Dividend, and the third and fourth Multiply together for your Divisor, and then Divide, and you shall find three Months, the just time that my friend ought to use it to return it with 8*l.* 12*s.* gain.

The proof of the last.

How much Money ought a Merchant to deliver after 8*l.* in the 100 for twelve Months, that in three Months he may gain 8 pounds twelve Shillings?

Answer. You may, if you please, work it by the Golden Rule of Three at twice: first saying, if three Months give 8$\frac{3}{4}$ pound, what 12 Months? You shall find 34$\frac{4}{5}$. Then say again, if 8 pounds come of 100 pounds, what shall come of 34 pounds 8 shillings, or 34$\frac{4}{5}$ Work, and you shall find the Answer to the Question, which is 430 pounds, and so much ought the Merchant to deliver.

But most briefly it is answered by the Reverse Rule of five Numbers, where I argue thus, saying, if 100 pounds be 12 Months in gaining of 8 pound, then but for three Months term only to take 8 pounds, 12*s.* must needs be a greater Summ:

l.	*Mo.*	*l.*	*Mo.*	*l.*
100	12	8	3	8$\frac{3}{4}$

to work it, set your numbers thus: multiplying the first into the second third for your Dividend, and the third and fourth for you Divisor: then divide, and you shall find as before 430 pounds, the true solution to your question.

CHAP

CHAP. XXXII.

Concerning Factors or Commission.

Mast. IN this Chapter the Author in the old Impression of this Book has given several Examples of Factorship (as he calls it) which would be of little use now, I shall therefore

1. Say something to Instruct the Reader how to Cast up the Commission, that any Summ comes to at any Rate *per Cent.* And

2. Incert a Table whereby the Commission of any Summ at any Rate may easily be discovered, by those that understand only Addition.

By the Rules of Practice just after the same way that the Interest of Money is at any Rate Computed as in the 9 Examples, Chapter 24th. may the Commission or Summ due to a Factor for his Buying or Selling be easily known.

Schol. Sir I very well remember the Rules, and to assure you thereof, I will do this Example.

A Factor having sold Merchandize to the value of *l.* 5769: 17: 7½ what does the Commission amount to for the Factor, supposing he agreed to have 2½ *per Cent.*?

I am taught by your Rule foregoing to take a Fortieth of the given Number for the Factors Commission, which is *l.* 144: 4: 11¼.

$\frac{1}{40}$ of *l.* 5769: 17: 07½

is = *l.* 144: 4: 11¼

So must the Factor have that, and

The Merchant *l.* 5625: 12: 08¼.

Mast. Well, I see you have rightly applied the Rules, so that I need not Instruct you any farther in that, but shall pass forward to the Table.

A TABLE, *shewing what the Commission of any Summ comes to.*

l.	Commission at *per Cent.* *l.* *s.* *d.* *q.*	Commission at 2½ *per Cent.* *l.* *s.* *d.*
1	00 00 04 3	00 00 06
2	00 00 09 2	00 01 00
3	00 01 02 2	00 01 06
4	00 01 07 1	00 02 00
5	00 02 00 0	00 02 06
6	00 02 04 3	00 03 00
7	00 02 09 2	00 03 06
8	00 03 02 2	00 04 00
9	00 03 07 1	00 04 06
10	00 04 00 0	00 05 00
20	00 08 00 0	00 10 00
30	00 12 00 0	00 15 00
40	00 16 00 0	01 00 00
50	01 00 00 0	01 05 00
60	01 04 00 0	01 10 00
70	01 08 00 0	01 15 00
80	01 12 00 0	02 00 00
90	01 16 00 0	02 05 00
100	02 00 00 0	02 10 00
200	04 00 00 0	05 00 00
300	06 00 00 0	07 10 00
400	08 00 00 0	10 00 00
500	10 00 00 0	12 10 00
600	12 00 00 0	15 00 00
700	14 00 00 0	17 10 00
800	16 00 00 0	20 00 00
900	18 00 00 0	22 10 00
1000	20 00 00 0	25 00 00
2000	40 00 00 0	50 00 00
3000	60 00 00 0	75 00 00
4000	80 00 00 0	100 00 00
5000	100 00 00 0	125 00 00
10000	200 00 00 0	250 00 00

A TABLE, *shewing what the Commission of any Summ comes to.*

l.	Commission at 3 *per Cent.* *l. s. d. q.*	Commission at 4 *per Cent.* *l. s. d. q.*	Commission at 5 *per Cent.* *l. s. d. q.*
1	00 00 03 01	00 00 04 02	00 00 05 00
2	00 00 06 02	00 00 08 01	00 00 10 00
3	00 00 09 02	00 00 12 03	00 00 15 00
4	00 00 12 03	00 00 16 02	00 00 20 00
5	00 00 15 00	00 00 20 00	00 00 25 00
6	00 00 18 01	00 00 24 02	00 00 30 00
7	00 00 21 02	00 00 28 01	00 00 35 00
8	00 00 24 02	00 00 32 03	00 00 40 00
9	00 00 27 03	00 00 36 02	00 00 45 00
10	00 00 30 00	00 00 40 00	00 00 50 00
20	00 01 00 00	00 01 00 00	00 01 00 00
30	00 01 10 00	00 01 20 00	00 01 30 00
40	00 02 00 00	00 02 00 00	00 02 00 00
50	00 02 10 00	00 02 20 00	00 02 30 00
60	00 03 00 00	00 03 00 00	00 03 00 00
70	00 03 10 00	00 03 20 00	00 03 30 00
80	00 04 00 00	00 04 00 00	00 04 00 00
90	00 04 10 00	00 04 20 00	00 04 30 00
100	00 05 00 00	00 05 00 00	00 05 00 00
200	00 10 00 00	00 10 00 00	00 10 00 00
300	00 15 00 00	00 15 00 00	00 15 00 00
400	00 20 00 00	00 20 00 00	00 20 00 00
500	00 25 00 00	00 25 00 00	00 25 00 00
600	00 30 00 00	00 30 00 00	00 30 00 00
700	00 35 00 00	00 35 00 00	00 35 00 00
800	00 40 00 00	00 40 00 00	00 40 00 00
900	00 45 00 00	00 45 00 00	00 45 00 00
1000	00 50 00 00	00 50 00 00	00 50 00 00
2000	01 00 00 00	01 00 00 00	01 00 00 00
3000	01 50 00 00	01 50 00 00	01 50 00 00
4000	02 00 00 00	02 00 00 00	02 00 00 00
5000	02 50 00 00	02 50 00 00	02 50 00 00
10000	05 00 00 00	05 00 00 00	05 00 00 00

The

The use of the foregoing Table.

Admit a Factor has sold Goods for his Employer, amounting to *l.* 5769: 17 07½; the Question is what his Commission or Employment comes to at 2½ *per Cent.*?

Schol. By your Table I perceive,

That against 5000, under 2½ is		*l.*	125	00 00
against	700	*l.*	17	10 00
	60	*l.*	1	10 00
	9	*l.*	0	04 06
	0 : 10 *s.* or ½1	*l.*	0	00 03
	0 : 7	*l.*	0	00 02½
	Summ	*l.*	144	04 11½

Here I find that the Answer is the same as in the Example before the Table.

Mast. You have done right. But suppose you were to find the Commission of *l.* 20605 at 5 *per Cent.*?

Schol. In the Column of 5 *per Cent.*

under *l.* 10000 is		*l.*	500	00 00
that again		*l.*	500	00 00
against *l.* 600		*l.*	030	00 00
l. 5		*l.*	000	05 00
Answer		*l.*	1030	05 00

Mast. Again, admit I would know the Commission of 1506 at 1½ *per Cent.*

Schol. Against *l.* 1000

and under 3 *per Cent.* is		*l.*	30	00 00
against	500	*l.*	15	00 00
against	6	*l.*	0	03 07½
Summ		*l.*	45	03 07½
Half of which, or the Answer			22	11 09½

CHAP. XXXIII.

Of Alligation.

Maſt. I will now proceed to ſhew you the Rule of Alligation; which hath this name, for that by it there are divers parcels of ſundry prices, and ſundry quantities alligate, or mixed together: whereby alſo it might be properly called the *Rule of Mixture*; and it hath great uſe in compoſition of Medicines, and alſo in mixtures of Metals, and many perhaps uſe it in mixture of Wines: but I wiſh that were leſs uſed. The order of this Rule is this.

General Rule.

When any Summs are propoſed to be mixed, ſet them in order one over another, and the mean price (whereunto you would reduce them) ſet on the left hand; then mark what Summs be leſſer than that mean price, and which be greater, and with a draught of your Pen link two numbers together, ſo that one be leſſer than the common number or mean price, and the other greater (for two greater or two ſmaller ought not to be linked together) and the reaſon is this, that one greater and one ſmaller, may be ſo mixed, that they will make the mean or common number very well: but two leſs can never make ſo many as the common number, being taken orderly: no more can two Summs greater than the mean, ever make the mean in due order, as it ſhall appear better to you hereafter. And as it is neceſſary to link every ſmaller (once at the leaſt) with one greater, and every greater with one ſmaller, ſo you are at liberty to link them oftner than once, and ſo may there be to one Queſtion many Solutions. When you have ſo linked or coupled them, then mark how much each of the leſſer numbers is ſmaller than the mean or common number, and that difference ſet againſt the greater numbers, which be linked with thoſe ſmaller on the right hand, and likewiſe the exceſs of the greater numbers above the mean, you ſhall ſet before the leſſer numbers which be combined with them.

Then

Then muſt you (by Addition) bring all theſe differences into one Summ, which ſhall be the firſt number in the Golden Rule, and the ſecond number muſt be the whole maſs that you will have of all thoſe particulars: the third Summ muſt be each difference by it ſelf, and by them ſhall be found the fourth number, declaring the juſt proportion of every particular in that mixture: As by theſe Examples I will make it plain. *Rule.*

There are four ſorts of Wine, of ſeveral prices, one of 6 pence a Gallon, another of 8 pence, the third of 11 pence, and the fourth of 15 pence the Gallon, of all theſe Wines would I have a mixture made to the Summ of 50 Gallons, that ſo the price of each Gallon may be 9 pence. Now I demand how much muſt be taken of every ſort of Wine? *A Queſtion of mixing of Wines.*

Schol. If it pleaſe you to work the firſt Example, that I may obſerve the applying of it to the Rule: then I hope I ſhall be able not only to do the like, but alſo to ſee the reaſon in the order of the Work.

Moſt. Mind then the placing of every kind of number in it.

The Prices ſeverally.		Differences.		
The common price. 9	6	6	As 12. 50:: 6.	25 Gallons of 6 *d.*
	8	2	As 12. 50:: 2.	$8\frac{1}{3}$ of 8 *d.*
	11	1	As 12. 50:: 1.	$4\frac{1}{6}$ of 11 *d.*
	15	3	As 12. 50:: 3.	$12\frac{1}{2}$ of 15 *d.*
	Summ = 12			

Here (you ſee) I have ſet down the ſeveral prices, which are 6, 8, 11, 15, and have linked together 6 with 15, and 8 with 11. The mean price 9, I have ſet on the left ſide, and the difference between it, and every particular price I have ſet on the right hand, not againſt the Summ (whoſe difference it is) but againſt the Summ that it is coupled or linked withall, ſo the difference between 15 and 9, is 6, which I have ſet, not againſt 15 but 6, that is linked with 15, and the difference between 6 and 9 (that is 3) I have ſet againſt 15. So likewiſe the difference between 8 and 9, is but 1, that I have ſet againſt 11, and the difference between 11 and 9 (which is 2) I have ſet againſt 8. Then I add all theſe four differences, and they

make 12, which I ſet for the firſt number in the Golden Rule: the ſecond number I make 50, which is the Summ of the Compoſition in Gallons that I would have, and the third Summ is every particular difference. Now having wrought by the Golden Rule, you find the number of Gallons that ſhall be taken of each ſort of Wine. And (if you pleaſe to examine the truth of the Work) add theſe four Summs together, and they will make 50, that is the total which I would have, as by this Example you may eaſily perceive.

The Proof of this Rule.

And to prove how the Prices do agree do this: Multiply the total Summ 50, by the common price 9, and it will make 450: then keep that Summ by it ſelf, and afterward multiply every ſeveral Summ of Gallons, by the price belonging to the ſame Gallons, and if that Summ do agree with this, which you have kept firſt, then is your Work well done. As here 25 is the number of Gallons of 6 pence price, multiply then 25 by 6, and it maketh 150, which you ſhall ſet down, then multiply $8\frac{1}{3}$ by 8, which is the price for the number of Gallons, and it will make $66\frac{2}{3}$: ſo again $4\frac{1}{6}$ multiplied by 11, doth make $45\frac{5}{6}$. And laſt of all, $12\frac{1}{2}$ multiplied by 15, maketh $187\frac{1}{2}$: and theſe added together do make 450, as in the Example annexed you may ſee: wherefore ſeeing it doth agree with the former Summ of 50, multiplied by 9, I may juſtly affirm this Work to be well done.

25	
$8\frac{1}{3}$	
$4\frac{1}{6}$	
$12\frac{1}{2}$	
50 Sum.	
150	
$66\frac{2}{3}$	
$45\frac{5}{6}$	
$187\frac{1}{2}$	
450	

The variation of this Queſtion.

And now to prove how you can do the like, I propound the ſame Queſtion, only requeſting you to uſe ſome other form of combining or linking the Summs.

Schol. That with your favour, and therefore I combine 8 with 15, and 6 with 11: and then the form will be as followeth.

Whereby amounteth the ſame in the total of the Differences as did before: and yet now the Differences be altered as the Combination is changed, whereof I underſtand the reaſon by your former

mer Work. And here appeareth no ſtrange thing, but that now I have $8\frac{2}{3}$ Gallons of 6 pence, and 25 Gallons of 8 pence: and 12 Gallons and $\frac{1}{2}$ of 11 pence, and ſo conſequently 4 Gallons and $\frac{1}{3}$ of 15 pence: ſo that multiplying $8\frac{2}{3}$ by 6, it maketh 50, and then 25 multiplied by 8, maketh 200: likewiſe $12\frac{1}{2}$ multiply'd by 11 gives $137\frac{1}{2}$: and $4\frac{1}{3}$ multiply'd by 15, maketh $62\frac{1}{3}$, which 4 Summs added into one, will make in the total 450, which agreeth with the multiplication of 50 (being the total Summ of Gallons) by 9 the common or mean price.

	50
	200
	$137\frac{1}{2}$
	$62\frac{1}{3}$
	450

Maſt. Seeing you conceive this work ſo well, I will propound another Example unto you of more variety in the Alligations or Combinings, as thus:

A Merchant being minded to make a Bargain for Spices, in a mixt maſs (that is to ſay) of Cloves, Nutmegs, Saffron, Pepper, Ginger, and Almonds: the Cloves being at 6 Shillings, Saffron at 10 Shillings, Pepper at 3 Shillings, Ginger at 2 Shillings, Almonds at 1 Shilling, and Nutmegs at 8 Shillings *per* pound. *A Queſtion of Spices.*

Now would he have of each ſort ſome, to the value of 300 pound in the whole, and each pound one with another to bear in price five Shillings: How much ſhall he have of each ſort?

Schol. That I ſhall try thus.

Firſt, I ſet down thoſe 6 ſeveral prizes, and at the left-hand I ſet the common price five Shillings. Then I link them thus, 1 with 10, 2 with 6, and 3 with 8: as in the Example following.

Total 18

Maſt. I had deſigned to have combined them in more variety: but I am content to ſee your own work firſt, and then more varieties in combination ſhall follow.

Schol. Then to continue as I began, I ſeek the difference between 1 and 5, (which is 4) and that I ſet againſt 10, then againſt 1 I ſet 5, which

5, which is the excess of 10 above 5, so I gather the difference between 2 and 5, which is 3, and that I set against 6, because it is combined with 2, and likewise the difference of 6 above 5, (which is 1) I set against 2. Then I take the difference of 3 and 5, which is 2, and that I set against 8: and before that 3, I set the difference of 8 above 5, which is 3. Then I gather together all these differences by Addition, and they make 18, which I set for my first Number in the Golden Rule, and so appeareth by those works, that of Almonds I must take $83\frac{1}{3}$ pound, of Ginger $16\frac{2}{3}$ pound, Pepper 50 pounds, of Cloves 50 pounds, of Nutmegs $33\frac{1}{3}$ pound, and of Saffron $66\frac{2}{3}$ pounds.

Then for trial hereof, I multiply every parcel	$83\frac{1}{3}$
by his several price, as $83\frac{1}{3}$ which is the Summ	$33\frac{1}{3}$
of Almonds, I multiply by 1 which is their	150
price.	300
Also $16\frac{2}{3}$ the quantity of Ginger, I multiply	$266\frac{2}{3}$
by 2, which is the price of it: and so each other	$666\frac{2}{3}$
in his kind, as this Table annexed doth represent,	
and then adding them all together I find the to-	1500
tal to be 1500, which also will amount by the	

Product of the gross Mass of 300, by the common price 5, wherefore it appeareth well wrought.

Mast. Now I will make the Alligation to prove your cunning somewhat better: but because you shall not think your self prosted too much, I will also note the differences, as by this Example you may see, where I have alligated 1 with 6 and 8; 2 with 8 and 10, 3 with 10.

Then I say,

The difference between 5, the mean price, and 1 is 4, which I put against 6 and 8, because 1 is combined with them.

Also the Difference between 5 the mean price, and 2 is 3, which I put against 8 and 10, 2 being combined therewith.

Then the Difference between 5 and 3 is 2, which I put against 10, 2 being joined therewith.

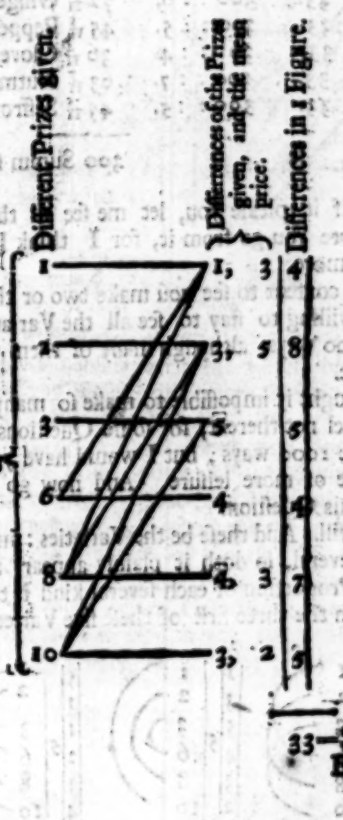
Then the Difference between 5 and 6 is 1, which I put against 1, 6 being combined therewith.

Also the Difference between 5 and 8 is 3, which I have put against 1 and 2, 8 being combined therewith.

Lastly, The Difference between 5 and 10 is 5, which I put against 2 and 3, 10 being combined therewith.

So

So the several Differences, as 1 and 3; 3 and 5, &c. in 1 Figure or Summ is 4, 8, 5, 4, 7, 5, the Summ of which is 33, and by that the several Quantities of the Spices is found, as you see by the following Proportions.



The mean price—5.

Then

Then to find the Quantity that muſt be taken of each kind of Spices to make a Heap of 300 pounds, worth 5 *s.* *per* pound, this is the Analogy.

			lb.	
33.	300 : :	4.	$36\frac{4}{11}$	Almonds.
33.	300 : :	8.	$72\frac{8}{11}$	Ginger.
33.	300 : :	5.	$45\frac{5}{11}$	Pepper.
33.	300 : :	4.	$36\frac{4}{11}$	Cloves.
33.	300 : :	7.	$63\frac{7}{11}$	Nutmegs.
33.	300 : :	5.	$45\frac{5}{11}$	Saffron.
			300	Summ for Proof.

Schol. But if it pleaſe you, let me ſee all the Variations of this Queſtion, before you go from it, for I think I could vary it two or three ways more.

Maſt. I am content to ſee you make two or three Variations: but ſhall not be willing to ſtay to ſee all the Variations: for it may be varied above 300 ways, although many of them would not well ſerve to this purpoſe.

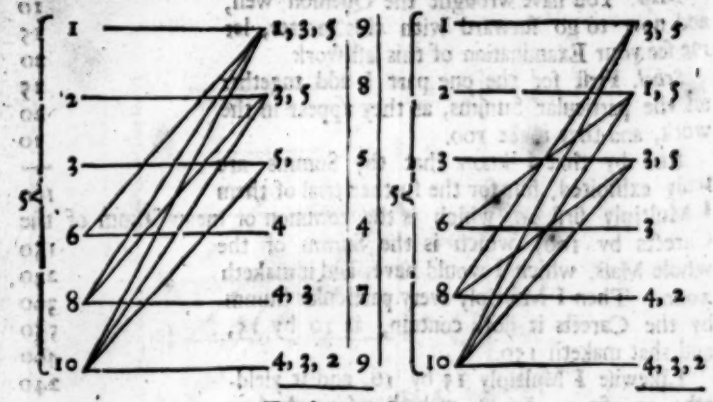
Schol. I thought it impoſſible to make ſo many Variations.

Maſt. Marvel not thereat, for ſome Queſtions of this Rule, may be varied above 1000 ways; but I would have you forget ſuch Fantaſies till a time of more leiſure. And now go forward with ſome Variation of this Queſtion.

Schol. So I will. And theſe be the Varieties: in which as the Combinations are ſeveral, ſo doth it plainly appear, that the Differences by which the Proportion of each ſeveral kind is taken are alſo ſeveral. And yet I ſee in the three firſt of theſe five Varieties, and in the one

Total=18 Total=18 Total=18

other



other foregoing, the Total Summ of the differences to be alike, that is to ſay, 18, whereby I perceive that the variety of their mixture doth depend on the variety of their differences ſeveral, and not of the variety of their Total Summ.

Maſt. In that you ſay right. And ſeeing you conceive it ſo well, I will make an end of this Rule, only exhibiting to you one Queſtion or two of the mixture of Metals, that by it you may compoſe others and exerciſe your ſelf therein alſo, becauſe the uſe of it ſerveth often in buſineſs of charge, not ſo much for Goldſmiths, as of Coinage. I demand of you this queſtion therefore; If a Mint-maſter have Gold of 22 Carects fine, and ſome of 23 Carects, ſome of 24: Again, ſome 15, ſome 16, and ſome of 18 Carects, and would mix them, ſo that he might have 100 Ounces of 20 Carects: How much muſt he take of each ſort? *Queſtion.*

Schol. To know that, I anſwer thus:

As 20. 100 :: 2. 10
20. 100:: 3. 15
20. 100:: 4. 20
20. 100:: 5. 25
20. 100:: 4. 20
20. 100:: 2. 10

Mast. You have wrought the Question well, and now to go forward with this matter, let me see your Examination of this last work.

Schol. First for the one part I add together all the particular Summs, as they appear in the work, and they make 100.

And by that I know that the Summs are truly exhibited, but for the further trial of them I Multiply first 20, which is the common or mean Summ of the Carects by 100, which is the Summ of the whole Mass, which I would have, and it maketh 2000. Then I Multiply every particular Summ by the Carects it doth contain, as 10 by 15, and that maketh 150.

Likewise I Multiply 15 by 16, and it yieldeth 240: So 20 by 18, maketh 360. And 25 by 22, yieldeth 550: likewise 20 by 23 produceth 460: and last of all, 10 Multiplied by 24, yieldeth 240: which Summs all put together make 2000, which doth agree with the like Summ before, wherefore I may conclude, that the work is right. And now (if it please you) I would endeavour to shew some varieties of this Question.

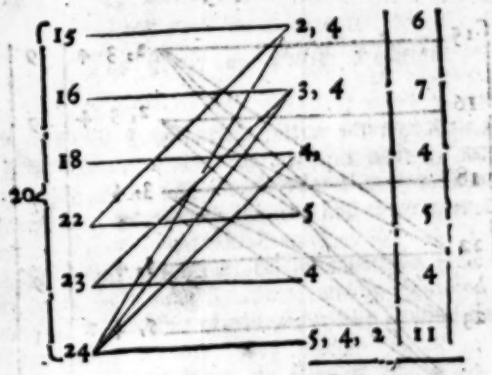
	10
	15
	20
	25
	20
	10
	—
	100
	150
	240
	360
	550
	460
	240
	—
	2000

Mast. Let me see them.

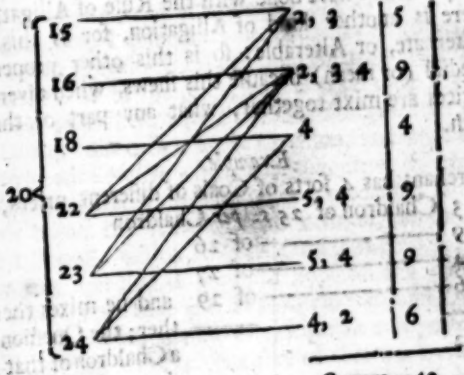
Schol. Here be 3 varieties more.

Variety 2.

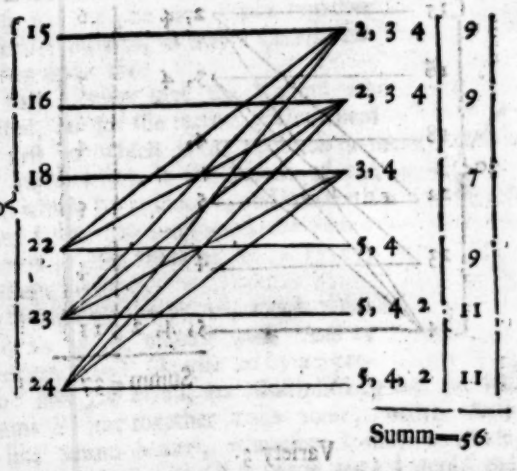
Variety 2.



Variety 3.



Variety 4.



And now I presume I have done with the Rule of Alligation.

Mast. There is another kind of Alligation, for as this is called Alligation Alternate, or Alterable: so is this other properly called Alligation Medial (or mean) because this shews, when divers simples of different prices are mixt together, what any part of that Compound is worth.

Alligation Medial.

Example.

A Coal-merchant has 4 sorts of Coals of different prices, *viz.*

13 Chaldron of 25 *s. per* Chaldron
29 —— of 26
34 —— of 27
46 —— of 29 and he mixes them together; the Question is what a Chaldron of that mixture is worth?
122

Rule.] As the Total quantity is to the Total value.
So is any particular quantity to the value of that Quantity,
So that the value of the Coals, at the prizes above being *l.* 166: 11: 00, The price of 1 Chaldron, is *l.* 1: 7: 3: $2\frac{68}{122}$, for

As 122. 166: 11 00:: 1. 1: 7: 3: $2\frac{68}{122}$.

CHAP.

CHAP. XXXIV.

The Rule of False Position.

Mast. NOW I will briefly teach you the Rule of False Position, which beareth this name, not for that it teacheth any fraud or falshood, but for that by false numbers taken at all adventures, it teacheth how to find those true Numbers you seek for.

Schol. So might any other Rule be called the Rule of Falsehood, for they work by wrong Numbers, and by them find out the right Numbers: as in the Rule of Alligation, the Rule of Fellowship, and the Golden Rule.

Mast. In the Golden Rule, the Rule of Fellowship, and the Rule of Alligation, although the Numbers that you work by, be not the true Numbers that you seek for, yet are they Numbers in just Proportion, and are found by orderly work, whereas in this Rule the Numbers are not taken in any Proportion, nor found by orderly working, but taken at all adventures.

And therefore, I sometimes being merry with my Friends, and talking of such Questions, do call unto them such Children or Idiots, as happened to be in the place, and so take their Answer, declaring that I would make them solve those Questions, that seemed so doubtful.

And indeed I did answer to the Question, and work the Trial thereof also by those answers which they happened by chance to make: which Numbers seeing they be taken as manifest false, therefore is this Rule called the Rule of false Position, or for briefness, The Rule of Falshood: which Rule take as followeth.

When any question is proposed appertaining to this Rule, first imagin any number that you list, which you must name the first position, and then work with it as the question importeth: and if you have missed, then is the last Number of that work either too great or too little: and how much it is so you shall note, as hereafter shall be taught you, and you shall call it the first Error. *Rule.*

Then begin again, and take another number, which shall be called the second position, and work by the question: if you have missed again, note the excess or default as it is, and call that the second Error.

Error. Then multiply croſs-wiſe the firſt poſition by the ſecond Error, and again, the ſecond poſition by the firſt Error, and note their products ſeverally: Then obſerve whether the two Errors were both alike, that is to ſay, both too much, or both too little: or whether they be unlike, that is, the one too much, and the other too little: for if they be like, then you muſt ſubſtract the one product from the other (I mean the leſſer from the greater) and the remainer ſhall be your dividend: ſo muſt you take the leſſer Error out of the greater, and the reſidue ſhall be the Diviſor. Now divide the dividend by that Diviſor, and the Quotient will ſhew you the true number that you ſeek for.

But if the errors be unlike, then you muſt add both thoſe products which you noted together, and take that whole number for the dividend, and add both errors together, and that Summ ſhall be the Diviſor, and the Quotient of that Diviſion ſhall give you the true number that the Queſtion requireth.

Schol. This Rule ſeemeth ſo unlike any other, that without ſome Example I ſhall not eaſily underſtand it.

A queſtion of Maſonry, the firſt Example.

Maſt. I will therefore propoſe ſeveral Queſtions and Examples, for the better underſtanding of the work hereof: and for the firſt take this. A Maſon was bound to build a Wall in 40 Days, and it was covenanted ſo with him, that every Day that he wrought, he ſhould have for his wages 2 Shillings 1 Penny, and every Day that he wrought not, he ſhould be amerced 2 Shillings 6 Pence: ſo that when the Wall was made, and the reckoning taken of the Days that he wrought, and of the other that he wrought not, the Maſon had clear but 5 Shillings 5 Pence for the Work. Now I demand how many Days he did work of thoſe 40, and how many he did not work.

Schol. I pray you expreſs the order of the work, that I may by imitation, and by comparing it with the Rule, be able again to do the like.

Maſt. This order ſhall you keep in the work of this Rule: firſt take ſome Number (as you liſt) at adventure; as for Example, I ſay, he played 12 Days, and wrought 28 Days. Now caſt you the wages of every day and ſee whether it will agree with the Summ of 5 Shillings 5 Pence to be coming to him.

Schol. The 28 Days that he wrought after 25 Pence the Day, yielderh 700 Pence: Then 12 Days that he wrought not, at 30 Pence

Pence each Day, doth amount to 360 Pence, which if I abate out of 700 Pence, there resteth 340: but you say he had not so much.

Mast. He had but 65 Pence, and by this supposition he should have had 340: therefore is this Summ too much by 275, which Summ I must set down as you see here, where first I have made a cross, and

First Supposition — 12 16 Second Supposition

First Error 275 † 55 † Second Error

at the upper corner to the left hand I have set the first position 12: and at the other corner under it I have set 275 which is the first error, with this Figure †, which betokeneth too much, as this line ——— without a cross line betokeneth too little.

On the right hand of the cross I have left two like rooms for the second position and its error. Therefore to prosecute the work, I suppose he played 16 days, and wrought 24.

Schol. I was a while in doubt why you named the days of his working, seeing they be not set in the figure: and I could not imagin how you knew them, or whether you did only suppose them at all adventures, as you did the days that he played: but now I gather, that seeing 40 days is the whole time limited, then the days that he played being mentioned, the rest of 40 must needs be the days that he wrought, and therefore 28 followed 12 of necessity, and 24 followeth 16 also of necessity, but yet I scarce perceive why you set not in the Figures as well 28 as 12.

Mast. It matters not which of them I take, so that in the second position I take the numbers of the same nature that is here both of working days, or both of idle, but now examin you this second position.

Schol. If he played 16 days, 16 times 30 pence, will be found 480 pence, and for 24 days that he wrought, every day yielding 25 pence, the Total is 600 pence: so that abating 480 out of 600, there resteth 120, and as you say, it should be but 65: therefore it is too much by 55: that must be set on the right hand of the Figure, at the lower part, under the 16, which is the second position, thus.

12 16

275 † 55 †

And as I gather by your words, it were all one if I did set 28 instead of 12, and 24 instead of 16.

Mast.

Mast. So it were. But this observe, that of what nature soever the two positions be, of the same nature is the Quotient. Therefore when the positions in this question are 12 and 16, which both being numbers of the playing days, the quotient shall declare the true number of playing days: whereas if the positions had been 28 and 24, which are supposed to be the working days, then would the Quotient have declared the true number of working days, and not of playing days, as it will do now. And therefore to continue the work of this question, and to find the true number of playing days, I must multiply cross-wise the first position by 55, that is the second error, and the product will be 660. Then I multiply 275 by 16, and it produceth 4400. Now because the errors are alike, that is to say, both too much, I must substract 660 out of 4400, and so remaineth 3740, which is the dividend. Again, I must substract the lesser error 55 out of 275, that is the greater error, and there will remain 220 which will be the divisor: then dividing 3740 by 220, the quotient will be 17. Wherefore I say, that 17 is the true number of days that the Mason played: and then it followeth that he wrought 23 days: and so is the question answered.

The proof of this Rule.

Now for the trial of this work, there needeth no other but this, to work with this number according to the question, and if it agree, then appeareth the number to be that which you would have.

And here now seeing he wrought 23 days, and must have for every day 25 pence, the whole Summ cometh to 575. Then again, seeing he played 17 days, and must abate 30 pence for every day, the whole Summ of the abatement will be 510 Therefore I substract 510 out of 575, and there will remain 65, which maketh 5 Shillings 5 pence, the clear wages of the Mason for his work according to the question.

Schol. Now I believe I understand the work and the Rule so well (especially by this proof) that I can be able to do the like: And for a proof, I take the same question, all save the last number, where I will suppose that he had 10 Shillings for his wages clear. And now to guess of the number of the days he wrought, I suppose first that he wrought 20 days: then I say if he wrought 20 days, his wages must be 500 *d.* then did he play other 20 days, for which must be abated 600 *d.* and then he loseth 100 *d.* And so I am at a stay, for it is not like to your former work.

Mast. You should have required of me some question, and not have taken a question of your own, until you were more expert in this Art, for you as well happen on an impossible question, as otherwise:

wise: but now to go forward, consider that this number is too little by 220, seeing he should gain by your supposition 120 pence, and in this position he loseth 100, those both make 220, which you shall set down for the first error, with this sign —, betokening too little as here doth appear.

And now for the rest go forward your self once again.

Schol. As my error hath shewed my folly, so it hath procured me better understanding.

Now therefore considering this position does not solve the question, I take another, supposing that he wrought 30 days. Then for his wages he must be allowed 750 pence, and for the 10 days which he wrought not, he must abate 300 pence, and so remaineth clear 450 pence, but it should be only 120 pence, therefore it is too much by 330, which I set down in the figure with the former position and its error, and the figure appeareth thus:

Now first, I multiply in cross ways 20 by 330, and it will be 6600.

Then again I multiply 30 by 220, and it will be 6600. Wherefore if I shall substract the one out of the other, there will remain nothing to be the Dividend.

20 30
—220 330†

Mast. In this you forget your self again; for in as much as the signs in the errors be unlike, therefore must you work by Addition, adding together those two products to make the Dividend, and also adding the two errors to make the Divisor. And because you shall no more forget this part of that Rule, take this brief remembrance.

Unlike require Addition;
And Like desire Substraction.

Schol. You mean, that if the errors have like signs, then must the Dividend, and the Divisor be made by Substraction, as is taught before; and if those signs be unlike (as in this last example they be) then must I by Addition form the dividend and the divisor. Therefore must I add 6600 to 6600, and it will be 13200, which will be the dividend. Then again I add 220 to 330 and it will be 550, which must be the divisor: wherefore dividing 13200 by 550, the quotient will be 24, whereby I know that the Mason wrought 24 days, and then it followeth that he played 16 days.

Mast. Examin your work, whether it be agreeable to the question or no.

Schol. For 24 days work the wages at 25 pence *per* day, must be 600 pence, and for 16 days which the Mason wrought not, there must be abated 480 pence, and then remaineth clear to the Mason 120, as the question importeth: wherefore it is evident that 24 is the true number of days that he wrought.

Mast. Although you seem now to understand this work, yet to acquaint you the better with this Rule, I think it good to propose to you some examples more before I make an end of it.

Schol. Sir, I thank you that you do so consider my proficiency in knowledg, for undoubtedly it is practice that maketh men prompt and expert in every Science and Art.

Mast. You say well, so that they follow some certain precepts to govern and rule their practice by, else may practice procure custom of error, and a repugnance to exactness of knowledge: namely, as long as the error is not plainly known to the vulgar sort. But return to your work.

A Question of Wares. The second Example.

A Servant bought of Velvet and Damask for his Master 40 yards, the Velvet at 20 Shillings a yard, and the Damask at 12 Shillings, and when he came home, his Master demanded of him, how much he had bought of each sort: I cannot tell (saith he) exactly; but this I know, that I paid for Damask 48 Shillings more than I paid for Velvet: now the Question is how many yards there is of each sort.

Schol. Although it seemeth difficult, yet I will try what I can do: for I remember you said, that it matters not how false the guess be, so it be somewhat to the question, and not an answer of a contrary matter.

Therefore first, I imagin that he bought 20 yards of Damask, for which he should pay after the former price 240 Shillings: then must he needs have of Velvet other 20 yards, (to make up the 40 yards,) and that would cost 400 Shillings. So that the total of the price of the Damask is less than the Summ paid for Velvet 160 Shillings, and should be more by 48. Therefore the first error is 208 too little. Then I begin again, and suppose he bought of Damask 30 yards, that cost 360 Shillings, then had he but 10 yards of Velvet which cost 200 Shillings: and now the price of the Damask is greater than the price of the Velvet by 160 Shillings, and should be but 48, therefore is the second error 112 too much, which I set, as here appears.

appears. Then I multiply in cross ways 208 by 30, and the product will be 6240. Also I multiply 112 by 20, and there will amount 2240. And in as much as the signs of the errors be unlike, I know I must work by Addition, therefore I add these two products together, and they make 8480, which is the Dividend: then I add also the two errors together, 208, and 112, and they make 320, which is the Divisor: wherefore dividing 8480 by 320, the quotient will be 26 ½, which is the true Summ of yards of Damask that he bought, and in Velvet 13 yards ½, and that appeareth by examination, thus: 26½ yards of Damask at 12 Shillings the yard maketh 318 Shillings: then in Velvet he had but 13 yards and ½ which cost 270 Shillings, at 20 Shillings the yard. Now substract 270 out of 318, and there will remain 48, which is the number of Shillings that the Damask did cost more than the Velvet.

20 30
208— 112

Mast. Now shall you have a question of another kind.

A question of debt, the third example.

There are three men that owe Money to me, and I have forgotten what the total Summ is, and what the particulars be.

Schol. Why, then it is impossible to know the debt.

Mast. Peace, you are too hasty, there is more help in it than yet you see. I have three several notes, whereby it appeareth that I did compare their debts together, and found the debt of the first and the second to amount to 47 pound, the debt of the first man and the third man did make 71 pound, and the second man his debt with the third, did rise to 88 pound. Now can you tell what every man did owe, and what was the whole Summ?

Schol. Nay in Truth, but as I perceive that it must be found by conjecture, so I will guess at it, supposing that the first man did owe 20 pound, and the second man 30, and the third ——

Mast. Nay stay there you are too far gone already: you may not suppose a several Summ for every man, for it is enough to suppose one Summ for the first man, and let the other rise as the question importeth. Therefore seeing you set the first man his debt to be 20*l.* the second man cannot owe 30 pound, for the Question says, that their debts added together did make 47 pound, so must the second man his debt be but 27 pound: Now the second debt with the third must make 88: therefore substract 27 out of 88, and there will remain 61, as the third man his debt. Then saith the Question, that the first and third mans debts do make 71: but by this supposition they make 81, that is 10 too much, which I must set for the first error. Now work you the second position.

Schol. I suppose the first mans debt to be 24 pound: then must the second mans debt be but 23 pound, seeing both make but 47 pound. And the second man his debt with the third, do make 88 pound, and the second man oweth but 23: therefore the third man must owe 65 pound. Now the third mans debt with the first, should make 71 pound, and they do make 89 pound, that is 18 pound too much, and that is the second error, which I set down with the first, and their positions in this form; and then I 20 24 multiply cross ways, 20 by 18, and it is 360. And 10 by 24 maketh 240. Also because the signs of 10 18 of the errors be like, I must work by Substraction; therefore I substract 240 out of 360, and there resteth 120, which is the Dividend; then I substract 10 out of 18 by the same reason, and so is the Divisor 8, which is found 15 times in 120: therefore I say that the first man did owe 15 *l.* and then the second man must owe 32 *l.* for those two make 47 *l.* and the third man's debt is 56: for so much remaineth if I abate 15 out of 71, or if I take 32 out of 88.

The fourth example.

Mast. For the fourth example, take this easie question for the variety in work. Two men having several Summs which I know not, do thus talk together: the first saith to the second, If you give me two Shillings of your Money, then shall I have three times so much Money as you. The second man answereth, It were more reason that our Summs were made equal, and so will it be if you give me three Shillings of your Money. Now guess what each of them had.

Schol. I imagin that the first had 9 *s.*

Note.

Mast. Be sure evermore that you take a likely Summ, as in this question, take such a number, that having 2 added unto it may be divided into three equal parts.

Schol. Why? I remember you said before, it matter'd not how false soever I guessed.

Mast. As for the possibility of the solution it is truth: but for easiness in work, the aptest numbers are most convenient.

Schol. I thought no less, and therefore I took 9 as an apt number to be parted into three: but I perceive I should have considered the partition after the Addition of two unto it, and then 7 had been more proper.

Mast. That is true, and then should the second man have 5: for although he have now but the third part of 9, that is 3, yet you must

must remember that he lent the first man 2, and so had he 5 at first.

Schol. Then to go forward: If the second man had three of the first man, then should he have 8 and the first man but 4; so had he double to the first man, yet he said in the question they should have equal, wherefore it appeareth that he hath 4 too much.

Therefore I note that error with this supposition, and guess again that the first hath 10 Shillings: whereunto I add 2 Shillings borrowed of the second man, and then he hath 12 Shillings, so the second man hath remaining but 4, whereunto if I add the 2 that he lent to the first man, it appears he had 6 Shillings at the beginning.

Then take 3 Shillings from the first man, and give to the second; and then hath the first man but 7, and the second 7 10
hath 9, which are not equal, but there are 2 too
many, wherefore I set down both the positions with
their errors, as before you see, and multiply cross, 4 2
so are the products 40 and 14: and because the signs be alike, I take 14 out of 40, and so resteth 26 to be divided: then likewise I take 2 out of 4, and there resteth 2, by which I divide 26, and the quotient will be 13, which is the Summ that the first man had. And so appeareth that 2 being added thereto, the Summ will be 15, so hath the second man but 5, and before he had 7: then take 3 from the first, and put to this 7, and so have each of them 10; therefore does 13 and 7 answer the question.

Mast. For the fifth example take this question. One man said to another, I think you had this year two thousand Lambs: so had I, said the other; but what with paying of tyth out of them, and then the several losses, they are much abated: for at one time I lost half as many as I have now left, and at another time the third part of so many, and the third time ¼ so many. Now guess you how many are left. *The fifth example: a question of Lambs.*

Schol. Because here is mention made of certain parts, I must take a number that may have all these parts, that is to say, ½, ⅓ and ¼, which will be 24, likewise 12 hath the same parts. Therefore I take first 12 to be the number that doth remain, so hath he lost 6, 4, and 3, that is 13, and the whole is 25, but it should be 2000.

Mast. You are deceived, for you have forgotten the 10 part for Tyth, which must be deducted, that is 200, so there remaineth but 1800, and now go on again.

Schol.

Schol. Then to find the error, I take 25 out of 1800, and there remaineth 1775 too few, which I set for the error. Then for the second position I take 24, whose half is 12, the third part 8, and the quarter 6, whereby ariseth 50, which is too little by 1750: therefore I set down both the positions, with their errors thus:

And multiply cross ways 1775 by 24, whereof cometh 42600. Also I multiply 1750 by 12, and there ariseth 21000. And because the signs are like, I substract the one from the other, and so remaineth the Dividend 21600. Then I substract 1750 out of 1775, and there resteth 25, by which I divide 21600, and the quotient is 864, whereof the half is 432, and the third part is 288, the quarter is 216; which being added together, will make 1800: and if you add thereto the tenth which was abated before, then will the whole Summ be 2000.

12	24
1775	1750

And now doth there come a question to my memory which was demanded of me, but I was not able to answer to it: And now methinks I could solve it.

864
432
288
216
1800

Mast. Propose your question.

A question of Sheep and tillage: the first example.

Schol. There is supposed a Law made, that (for furthering of tillage) every man that doth keep Sheep, shall for every ten Sheep ear and sow one Acre of ground: and for his allowance in Sheep pasture, there is appointed for every four Sheep one Acre of Pasture. Now is there a rich Sheep-master which hath 7000 Acres of ground, and would gladly keep as many as he might: by that Statute I demand how many Sheep he shall keep.

Mast. Answer to the question your self.

Schol. First, I suppose he may keep 500 Sheep, and for them he shall have in Pasture after the rate of four Sheep to an Acre, 125 Acres, and in Arable ground 50 Acres, that is, 175 in all: but this error is too little by 6825. Therefore I guess again that he may keep 1000 Sheep, that is, in Pasture 250 Acres, and in tillage 100 Acres, which make 350: that is too little by 6650. Both these errors with their positions, I set down as you see, and multiply them cross 6825 by 1000, and it maketh 6825000, also I multiply 6650 by 500, and there cometh 3325000, which Summ I substract out of the former, and there remaineth

500	1000
6825	6650

3500000

3500000 for the Dividend: likewise I substract the lesser error out of the greater, and there resteth 175, by which I divide 3500000 (the Dividend aforesaid) and the quotient is 20000, so that by this rate he that hath 7000 Acres of ground may keep 20000 Sheep.

Mast. You have done well, notwithstanding both this last question and the next before might be wrought without the second position by the Rule of proportion, as thus: When in this question you found in the first error that for 500 Sheep there must be 175 Acres, then might you reduce it to the Golden Rule thus: 175. 500:: 700. 20000 *i.e.* If 175 Acres will admit in allowance 500 Sheep, then 7000 will have 20000. And so by one position, with the help of the Golden Rule may you answer that question. *Another way of working.*

Likewise for the question of Lambs, when you had found that 12 came of 25, you might have said,

If 25 do leave but 12, what shall 1800 leave? and it would appear to be 864, for 25. 12:: 1800. 864.

Schol. Sir, I thank you for this aid, for it doth much shorten the work of this Rule.

Mast. Yet again I will shew you another way to answer to this last question without the Rule of false position, and that by the Rule of Fellowship, for it appeareth in the proposing of the question, that ten Sheep must have in pasture 2 Acres and ½, and for them must there be eared but one Acre: so it followeth, that for 2 Acres eared, there must be 5 set to Pasture: and if you put them both into one Summ, they will make 7. Therefore look what proportion 7, doth bear to 5 and to 2, such proportion shall any total in this question bear to the pasture ground, and the eared ground. *Another way yet.*

Now answer to this Question.

There is a Cistern with four Cocks, containing 72 barrels of water: and if the greatest Cock be opened, the water will all run out in six hours; at the second Cock it will require eight hours: at the third Cock it will evacuate in no less than nine hours: and the smallest will require twelve hours. Now I demand in what time the Cistern will be emptied if all the Cocks be set open at once. *A question of water: the seventh example.*

Schol. First, I imagin it will be empty in two hours.

Mast. Then must there run out by the first Cock ⅓ of the water, that is 24 Barrels; (for if two hours require 1 Cistern, then 6 hours will require ⅔ or ⅝) and by the second Cock ¼, that is 18, and by the third Cock ⅑, that is 16 Barrels, and by the smallest Cock ⅛, that is 12 Barrels, all which Summs put together, do make 70, as by their

Addition

Addition it doth appear, but it should be 72, therefore the error is too few, by 2.

Schol. Then I begin again, because I think I understand the work, and put three hours for the due time: so shall there run out at the greatest Cock $\frac{1}{3}$, that is 96 Barrels, and at the second hole $\frac{1}{4}$, that is 27, and at the third Cock $\frac{1}{6}$, that is 24, and at the smallest hole $\frac{1}{12}$, that is 18 Barrels, which all together make 165, and should be but 72, so is it too much by 33: therefore I set the errors in order with their positions, and work by multiplication cross, saying, 2 times 3 is 6, and 2 times 33 maketh 66, and because the signs are unlike, I must add these 2 totals together, which make 72: also I add the two errors, and they make 35, by which I divide 72, and the Quotient riseth $2\frac{2}{35}$, whereby I see that all the Cocks being set open, the water will run out in 2 hours, and $\frac{2}{35}$ of an hour.

224
18
16
12
70
2 3
2 — 33†

Mast. Variety maketh you to grow expert in the Rule. Therefore I will exercise you somewhat more with a Question or two.

Hiero King of the *Syracusans* in *Sicilia* had caused to be made a Crown of Gold of a wonderful weight, to be offered for his good success in Wars: in making whereof the Goldsmith fraudulently took out a certain portion of Gold, and put in so much Silver for it, that there was nothing abated of the full weight, although there was much of the value diminished.

Which at length being discovered the King was much displeased: and being desirous to know the truth without breaking of the Crown, proposed the matter to *Archimedes*, which although he could not presently answer, yet he had hopes to solve the matter, and as he chanced to enter into a Vessel full of Water to wash himself, he observed, that as his Body entred into the Water it run over the Tub, whereby his ready wit of such small effects conjecturing greater works, conceived by and by a solution to the King's Question, and therefore rejoycing exceedingly, more than if he had gotten the Crown it self; he thereupon caused two massie pieces, one of Gold, and another of Silver, to be prepared, of the same weight that the said Crown was of: and considering that Gold is heavier than Silver, and therefore Gold of like weight with Silver must needs take up less room, by reason it is of a more compact and solid substance, he was assured, that putting the Mass of Gold into a Vessel brim full of Water, there would not so much Water

ter run out, as when he should put in the Silver Mass of the like weight. Wherefore he tried both, and noted not only the quantities of the Water at each time, but also the difference or excess of the one above the other, whereby he learned what proportion in quantity or bulk is between Gold and Silver of equal weight. And then putting the Crown it self into the Vessels of Water brim full (as before) marked how much Water did run out then, and comparing it with the Water that run out when the Gold was put in, noted how much it did exceed that: and likewise comparing it to the Water that ran out when the Silver was put in, marked how much it was less than that, and by those proportions found out the just quantity of Gold that was taken out of the Crown, and how much Silver was put in stead of it. But seeing *Vitruvius* which writeth this Story, doth not declare the particular work of this matter, it shall be no inconvenience to suppose an Example. And for instance, I suppose the weight of the Crown to be 8 pound, and so of each the other two Masses. And when the Mass of Gold was put into the Water, I imagine that there ran out two pound of Water: and when the Mass of Silver was put in, I suppose there ran out 3 pound $\frac{1}{3}$. Again, when the Crown was put in, there ran out two pound $\frac{1}{4}$: Now to know what quantity of Silver was in the Crown, work by the Rule of False Position, and imagin that there was two pound of Silver, then there must be six pound of Gold, then say thus by the Rule of Proportion. If eight pound of Gold do expel two pound of Water, what shall six pound expel? and it will be 1 pound $\frac{1}{2}$. Again, for the Silver; if eight pound of Silver expel three pound $\frac{1}{3}$ of Water, what shall two pound of Silver put out? it will be $\frac{7}{8}$, now add those two weights of Water together, *viz.* 1 $\frac{1}{2}$ lb. and $\frac{7}{8}$ lb. and they will make two pound $\frac{3}{8}$, and it should be by the supposition two pound $\frac{1}{4}$, so is it too much by $\frac{1}{8}$

Schol. Now I understand the work I think, therefore I pray you let me work the rest of the Question. And because this first Supposition did err, I note that Position and its Error, and take a new Position, esteeming the Silver to be but one pound, so must there be in Gold 7 pound. Then I say, If eight pound of Gold require two pound of Water, what shall seven pound require? and it will be 1 pound $\frac{3}{4}$. Again, if 8 *l.* of Silver expell $\frac{1}{3}$ pound $\frac{1}{3}$ of Water, what shall 1 pound expell? and it will be $\frac{7}{16}$. Now I add those two Summs together, and they make two pound $\frac{3}{16}$, and they should make 2 pound $\frac{1}{4}$, so is it too little by $\frac{1}{16}$. Therefore I set their Positions with their Errors

in order as here followeth; and then I multiply cross ways 2 by $\frac{1}{11}$ and it maketh $\frac{1}{5}$: Likewise 1 multiplied by $\frac{1}{5}$ maketh $\frac{1}{5}$. And because the signs be unlike, I must add these two Products which make $\frac{4}{5}$ or $\frac{4}{5}$ and that is the Dividend.

2 1
$\frac{1}{5}$ $\frac{1}{11}$

Again, I must add $\frac{1}{5}$ to $\frac{1}{11}$, and it will be $\frac{16}{55}$, that is the Divisor. Now I divide $\frac{4}{5}$ by $\frac{16}{55}$ and the Quotient will be $\frac{55}{20}$, that is $1\frac{1}{5}$: whereby I know that there was put 1 pound and $\frac{1}{5}$ of Silver into the Crown, and so much Gold taken out for it.

Mast. Prove it now by Examination, according to the Question.

Schol. If there were 1 pound $\frac{1}{5}$ of Silver, then was there of Gold 6 pound $\frac{3}{5}$. Now say I by the Rule of Proportion: if 8 pound of Gold expell two pound of Water, what shall 6 pound $\frac{3}{5}$ expell, it will be 1 pound $\frac{3}{10}$.

Again, if 8 pound of Silver expell three pound $\frac{1}{2}$ of Water, what shall 1 $\frac{1}{5}$ expell? It will be $\frac{7}{15}$. Now must I add together 1 pound $\frac{3}{5}$ and $\frac{7}{12}$, and they will make 2 pound $\frac{8}{60}$, that is, 2 pound $\frac{1}{4}$, according to the Supposition of the Question: whereby I perceive the Question to be well done. And I cannot but rejoyce at this excellent Invention, so that I am now much more desirous to be perfectly instructed in every part thereof, and, whether the Proportion between Water and Gold be such, that for 8 *l.* of Gold put into a Vessel full of Water, there shall run out two pound of Water, and for as much Silver, whether 3 pound $\frac{1}{2}$ of Water would run out.

Mast. I perceive your meaning, and conjecture your imagination to be thus, that if you knew the exact Proportion between Gold and Silver, and Water, both in their weight and quantities, then you could easily find out the mixtures of them, which thing does not properly fall within our Cognizance in this place.

A Question of the proportion of Gold, Silver, and Quicksilver unto Water.

But seeing you are so desirous to know these things, I will tell you something whereby you may practise your Art in finding it, and propose it in form of a Question. Gold beareth a greater proportion to Water than Silver doth, and their two Proportions be in proportion together, as 48 to 25. But to help you somewhat in this matter, note that the proportion of Quick-silver unto Water, is the just Geometrical mean proportional between the proportion of Gold and Silver unto Water.

And this Proportion is $\frac{200}{21}$. Now if you will know the just numbers of these 3 Proportions, then must you find out 3 numbers in Progression Geometrical, whereof the middlemost must be $\frac{200}{21}$, and the

the first must be unto the last, as 25 to 48. And thus I will leave you to find those Numbers when you be at leisure.

Schol. Sir, I thank you heartily for thus much, for now I see the possibility to find them out. However, because this Question seemeth strange, if it might please you to instruct me somewhat in the order of working it, I should the more easily find the true working.

Mast. You desire too much, and will study for nothing: therefore to oblige you to study the better, I will leave this doubt wholly to your own search: But as touching the generality of the Rule, *Archimedes* needed not to take two Masses of Gold and Silver equal in weight with the Crown, for the Proportion might as well be found in any other weight, yea, although the Mass of Gold were of one weight, and the Mass of Silver of another. As for Example: if the Crown were of 8 pound weight as I did suppose, and I have not so much other fine Gold, but only one pound, and trying that by Water, and finding that it doth expell but $\frac{3}{4}$ of an ounce of Water, by it I may inferr, that 8 pound of Gold would expell 6 Ounces of Water. And likewise of Silver, whereof if I had but two pound, and find that it doth expell three Ounces of Water, then might I affirm, that 8 pound would expell 12 Ounces, that is, one pound weight: and it is as good as if the three Masses were all of one weight. And thus for this time I will make an end of this other part of Arithmetick.

Schol. Although I cannot sufficiently thank you for this, yet I hope you forget not your Promise of teaching the Extraction of Roots, whereof hitherto I have learned nothing.

Mast. I will not break my Promise, but intend (God willing) to perform it, and in the mean time pray be mindful to retain what you have learnt, so will it both conduce to your Profit and Diversion.

FINIS.

DECIMALS made Eaſie:

CONTAINING,

I. An Explanation of Fractions in General, more eſpecially Decimals, and how to Read or Write any Fraction whatſoever.

II. Reduction, Addition, Subſtraction, Multiplication, Diviſion, and the Rule of Proportion in Decimals, with the Demonſtration of each, and its Relation to the Rules for working Vulgar Fractions.

III. The Reaſon why Decimals are wrought as whole Numbers.

IV. The Excellency of Decimal Fractions above any other Fraction that can poſſibly be invented, proved by ſeveral Inſtances.

V. The particular uſe thereof in Computing the Intereſt (Simple or Compound, or Diſcompt) of Money, and Rules for Purchaſing or Selling an Eſtate, whether in Fee, or for Lives or Years, demonſtrated in the Solution of the moſt uſeful Caſes, with Proper Tables relating thereto.

VI. The moſt eaſie Method of Extracting the Square and Cube-Roots of Numbers, whole or broken; alſo the uſe thereof in ſolving ſeveral Geometrical Queſtions.

Wherein are many things never before made Publick.

Uſeful for Maſters and Scholars; alſo for Lawyers and Scriveners, or as a Foundation for any one that intends to acquire conſiderable knowledge in the Mathematicks by his own Induſtry.

By *E. HATTON*, Philomercat.

Nemo Arithmetices ignarus hic ingrediatur.

LONDON:

Printed by *J. H.* for *Charles Harper*, at the *Three Flower de Luce's*, in *Fleet-ſtreet*, and *William Freeman*, at the *Bible*, againſt the *Middle-Temple-Gate*, in *Fleet-ſtreet*. 1699.

DECIMALS made Easie:

CONTAINING,

I. An Explanation of Fractions in General, more especially Decimals, and how to Read or Write any Fraction whatsoever.

II. Reduction, Addition, Substraction, Multiplication, Division, and the Rule of Proportion in Decimals, with the Demonstration of each, and its Relation to the Rules for working Vulgar Fractions.

III. The Reason why Decimals are wrought as whole Numbers.

IV. The Excellency of Decimal Fractions above any other Fraction that can possibly be invented, proved by several Instances.

V. The particular use thereof in Computing the Interest (Simple or Compound, or Discompt) of Money, and Rules for Purchasing or Selling an Estate; whether in Fee, or for Lives or Years, demonstrated in the Solution of the most useful Cases, with Proper Tables relating thereto.

VI. The most easie Method of Extracting the Square and Cube-Roots of Numbers, whole or broken; also the use thereof in solving several Geometrical Questions.

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Decimal Arithmetick.

CHAP. I.

An Explanation of Fractions in General, and how to Read or Write the ſame.

1. A Fraction is part of a Unit, and as no Number can poſſibly be ſo great, or contain ſo many Units but that it may be augmented or encreaſed by adding other Units thereto: ſo no Fraction can be ſo little, *i. e.* no Unit can be divided into ſo many parts but that thoſe parts may ſtill be ſubdivided or made leſs; ſo that the parts of a Unit are infinite as well as the number of Units are. *Vid. Le Grand's Philoſoph. Part* 2. *Ch.* 15. *What parts a Unit may be divided into.*

2. All Fractions have 2 parts, *viz.* firſt the number of parts that the Unit is divided into, and ſecondly, That which ſhews how many of thoſe parts a Fraction contains.

3. The number of parts that a Unit is divided into is called the Denominator, and is wrote under the Numerator: And the number of thoſe parts contained in a Fraction is called the Numerator, and is wrote over the Denominator thus, in $\frac{3}{4}$; *What the Numerator and Denominator of a Fraction.*

$\frac{3}{4}$ —the Numerator.
$\frac{}{4}$ —the Denominator.

So that this Fraction ſhews the Unit is divided into 4 equal parts, and that there are 3 of thoſe parts in the Fraction; ſo is the Fraction to be read *three fourth parts*; and by the ſame Rule

$\frac{1}{2}$ of any thing is to be read One half.
$\frac{2}{3}$ is Two third parts.
$\frac{3}{4}$ is Three fourth parts.
$\frac{4}{5}$ is Four fifths.
$\frac{5}{6}$ is Five ſixths.
$\frac{7}{8}$ is Seven eighths.
$\frac{9}{10}$ is Nine tenths of any thing.

And

And $\frac{3764}{10000}$ is 3 thouſand ſeven hundred and ſixty four ten thouſandth parts; for in all Fractions you are firſt to read the Numerator and then the Denominator.

4. Of Fractions there are two diſtinct kinds, Vulgar and Decimal.

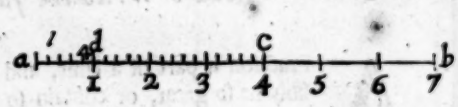
Of Vulgar Fractions there are 4 ſorts, *i. e.*

The ſeveral kinds of Fractions.

1. A Simple or ſingle Fraction } Proper.
2. A Compound Fraction —— }
3. A Simple Fraction } Improper.
4. A Compound Fraction }

5. A Simple Proper Fraction is any of thoſe foregoing, which is immediately the Fraction of a Unit, as the Line *A B* being a Unit, or 1 divided into 7 equal parts, the line *A. C.* is 4 Sevenths, or $\frac{4}{7}$ of the ſaid Line or Unit; and hence it is plain, 1. That any number of Seventh Parts under 7 are Simple Fractions whoſe Denominators are 7. 2*dly*, That $\frac{7}{7}$, or where-ever the Numerator of a Fraction is equal to the Denominator, that Fraction is a Unit or One. But

A Simple proper Fraction, what.



6. A Compound Fraction is not immediately the the Fraction of a Unit as any of the 7th parts of the Unit or Line foregoing, but it is the Fraction of the Fraction (or part of another part) of a Unit, as the Line *a* 4 is $\frac{4}{5}$ of the Line *a d*, and the Line *a d* being $\frac{1}{7}$ of the Line *a b*, therefore the Line *a* 4 is $\frac{4}{5}$ of $\frac{1}{7}$ of *a b*, or a Unit, which $\frac{4}{5}$ of $\frac{1}{7}$ is a Compound Fraction.

A Compound Fraction, what.

Hence it is evident, that a Compound Fraction may be reduced to a Simple, for if it were required to know what part of the Line *a b*, the line *(a l)* is; it muſt be conſidered that the line *a d* being divided into 5 parts, and that there is 7 times *a d* in the line *ab*, there is therefore 7 times 5 of the parts *(a l)* in the line *a b*, ſo that *a l* is $\frac{4}{35}$ part of the line *(ab)* and conſequently the line *a* 4 is $\frac{4}{35}$ of the line *a b*, ſo that the Simple Fraction $\frac{4}{35}$ is equal to the Compound Fraction $\frac{4}{5}$ of $\frac{1}{7}$ of *a b*.

Compound may be reduced to Simple Fractions.

7. Having thus ſhewed you the Nature of a Simple and a Compound Fraction I come to diſtinguiſh between *Proper* and *Improper* Fractions.

A Proper Fraction is either a part or such a Number of parts of a Unit as is less than a Unit, so that the Numerator of this Fraction is always less than the Denominator, as $\frac{5}{7}$ or $\frac{23}{35}$, &c. of the line foregoing is a Proper Fraction thereof, because it is less than the whole line (*a b*.) But

A Proper Fraction, what.

An Improper Fraction has the Numerator greater than the Denominator as $\frac{11}{7}$ of the line (*a b*) which is the whole line *a b* and the line (*a c*) added together, which being more than the line (*a b*) is improperly said to be a part thereof; so also $\frac{28}{7}$ of the line *a b* is an Improper Fraction containing just 4 times the line *a b*.

An Improper Fraction, what.

And hence it follows, that an Improper Fraction may be reduced to a whole Number, or to a whole Number and a Fraction, called a Mix'd Number: for in $\frac{28}{7}$ there is 4 Units, *i. e.* in 28 times (*a d*) there is 4 time (*a b*) which value of an Improper Fraction is always therefore found by dividing the Numerator by the Denominator.

An Improper Fraction reducible to a whole Number.

Likewise from the Division of the line (*a b*) and what has been said, it is plain that one Fraction may be equal to another, and yet of different Numerators and Denominators, as the Fraction $\frac{20}{35}$ is equal to $\frac{4}{7}$, for the line *a b* being divided into 35 parts, (*a c*) is 20 of those parts. or if the line *a b* is divided into 7 parts, then the line (*a c*) is equal to 4 of those 7 parts; so that $\frac{4}{7}$ is the same value with $\frac{20}{35}$, because 20 contains 4 so often as 35 contains 7.

Fractions may be equal tho' of different parts.

From whence it may be gathered, That of 2 Fractions, that is the greater value which has its Numerator neareſt to the value of its Denominator, as I know that $\frac{11}{35}$ is of less value than $\frac{1}{3}$, for 7 contains 4 not twice, but 35 contains 15 above twice, so that $\frac{7}{7}$ or $\frac{35}{35}$ being a Unit (as was said in Paragraph the 5th foregoing) $\frac{1}{3}$ must be nearer a Unit, and consequently greater than $\frac{11}{35}$, for proof of which it is plain by the line (*a b*) above that the line *a c* being $\frac{1}{3}$, is more than the line (*a 3*) which is $\frac{11}{35}$, *Vide Demonstration of Division following.*

To know which of any two Vulgar Fractions is the greateſt.

II. Having shewed how Fractions in general, whether Vulgar or Decimal may be read, and also so much of their Nature, as was necessary to the right understanding thereof; I shall next proceed to shew the property of *Decimal Fractions* more particularly.

A Decimal Fraction is any Fraction whose Denominator is 10 or 100, or some Number that is a Unit, with Cyphers toward the Right hand: And they are called Decimal Fractions, because the Denominator is always either 10, or a Number produced by Multiplying 10 in it self continually, as 10, 100, 1000, 10000, 100000, &c.

Definition of Decimals.

2. By this definition, it is plain that the 2 laſt Fractions under Paragraph the 3d. *viz.* $\frac{3}{10}$ and $\frac{125}{1000}$, and ſuch like are Decimal Fractions; ſo that though the nature of a Decimal Fraction with reſpect to its being a Fraction, be in effect the ſame with that of a Vulgar Fraction: Yet conſidering the nature of the Denominator, the working of Decimal Fractions is much different from that of Vulgar.

3. There is likewiſe a difference in the manner of writing Decimal Fractions from that of Vulgar, For though a Decimal Fraction be ſuppoſed to have a Denominator, yet that Denominator is ſeldom or never put down, For

Numeration of Decimals.

4. If it be conſidered (as was aſſerted in the firſt Paragraph of this general head) That the Denominator is always a Unit, with a Cypher or Cyphers toward the Right hand; it will be very eaſie (the variety being but little) to repreſent the Denominator of a Decimal Fraction by a point Comma, or other mark to be in the ſame place that the Unit of the Denominator would be, if the Fraction were wrote Vulgar Fraction-wiſe: Thus if $\frac{3}{10}$ or $\frac{46}{100}$ were to be expreſſed without their Denominators 10 and 100; it is eaſily done by prefixing a point to the Left hand, the 3 which is in the tens place, ſhews that the Denominator is 10, thus, .3; ſo likewiſe a point being in the third or hundreds place of 46, ſhews the 46 to be 46 hundred parts thus: .46; likewiſe $\frac{34}{1000}$ is wrote without the Denominator thus: .034, a Cypher being added to make the point poſſeſs the thouſands place, becauſe the Denominator is 1000; alſo $\frac{16}{10000}$ is wrote thus: .0016, two Cyphers being placed to the Left hand, that ſo the point may poſſeſs the tens of Thouſands place, which ſhews that the Denominator is 10000.

How to read or write a Decimal.

5. Hence it is not at all difficult to read or write any Decimal without a Denominator by conſidering

1. That the Numerator is firſt to be read or written down as in whole Numbers, and

2. That the parts the Unit is divided into is always ſhewn by the place wherein the point ſtands, thus; .356 is three hundred fifty ſix thouſand parts, *i. e.* The 356 being read, as in the Rules of Numeration of Whole Numbers, the Point ſtanding in the thouſands place ſhews the 356 to be 356 thouſandth parts; and by the ſame Rule may any of the Decimals in the following Table be read.

A TABLE *shewing how to Read or Write Decimals.*

Fifty four Thousand	Three Hundred Twenty One								
								1	Thousand Million
							1		Hundred Million
						1	2		Ten Million
					4	3	2	1	Million
			5	4	3	2	1		Hundred Thousand
				4	3	2	1		Ten Thousand
					3	2	1		Thousand
						2	1		Hundred
							1		Ten

} Parts.

6. This Table is eaſie to be underſtood, The *words* at the top ſhewing the way of reading the Numerator or Figures actually expreſſed, and *thoſe* to the Right hand ſhewing the place the point poſſeſſes, or how many parts the Unit is divided into. Thus the one in the loweſt line being in the Units places is one, and the point being in the Tens place ſhews it to be one tenth, but the 1 in the uppermoſt line is one Thouſand Millionth part; becauſe though the Figure actually expreſſed is one (as the loweſt) yet the point being in the Thouſand Millions place, the 1 is 1 Thouſand Million parts and ſo of the reſt. *Uſe of the Table.*

And thus much for the Nature of Fractions in general, and how to write or read a Decimal Fraction without its Denominator, what farther Demonſtrations are neceſſary to the full underſtanding thereof, I ſhall ſhew in the following Rules in the proper places.

CHAP. II.

Reduction of Decimals.

Why Reduction of Fractions is to be learnt first.

AS Vulgar Fractions must first reduced to a common Denominator before they can be added, &c. so in Decimals Reduction is to be learnt before Addition, Substraction, &c. because all Decimal Fractions of Money, Weight and Measures (Liquid and Dry) have their Original Derivation, and are produced with Vulgar Fractions; so that those Vulgar Fractions must first be reduced into Decimals before those Decimals can be added, Substracted, &c.

One Vulgar Fraction reducible to another.

2. Vulgar and Decimal Fractions being of the same nature with respect to the Fraction compared to a Unit; it follows, that by the same Rule one Vulgar Fraction is Reducible to another, it is also Reducible to a Decimal.

3. Now one Vulgar Fraction may be reduced to another of the same value (though of different Numerators and Denominators) by this proportion.

As the Denominator of any given Fraction
Is to its Numerator;
So is any other Denominator whatsoever
To its Numerator;
which will make a Fraction equal to that given.

For instance, I would Reduce $\frac{3}{4}$ to a Fraction whose Denominator is 24; to find a Numerator to that Denominator, I say

Denom. Numer. Denom. Numer.
4. 3 :: 24. 18.

That is: As 4 (the Denominator given) is in proportion to 3, its Numerator: So is 24 the Denominator of the Fraction required, To 18 its Numerator, which $\frac{18}{24}$ is in value equal to $\frac{3}{4}$.

And by the same Rule, Vulgar Fractions, may be reduced to Decimals.

4. So that (as in the 2d. step above) if you would find the Numerators to Fractions whose Denominators are 10, 100, 1000, &c. which is all one with reducing a Vulgar to a Decimal Fraction, you may do it by the foregoing proportion.

For instance, To reduce $\frac{3}{4}$ to a Decimal Fraction whose Denominator is 100.

As

As 4. 3 :: 100. 75

That is, As 4 is in proportion to 3 its Numerator; ſo is 100 to 75 its Numerator, which is expreſt without its Denominator 100, by placing a point in the hundreds place, as in the laſt Chapter is taught:

So is ¾ equal to the Decimal .75

And by the ſame Rule the Vulgar Fraction ½ may be reduced to the Decimal ⅝ or .5, or the Vulgar Fraction ¼ to the Decimal .25 for

Firſt, As 2. 1 :: 10. .5

Secondly, As 4. 1 :: 100. .25, that is,

As 2 is to 1, ſo is 10 to 5; or as 4 is to 1, ſo is 100 to 25; ſo that the Vulgar Fraction ½, is equal to the Decimal .5 And the Vulgar Fraction ¼, is equal to the Decimal .25, *&c.*

What to do when remainers happen in Dividing.

5. But it many times happens that the Numerator of the Fraction given (when multiplied by 10, 100, 1000, *&c.*) cannot be divided by the Denominator, without having a Remainer, in which caſe it is the beſt way to Multiply the Numerator of the Fraction given by 10000 or 100000, and divide that product by the Denominator given, ſo ſhall the Decimal Fraction exhibited, be much nearer the truth, than if you had Multiplied the Numerator of the Vulgar Fraction given by 10, 100 or 1000, which Rule muſt always be obſerved, eſpecially in producing Decimals, that are afterwards to be Multiplied by any whole Number, that conſiſts of above 1 place.

1. *For inſtance*, If the Vulgar Fraction ⅖ were to be reduced to a Decimal whoſe Denominator is 10.

The Decimal will be found ⅘ or .4

2. If to a Decimal whoſe Denominator is 100.

The Decimal will be found .42

3. If to a Decimal whoſe Denominator is 1000.

The Decimal will be found .428

4. If to a Decimal whoſe Denominator is 10000.

The Decimal will be found .4285, *&c.*

So that taking the ⅖ for the Fraction of a Pound Sterling, if in reducing the ſame to a Decimal, you make the Denominator of that Decimal but 10, you omit or looſe .0285 *l.* which is 6¾*d*; if you make the Denominator of the Decimal but 100, you omit or looſe .0085 which is 2*d.* and if you make the Denominator of the Decimal 1000 you omit, or looſe .0005 which if to be multiplied but by 2, the loſs by that omiſſion will be near 1 Farthing.

6. Hence

What to do when nothing remains and all the dividend is not divided.

6. Hence it is evident that in reducing Vulgar Fractions to Decimals the safest way is to find a Numerator to a Denominator that is at least 10000; and if in dividing the product of the Numerator given Multiplied into 10000 by the Denominator given it happens that nothing remain before you have divided all the Figures in the Dividend, you need not divide any farther, but take the Quotient for the answer, and omit the Cyphers undivided in the Dividend; *for instance*, If you would reduce $\frac{1}{8}$ to a Decimal Fraction whose Denominator is 10000.

As 8. 1 :: 10000
1

8) 10000 (.125
8
20
16
40
40
0

Here you see, That I make no use of the Cypher next the Right hand in the Dividend, because there is nothing remains when I have divided 1000; from whence I gather that in order to the true valuing the Figures in the Quotient, to know when you may, and when you may not put Cyphers towards the Left hand, the Figures therein observe this

General Rule.

Why Cyphers to the Right hand in a Decimal signifie nothing, which is a Rule for reducing a Decimal to its lowest Terms.

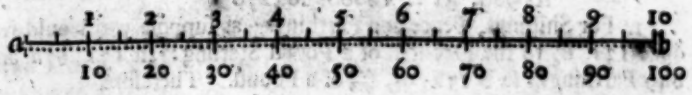
That as many Cyphers as are divided of those you assign for the Denominator of your Decimal Fraction, so many Decimal places must you have in the Quotient, and if there is not so many places in Figures, you must make up the number by prefixing Cyphers towards the Left hand: This Rule you will find useful in the Reduction of Coin, Weight and Measure to Decimals, as will appear by and by.

7. From the sixth or last step it is plain, That a Cypher or Cyphers next the Right hand in the Numerator of a Decimal does neither augment nor diminish the value of that Decimal, for otherwise the omitting

omitting the Cypher in the Units place of the dividend above would make the Quotient (125) too little: But since the places next the Right hand in the Denominator of a Decimal are always Cyphers, therefore Cyphers in the Numerator must be of no value, for $\frac{1250}{10000}$ is of no greater value than $\frac{125}{1000}$ or $\frac{12}{100}$ or $\frac{1}{8}$, and consequently the .125 above is of the same value with .1250.

To Demonstrate this yet more plain.

Suppose the line following be a Unit divided into 10 parts, and each of those into 10 makes 100 parts:



It is undeniably true, that the line (*a b*) being divided into 100 equal parts, the line (*a* 20) which is 20 hundred parts of the line (*a b*) is equal to (*a* 2) which is 2 Tenth parts of that line (*a b*) so 30 hundreds is no more then 3 Tenths, *&c.*

8. And as the same Rule serves both for reducing one Vulgar Fraction to another, and to a Decimal: so likewise may you reduce a Decimal to a Vulgar Fraction. *To reduce a Decimal to a Vulgar Fraction.*

For Instance, If it were required to reduce .125 to a Vulgar Fraction whose Denominator is 16, what must be the Numerator? by the third step of this Chapter.

As 1000. 125:: 16. 2.

So that $\frac{2}{16}$ is a Vulgar Fraction equal to the Decimal .125. and so much for Reduction in General, I proceed to shew next

I. How to reduce the parts of Coin, Weight and Measure to Decimals, and

II. To find the Value of any Decimal, whether the same be of Coin, Weight, *&c.*

9. First for reducing Farthings, Pence, or Shillings, or Shillings and Pence, Pence and Farthings, or Shillings Pence and Farthings into the Decimals of a Pound; you must first reduce the same to the Vulgar Fraction of a Pound, and then that Vulgar to a Decimal, by the third step of this Chapter. *A Rule to reduce Shil. Pence, &c. to the Decimal of a Pound.*

For

For Instance, If you would reduce 9 *d.* into the Decimal of a Pound, consider that 9 *d.* is $\frac{9}{240}$ of a Pound for 1 Penny is $\frac{1}{240}$ of a Pound: then by the ſaid third ſtep, obſerving alſo the fifth and ſixth,

As 240. 9 :: 10000. .0375

So that the 9 Pence being multiplied by 10000, and that product divided by 240, the Quotient is 375, and by the General Rule under the ſixth ſtep, is to have a Cypher before it, which is .0375, the true Decimal of 9 Pence.

10. For Pence and Farthings, as $10\frac{1}{4}$ Pence *&c.* reduce the Pence and Farthings into Farthings, as $10\frac{1}{4}$ *d.* is 41 Farthings; now 1 Farthing being $\frac{1}{960}$ of a Pound, therefore 41 Farthings is $\frac{41}{960}$, and conſequently by the third ſtep of this Chapter.

As 960. 41 :: 10000. .04270

11. For Shillings, Pence and Farthings; as ſuppoſe you would reduce 13 *s.* $6\frac{1}{4}$ to the Decimal of a Pound Sterling: In 13 *s.* $6\frac{1}{4}$ *d.* are 649 Farthings, ſo is 13 *s.* $6\frac{1}{4}$, $\frac{649}{960}$ of a Pound. Therefore

As 960. 649 :: 1000. .676

12. As for reducing any number of Shillings it may be done by the ſame Rules as in the 9th ſtep, *&c.* firſt reducing the Shillings into the Vulgar Fraction of a Pound, and then working by the Rule of Proportion, as 6 *s.* is $\frac{6}{20}$ of a Pound, and as 20 is to 6, ſo is 100 to .30 the Decimal of 6 *s.* But the moſt conciſe way of *Reducing any Number of Shillings to the Decimal of a Pound*, is by this

Rule.

A ſhort way for reducing Shill. to the Decimals of a Pound.

If the Shillings are an even Number, half thereof is the Decimal: or if they are an odd Number, put a Cypher toward the Right hand thereof; and then half that Number is the Decimal ſought; thus:

s.

The Decimal of 2 —— is —— .1 of a Pound Sterling.
4 —— —— .2
6 —— —— .3
8 —— —— .4 *&c.*

Alſo of —— 3 or 30 is = .15 of a Pound Sterling.
5 or 50 = .25
7 or 70 = .35
9 or 90 = .45 *&c.*

13. By

13. By the Rules above, any Number of Pence or Farthings under 1 *s.* are reducible to the Decimal of a Shilling, *&c.* as 3 *d.* being $\frac{3}{12}$ of a Shilling, the Decimal is .25 of a Shilling, ſo likewiſe 6 pence or $\frac{6}{12}$ is .5 of a Shilling.

Alſo 1 Farthing is $\frac{1}{4}$ of a Penny or .25 of a Penny in a Decimal; and $\frac{1}{2}$ Penny is .5 thereof in a Decimal, and ſo of the reſt.

14. Having ſhewed how to reduce any Denomination of Money to the Decimal of a higher Denomination, I ſhall next ſhew how to reduce the parts of weight to Decimals; and firſt of *Avoirdupoiſe* weight, for reducing any number of Ounces into the Decimal of a pound, as 7 ℥ which is $\frac{7}{16}$ of a pound, and therefore in a Decimal is .4375 for *Ounces to Decimals of a Pound.*

As 16. 7 :: 10000. .4375 of a ℔.

Alſo by the ſame Rule 3 Ounces, 8 Drams, or 56 Drams is $\frac{56}{256}$ ℔, (256 Drams being a pound) which Vulgar Fraction in a Decimal is .21875 of a pound, for

As 256. 56 :: 100000. .21875

Alſo for reducing quarters of hundreds and pounds into the Decimals of a hundred weight of 112 ℔, as 14 pound is $\frac{14}{112}$ of a hundred, and that in a Decimal is .125 *C.* for *Quarters of Hund. and Pounds into Decimals of a Hundred.*

As 112. 14 :: 1000. .125

Likewiſe 1 quarter 21 pound is by the ſame Rule .4375 *C.* for 1 *q.* 21 ℔ is 49 ℔ or $\frac{49}{112}$ of a *C.* And

As 112. 49 :: 10000. .4375 of a hundred.

15. For reducing *Troy* Weight as Penny-weights, Grains, *&c.* to the Decimal of an Ounce, or a Pound, obſerve the ſaid Rule for bringing firſt the ſame into Vulgar Fractions, and then thoſe Fractions into Decimals. Thus 15 penny weight is $\frac{15}{20}$ of an Ounce, or .75 of an ℥, for *Troy weight reduced to Decimals.*

As 20. 15 :: 100. .75

Alſo by the ſame Rule 7 penny weight 12 grains, or 180 grains is $\frac{180}{480}$ of an Ounce, or in a Decimal, .375 ℥, for

As 480. 180 :: 1000. .375

And by the ſame Rules it is very eaſie for a mean Capacity to underſtand the reducing the parts of Liquid, or Long Meaſure into the Decimals of ſuperior Denominations.

II. Thus having ſhewed how to reduce the parts of Coin, Weights, *&c.* to Decimals, I come now to ſhew the Method of finding the value of any Decimal, for which take this

General Rule.

A Rule to find the value of any Decimal.

Multiply the Decimal given by such a Number of Units of the next inferiour Denomination, as is equal to a Unit of the Denomination your Fraction is of, and from the Right hand the product cut so many places as are contained in the Decimal given; so those Figures to the Left hand, such point or dash of Separation, are Units of the said next lower Denomination, and those to the Right hand are parts of one of those Units; Examples will Demonstrate this:

Example, 1. of Coin.

What is the value of .775 of a Pound Sterling?
See the Work according to the Rule.

$$\left.\begin{array}{r} .775 \\ \text{Shillings in a Pound} = 20 \end{array}\right\} \text{Multiply}$$

$$\left.\begin{array}{r} \text{Product } 15.500\,s. \\ \text{Pence in a Shilling} = 12 \end{array}\right\} \text{Multiply}$$

6.000 Product, Answer *l.* 00: 15: 06:

The Relation this Rule has to that for finding the value of a Vulgar Fraction.

2. The Rule to find the value of a Vulgar Fraction is, 1st. *To Multiply the Numerator of the Fraction, by such a Number of Units of the next inferiour Denomination, as is equal to a Unit of that Denomination your Fraction is of,* which is the very same with the Rule above, the Decimal given being the Numerator of the Fraction given. 2dly, in Vulgar Fractions, the last product is to be divided by the Denominator of the Fraction, which is the same with cutting off Figures from the product, as in the Rule above; the Denominator of a Decimal being only 1 with Cyphers, as 10, 100, 1000, *&c.* by any of which Numbers another may be divided, by cutting as many figures from the Dividend, as there are Cyphers in the Divisor, so those figures toward the Left hand are the Quotient, and those to the Right hand the dash are the remainer.

3. But

3. But to make this more apparent, the Decimal Fraction above, *viz.* .775 in a Vulgar Fraction is $\frac{186}{240}$, by the 8th ſtep of this Chapter, now 186 multiplied by 20, and divided by the Denominator 240, the Quotient is 15 *s.* as above, and the remainer 120 *s.* reduced into Pence by multiplying by 12, is 1440 Pence, which divided by 240, the Quotient is 6 *d.* and nothing remains.

$\frac{186}{240}$ *l.*

186 } Multiply
20 }

240) 3720 (15 Shilling
1

132

120 } Multiply
12 }

240) 1440 (6 Pence
1

0 remains.

Whereby it is evident, That whoever knows how to find the value of a Vulgar Fraction, may likewiſe by the ſame Rule find the value of a Decimal.

The Demonſtration of the laſt General Rule for finding the value of a Fraction.

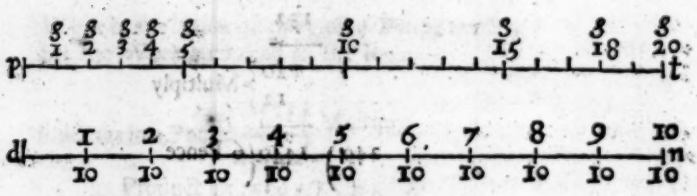
4. Admit the lines (*p t*) repreſent 1 Pound Sterling, divided into 20 equal parts, which parts will each repreſent 1 Shilling. Now ſuppoſe there is a given Fraction, whoſe value is required as .9 of a Pound: Draw another line of the ſame length with the former, and parallel thereto as the line (*D N*), which divide according to the nature of the Fraction, whoſe value you ſeek; as here into 10 equal parts according to your Denominator, and right againſt .9 of thoſe parts, in this laſt line ſtands 18 Shillings in the line of Shillings, which ſhews that .9 of a Pound Sterling is in value 18 Shilling. And the like of any other Fraction.

For, As 10 the whole line (*d x*) or Denominator of the Fraction given, is in proportion to 20 Shillings the whole line (*p x*): So is 9 the Numerator of the Fraction given (as in the lower line (*d* 9) to 18 Shillings, or the line (*p* 18) in the upper line, the value required; and therefore do you Multiply and Divide as in the Rules directed.

10. 20 :: 9

20

10) 18.0 Shillings the Anſwer.



And this Reaſon ſerves not only for Money, but alſo Weight, Meaſure, *&c.* being general as the Rule is.

A Rule to find the value of the Decimal of a Pound Sterling by inſpection.

5. Double the Digit ſtanding next the point in the Decimal given, and (if the next Figure toward the Right hand that be 5 or more) add 1 to the product; then for the figure ſtanding in the ſaid ſecond place, reckon ſo many tens of Farthings, as the ſame is more than 5; or according to the figure if leſs than 5, and reckon the figure in the third place ſo many Farthings, which (as often as they are above 25) make leſs by 1.

The Reaſon of this Rule Demonſtrated with an Example.

1. That place next the point in a Decimal Fraction, is called the *Prime's* place, becauſe it is the firſt Quotient of the Unit divided by 10; ſo that any figure there, is ſo many tenth parts of a Unit; for inſtance

inſtance in the Fraction .775 foregoing the 7, next the point is 7 tenths of a pound or 14 Shilling, and therefore does the Rule direct to double the figure in that place.

2. The Reaſon why the Rule ſays you are to add 1 to the laſt product when the figure in the ſecond place from the point is 5 or more, is becauſe 5 in that place, *i. e.* 05 is always 1 Shilling, becauſe it is $\frac{5}{100}$ of $\frac{1}{20}$, or $\frac{1}{4}$ of $\frac{1}{20}$ of a pound, now $\frac{1}{20}$ of a pound being 2 *s.* $\frac{1}{2}$ of that muſt needs be 1 *s.* which 1 in the Example makes the former product 15 *s.*

3. As the place next the point is ſo many tenth parts, and the ſecond from the point ſo many hundred parts of a Unit, or Tenth parts of $\frac{1}{10}$: So that figure in the third place from the point is ſo many tenth parts of $\frac{1}{100}$, or a thouſand parts of a Unit, as 5 in the Example is 5 tenths of 1 tenth of 1 tenth of a Unit, which is 5 thouſandth part of a Pound.

4. So that what is ſaid in the Rule of calling the figure in the ſecond place above, or under five, ſo many tens, and that in the third place from the point ſo many Farthings (as in the Example 25 Farthings), ſuppoſes 1000 Farthings in a Pound Sterling, and there being but 960 in a Pound, therefore the Rule muſt be ſomething Erroneous. But that is rectified well enough, by deducting (for the 40 Farthings that 1000 exceeds 960), 1 at every 25; for if 1000 Farthings is 40 above 1 Pound, then 500 is 20 above, and 250, 10 too much, or 25 Farthings 1 too much.

5. So that in the Example of .775 *l.* the .7 is 14 *s.* and the .07 being above 5, the 14 *s.* muſt be made 15 *s.*

Then the 2 above 5 in the ſecond ſeven from the point, being ſo many 10 Farthings is 20, and the 5 in the third place is 5 Farthings, which together is 25 Farthings, which by the 4th. ſtep of this Demonſtration being 1 too much, muſt be made 24 Farthings, or 6 *d.* ſo is the value (by inſpection) of .775 *l.* — 15 *s.* 6 *d.* which is evident by the Example next following the laſt General Rule.

6. The ſame General Rule above ſerves alſo for reducing or finding the value of a Decimal, of *Avoirdupoiſe* Weight, or *Troy* Weight, Meaſure, *&c.* as will appear in the Examples following.

Example 2.

What is the value of .3775 of a Hundred Weight, or 112 Pounds?

See

See the Opperation.

Multiply { .3775 of a Hundred / 4 Quart. in a Hundred. }

Multiply { Quar. = 1.5100 parts of a Quarter. / 28 Pounds in 1 Quarter. }

408
102

Multiply { Pounds = 14.2800 parts of 1 Pounds / 16 ℥ in 1 Pound }

168
28

Ounces = 4.4800 parts of an Ounce

qu. lb ℥

So the Answer is 1: 14: 4 $\frac{48}{100}$ or $\frac{12}{25}$.

Example 3.

What is the value of .3125 of a Pound *Troy*? See the Work.

℥ *dw.*

Answer 3: 15.

.3125 of a Pound
12 Ounces in 1 pound } Multiply

3.7500 product = 3 ℥ .75
20 Penny weight in an ℥ } Multiply

15.0000 Penny weights.

Example 3.

Example 4.

What is the value of .5125 of a Barrel of Ale?

.5125 of a Barrel
4 Firkins in a Barrel } Multiply

Firkins = 2.0500 parts of a Firkin
8 Gallons in a Firkin } Multiply.

.4000 parts of a Gallon, or $\frac{4}{10}$

Firkin Gallon.
Answer 2 : 00.4

CHAP. III.

Addition of Decimals.

IN Addition of Decimals there is no difficulty, observing, *to place the points, and places there from, one under another, and to cut from the Right hand the Summ as many places as are in that Decimal that has most places in those given.* As in the several Examples following. *Rule.*

Example 1.

To .4765 Add .073 and .1172
By the same Rule that Numbers of one Denomination are added the Summ of the 3 Decimals is, .6667 *l.* or 13 *s.* and 4 *d.* which is thus proved.

.4765
.073 } Add
.1172
.6667 Summ Total.

Parts

			s.	d.
Parts of 1 Pound	.4765	is =	09 :	06¼
	.073	is =	01 :	05⅓
	.1172	is =	02 :	04½
Total =	.6667	=	13 :	04.

which 13 *s.* 4 *d.* is both the true Summ of the Shillings and Pence given, and also the value of the Decimal .6667.

Example 2.

To 376.04 Add .57983 and .051

Add	376.04
	0.57983
	0.051
	376.67083 Total.

Example 3.

To 476 Add 32.9, and .87 and .775

476.0	Add
32.9	
0.87	
0.775	
510.545 Summ	

By observing the manner of placing and adding the Numbers in the 3 Examples above, you may easily add any Decimal Fractions whatsoever; so that I shall proceed to shew

The

The Relation the Rule for Adding Decimals has to that for Vulgar Fractions.

To add Vulgar Fractions or mixt Numbers together. The Rule is, *To reduce the Fractions to a common Denominator, and then the Summ of the Numerators divided by the common Denominator, quotes the Summ or Answer.*

So it is in Decimals, which have always a common Denominator, for as much as Cyphers to the Right hand of the Numerator of a Decimal does not alter its value by the 7th step of Chap. 2. So that the Decimal Fractions in the last Example in a Common Denominator stand thus: which are the same with those above, the Denominator to that Fraction that has most places, being always the common Denominator, as here 1000, by which 2545 (the Summ of the Numerators) being divided the Quotient is 2.545 the true Summ sought, so that the Rule for adding a Decimal is the very same with that for adding a vulgar Fraction, but the operation for Decimals is much easier for 2 Reasons. 1. Because they need no reducing to common Denominators; and 2. Because the Denominator or Divisor whereby the Summ of the Numerators is divided, being 10, 100, or 1000, *&c.* divides any other Summ without Work, only by cutting off as many places toward the Right hand the Dividend, as there are Cyphers in the Divisor, as in the Example 2545 is divided by 1000, by only cutting off 545, so is the Quotient 2.545, or $2\frac{545}{1000}$.

.900
.870
.775

2.545 the Summ (as before) of the Fractions.

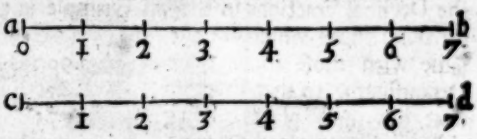
The Demonstration of Addition of Fractions.

If the Summ of the Numerators and Denominators of Fractions would give the true result or aggregate, as if $\frac{2}{3}$ and $\frac{1}{4}$ were $\frac{3}{7}$, then this Rule would not seem so difficult as many fancy it: But they think it strange that the Summ of 2 or 3 Fractions cannot be known, without the trouble of first reducing the Fractions to one Denominator, and afterward dividing the Summ of the Numerators thereby: I shall therefore shew first why Fractions are first to be reduced to a common

Denominator; and ſecondly why the Summ of the Numerators is to be divided thereby.

The neceſſity of reducing Fractions to a Common Denominator, before Addition.

1. It is impoſſible to add Fractions that have different Denominators together, ſo as to give the true Total; becauſe the Unit in each Fraction being divided into a different number of parts, it cannot be expreſſed without reducing thoſe different parts into one and the ſame, how the Summ of the Numerators bears proportion to the Denominators: But if the Denominators are alike, then the Summ of the Numerators is the Summ of the Fractions, if divided by one of the Denominators thus: $\frac{3}{7}$ and $\frac{5}{7}$ is $\frac{8}{7}$, becauſe in both Fractions the Unit is divided into 7 parts, and the Summ of 3 and 5 parts in either Line is 8 parts, which is one whole Line and $\frac{1}{7}$ part more, and therefore is the Summ of the Numerators always to be divided by the Denominators, that ſo it may be known how many Units the Total of the parts contains: As in the Example above, likewiſe $\frac{4}{7}$ and $\frac{6}{7}$ is $\frac{10}{7}$, now to know how many times $\frac{7}{7}$ or 1, equal to the Line *a b*, is in $\frac{10}{7}$ the way muſt be to divide the Number of parts in the Summ of the Numerators (as here 10) by the number of parts that the Unit is divided into (which is here 7) and the Quotient ſhews the Units, and the remainer the parts of a Unit, as $\frac{10}{7}$ of the Line *a b* or *c d* is once the whole Line (or $\frac{7}{7}$) and 4 parts, or the diſtance from *a* to 4, or *c* to 4.



Why the Summ of the Numerators are to be divided by the Denominator.

I ſhall ſhew next before the uſe of Decimals, why Decimals are added as whole Numbers.

CHAP.

CHAP. IV.

Subſtraction of Decimals.

Subſtraction of Decimals is performed in every reſpect as Subſtraction of whole Numbers of one Denomination, obſerving the Rule given in Addition for the placing of the Decimals, and pointing out the Decimal places of the remainer, which muſt be always the ſame with that of the moſt places of the Decimals given. *Rule.*

Example 1.

From .976 take .037647?

From — .976000
Take — .037647 } Add
Reſts — .938353 }

Proof — .976000 Summ.

Example 2.

From 376.4 take .0075?

From 376.4
Take — .0075 } Add
Reſts 376.3925 }

Proof — 376.4 Summ

Example 3.

From 376 take .376

From	376.0	
Take	.376	Add
Rests =	375.624	
Proof	376 =	Summ

Example 4.

From 376 take 1.47689

From	376	
Take	1.47689	Add
Rests =	374.52311	
Proof =	376.00000 =	Summ

The Relation the Rule for Substraction of Decimals has to that of Vulgar Fractions.

In Vulgar Fractions the Rule to Substract one from another, is first to *Reduce them to one and the same Denominator, and then the difference of the Numerators placed over the Common Denominator is the Remainer or Answer.*

And the very same is to be observed in Substraction of Decimals: they needing no Reduction to a Common Denominator (as was said in Addition) being so already, so that the difference between the Numerators, which are the Decimals given, is the Answer; which shews that the nature of the operations in Decimals and Vulgar Fractions are the same; though the Work of Decimals be less troublesome, for the reason mentioned in this part, in Addition foregoing.

The

The Demonſtration of Subſtraction of Fractions.

As Fractions cannot be added (for the reaſons foregoing in Addition) unleſs they are reduced to a common Denominator, ſo neither can the difference be diſcovered without that: For if the difference between 2 Fractions of different Denominators were required, as of $\frac{2}{3}$ and $\frac{1}{4}$, the Difference of the Numerators is 1, but that 1 is neither $\frac{1}{3}$ nor $\frac{1}{4}$, ſo that neither of thoſe Denominators will ſerve, there muſt therefore be ſuch a Denominator found as will ſerve both Fractions, which cannot be done unleſs the Numerators are altered, and made proportionable to that new or common Denominator. *The reaſon of reducing Fractions to a Common Denominator.*

As here, If you Multiply 3 (one of the Denominators given by 4, it will produce 12, and the other Denominator 4 multiplied by 3 is alſo 12, now if you Multiply the reſpective Numerators by the ſame Number you multiplied its Denominator, it will produce a new Numerator, which will have ſuch proportion to the new Denominator, as the Primitive Numerator had to the Primitive Denominator.

Thus becauſe I multiplied the Denominator 3 given by 4, I Multiply its Numerator 2 by the ſame Number, and the product is 8, ſo the new Fraction is $\frac{8}{12}$ equal to $\frac{2}{3}$ given; ſo likewiſe the other Denominator 4, multiplied by 3, and its Numerator by the ſame Number produces $\frac{9}{12}$, equal to $\frac{3}{4}$.

Now the Fractions being thus in a common Denominator as $\frac{9}{12}$, and $\frac{8}{12}$ the Difference is undeniably $\frac{1}{12}$; becauſe in each of theſe new Fractions the Unit is divided into one and the ſame number of parts, and the proportion of 9 the new Numerator is to 12 its Denominator; as the Numerator 3 in the Fraction given is in proportion to its Denominator 4: And likewiſe 8 is to 12 in the other new Fraction, as 2 is to 3 in the other Fraction given, and conſequently being in the ſame proportion the Fractions muſt be of the ſame value with thoſe given, becauſe the value of Fractions conſiſts in the difference of the Numerator and Denominator. *The Reaſon of the Difference between two Fractions.*

So the difference between 9 and 12 in the new Fraction is $\frac{3}{12}$, which is equal to $\frac{1}{4}$, the difference between the 3 and 4 in the Fraction given: ſo that $\frac{9}{12}$ and $\frac{3}{4}$ is of the ſame value becauſe $\frac{3}{12}$ makes each of them a Unit, and ſo of any other Fraction.

The reaſon why Decimals are ſubſtracted as whole Numbers you have next before the uſe of Decimals.

CHAP.

CHAP. V.

Multiplication of Decimals.

Rule. Multiply your Decimals in every reſpect, as though they were whole Numbers, and cut off for Decimals ſo many places toward the Right hand of the product, as there are Decimal places both in the Multiplicand and Multiplier.

And if ſo many places be not found in the product, you muſt make up the Number, by placing Cyphers toward the Left hand next the point.

Example 1.

Multiply 34.25 } A mixt Number by a mixt.
by 3.2 }

6850
10275

109.600 product

To Multiply Shillings and Pence by Shilings and Pence, &c. by Decimals, &c.

Under this Example of Multiplying a mixt number by a mixt will fall the Solution of thoſe pretended Problems, propoſed by ſome who fancy them ſufficient Teſts to try the abilities of an Accomptant, though there is nothing more eaſie to thoſe that underſtand either Vulgar Fractions or Decimals: the Queſtions are of this Nature; ſay they.

What is the product of 3 *s.* 4 *d.* by 6 *s.* 6 *d.*

The

The Anſwer by Decimals is thus:

s.	*d.*		*s.*	
3	3	Equal to	3.25	} Multiply
6	6	is Equal to	6.5	}

1625
1950

21.125 Shillings for Anſwer.

The value of the Decimal in the product .125 of a Shilling, is $1\frac{1}{2}$ *d* ſo that the Anſwer is 21 *s.* $1\frac{1}{2}$ *d.*

This and the like Queſtions are performed by Vulgar Fractions thus: *By Vulgar Fractions.*

$3\frac{3}{12}$ *s.* by $6\frac{6}{12}$.

Theſe Numbers in Fractions are $\frac{39}{12}$ by $\frac{78}{12}$, and their product (as by the Rules given for Vulgar Fractions is $\frac{2028}{144}$ *s.* or 21 *s.* $1\frac{1}{2}$ *d.* as before.

But ſome are apt to take the product of 39 *d.* by 78 *d.* which is the Numerator, 3042 for the Anſwer in Pence, and think the Rules of Reduction are not right, becauſe that product is not the true Anſwer.

I am ſure if that were the Anſwer, the Rules for Multiplication both of Vulgar Fractions and Decimals muſt be wrong; which cannot be, as is eaſily ſhown by undeniable Reaſon and Demonſtrations; (ſome of which follows) but that it is not applicable to any Rule in Reduction to take the 3042 for the anſwer in pence I am very well aſſured, nor is there any foundation for ſuch an opinion either from Rule or Reaſon, 3042 pence being juſt 12 times the true anſwer 21 *s.* $1\frac{1}{2}$ *d.* becauſe they divide the 3042 by 12 inſtead of 144, as appears plain from the method of working the Queſtion by Vulgar Fractions above.

Example 2.

Example 2.

Multiply 34.25 } A mixt Number by a Decimal.
by .32 }

6850
10275

10.9600 Product.

Example 3.

Multiply 34.25 } Mixt Numb. by a whole Numb.
by 32 }

6850
10275

1096.00 Product.

Example 4.

Multiply .3425 } A Decimal by a whole Number.
by 32 }

6850
10275

10,9600 Products.

Example 5.

Example 5.

.3425 } A Decimal by a Decimal.
.32 }

6850
10275

.109600 Product.

A ſecond Example of a Decimal by a Decimal.

.1723
.012

3446
1723

.0020676 Product.

The Relation that Multiplication of Decimals has to that of Vulgar Fractions.

The Rule to Multiply Vulgar Fractions together, is to Multiply the Numerators together for a new Numerator, and the Denominators together for a new Denominator.

And in the very ſame manner we Multiply Decimals, to inſtance in the laſt Example, $\frac{1723}{10000}$ by $\frac{12}{1000}$ the product of the Numerators 1723, and 12 is 20676; and the product of the Denominator 10000 by the other 1000 produceth 10000000; ſo that the new Numerator 20676 being placed over the new Denominator 10000000, the Fraction which is the product is $\frac{20676}{10000000}$, which expreſſed without its Denominator, as by the 4th ſtep, 2d. General head, of Chap. 1ſt. is .0020676 = the product above.

The Demonſtration of Multiplication of Fractions.

It may ſeem ſtrange to ſome that the product is not greater than the Multiplier, or Multiplicand, ſince it is always greater in whole Numbers.

Why the Product in Multiplication of Fractions is leſs than either of the Factors.

But the Reaſon is plain, ſince the multiplying Units encreaſes thoſe Units *ad infinitum*; but the Multiplication of parts of a Unit does ſtill decreaſe, or leſſen the parts to infinite littleneſs; ſo that if a Fraction be multiplied by 1, it produceth the Fraction given, as $\frac{1}{10}$ multiplied by 1 produceth $\frac{1}{10}$; but if that $\frac{1}{10}$ be multiplied by leſs than 1 (*i. e.* by any Fraction) then it follows that the product muſt be proportionably leſs than $\frac{1}{10}$, as the Multiplier is proportionably leſs than 1: Thus if $\frac{1}{10}$ be multiplied by $\frac{5}{10}$, or half of 1, the product will be as much leſs than $\frac{1}{10}$ (the Fraction given) as the Multiplier $\frac{5}{10}$ is leſs than a Unit: as $\frac{5}{10}$ by $\frac{5}{10}$ produces $\frac{25}{100}$ (or $\frac{1}{4}$) which is $\frac{1}{2}$ the Multiplier, becauſe that Multiplier is but half of 1.

The Multiplicand and Multiplier are a Compound Fraction of Unity.

Hence it is evident, That to Multiply any Number by a Fraction, is nothing but to take ſuch a part of the Multiplicand for the product, as the Multiplier is of 1: As to Multiply 24 (or any other Number) by $\frac{5}{10}$ (or $\frac{1}{2}$) is to take $\frac{5}{10}$ or $\frac{1}{2}$ of 24, which is 12 for the product, and by the ſame Rule $\frac{1}{2}$ multiplied by $\frac{1}{4}$ produces a quarter of $\frac{1}{2}$, which is $\frac{1}{8}$, equal to $\frac{1}{4}$ of $\frac{1}{2}$ of 1.

Hence it is alſo evident, That to Multiply $\frac{1}{3}$ by $\frac{1}{4}$ being only to take $\frac{1}{4}$ of $\frac{1}{3}$ of a Unit: The Multiplicand and Multiplier are one Compound Fraction of a Unit.

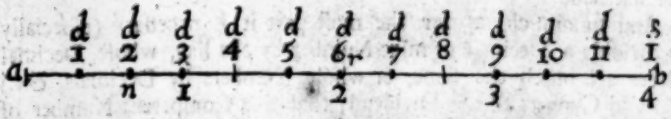
And to reduce any Compound Fraction to a ſimple one of the ſame value, is to Multiply the Numerators together for a new Numerator, and alſo the Denominators for a new Denominator, according to the Rule in the relation laſt foregoing.

The reaſon of reducing Compound to ſimple Fractions.

Now, that ſuch Multiplication will reduce Compound Fractions to ſimple ones of the ſame value, appears beſt by an inſtance of what is undeniable and plain, for Example, It is plain and beyond diſpute, that $\frac{2}{3}$ of $\frac{3}{4}$ of 1 Shilling Sterling is 6 pence; For the Numerators 2 and 3 multiplied together is 6, and 3 by 4 the Denominators is 12, ſo $\frac{6}{12}$ is the product, now $\frac{1}{12}$ of a Shilling being 1 Penny, and conſequently $\frac{6}{12}$ ſix pence proves the Rule to be true.

Or

Or to make this yet more apparent, The line *a b* is one Shilling divided into 12 equal parts or pence.



Likewiſe the ſame line divided into 4 equal parts, and each of thoſe 4 into 3 equal parts; then will $\frac{1}{3}$ of $\frac{1}{4}$ be $\frac{1}{12}$ of the whole line *(a b)* for in 1 fourth there are 3 parts, and conſequently in 4 fourths (or the whole line, or Unit) there muſt be 12 of theſe parts; which ſhews the Reaſon why the Denominators are to be multiplied together for the Denominator of a ſimple Fraction.

And the Reaſon why the Numerators are multiplied together (as in the Fraction above, $\frac{1}{3}$ of $\frac{1}{4}$) is becauſe $\frac{1}{3}$ of $\frac{1}{4}$ of the line *(a b)* as *(a n)* is but $\frac{1}{3}$ part of the Fraction $\frac{2}{3}$ of $\frac{1}{4}$ (as the line *(a r)* therefore $\frac{2}{3}$ of $\frac{1}{4}$ or *(a r)* or $\frac{2}{12}$ is 2 times *(a n)* or *(a n)* which ſhews why the Numerator 2 is to be multiplied by 2, to give the Number of the twelfth parts in the ſimple Fraction of the line *(a b)* required.

The reaſon why Multiplication of Decimals is performed as whole Numbers, is ſhewn next before the uſe of Decimals.

CHAP. VI.

Diviſion of Decimals.

DIviſion by Decimals is performed in every reſpect as that of whole Numbers. So that there's no difficulty in it; and after Diviſion is made, take the following Rule for giving the true value of the Quotient.

A General Rule for the Quotient.

For knowing what places in the Quotient are Decimal, take the difference between the Decimal places in the Dividend, and thoſe in the Diviſor, and ſo many as that difference is, you muſt ſeperate by

The Reaſon of this, is ſhewn in the Demonſtration of Diviſion.

by a point from the Quotient toward the Right hand thereof, and if there are not ſo many, make up the Number by Cyphers toward the left hand.

And foraſmuch, as for the moſt part it is neceſſary (eſpecially in dividing a Decimal or mixt Number by the like, whoſe Decimal places are much the ſame, or whole Numbers by Decimals, *&c.*) To add Cyphers to the Dividend, that ſo a Competent Number of Decimal places may come out in the Quotient; therefore to know in all caſes what Cyphers to add to the Dividend in places toward the Right hand, take this Rule.

A General Rule for the Dividend.

The Reaſon of this is ſhewn in the Demonſtration of Diviſion of Decimals.

To the Number of Decimal places in the Diviſor, add the Number of Decimal places that you would have in the Quotient (as 3 is ſufficient if the Quotient is not to be multiplied by any thing, and there happen to be a remainer in dividing) and as many as that Summ is, make ſo many Decimal places in the Dividend, by adding Cyphers if requiſite.

Example 1.

To divide a mixt Number by a mixt, as 109.6 *by* 3.2?

```
3.2) 109.6000 (34.250
     ......
      96
     ----
      136
      128
     ----
       80
       64
     ----
      160
      160
     ----
        00
```

Note,

Note, This proves the first Example in Multiplication of Decimals.

Example 2.

To divide a mixt Number by a Decimal, as 10.96 *by* .32.

```
.32) 10.9600 (34.25
      96...
     ------
      136
      128
     ------
       80
       64
     ------
      160
      160
     ------
        0
```

Note that this proves the second Example in Multiplication of Decimals; and though I add 3 Cyphers, that so I may have 3 Decimal places in the Quotient according to the last General Rule; yet because nothing remains before I have divided the Cypher in Units place, I therefore Cancel it as useless, and according to the first General Rule in Division, I say the difference between 4 Decimal places in the Dividend, and 2 in the Divisor is 2, therefore I seperate 2 of the first figures toward the Right hand the Quotient for Decimals.

Example 3.

Example 3.

To divide a whole Number by a whole Number, so as to have a Competent Number of Decimals in the Quotient, as 1096 *by* 32.

```
32) 1096.000 (34.25
    96...
    ----
     136
     128
     ---
      80
      64
      --
     160
     160
     ---
       0
```

Note, This proves the truth of the third Example in Multiplication.

Example 4.

To divide a mixt Number by a whole Number that is greater than the Dividend, as 10.96 *by* 32.

```
32) 10.9600 (.3425
    96...
    -----
     136
     128
     ---
      80
      64
      --
     160
     160.
     ----
       0
```

Example 5.

Example 5.

To divide a Decimal by a Decimal, as .0020676 *by* .1723.

```
.1723) .0020676 (.012
       1723
       ----
        3446
        3446
        ----
           0
```

Note, This proves the truth of the last Example in Multiplication of Decimals.

Example 6.

To divide a Decimal by a whole Number, as .9 *by* 324.

```
324) .9000 (.0027
     648
     ----
      2520
      2268
      ----
       252 Remains.
```

Note, That in the two last Examples, the difference between the Decimal places in the Dividend and Divisor, is more than there are places in the Quotient: Therefore according to the first Rule, I prefix Cyphers toward the Left hand in the Quotient to make up such Number.

The Relation that Division of Decimals has to that of Vulgar Fractions.

Decimals may be Divided as Vulgar Fractions and the contrary; as by the Examples following.

There is no Rule in Decimals performed more contrary to Vulgar Fractions than Division, not but that Division of Decimals may be done the same way with that of Vulgar Fractions, or Division of Vulgar Fractions

Fractions the same way that Decimals are; but then in Decimals there will always be a superfluous number of Cyphers, both in the Numerator and Denominator, if worked as Vulgar Fractions, the Fractions (when Division is performed *By multiplying the Numerator of the Divisor into the Denominator of the Dividend for the Denominator of the Quotient, and the Denominator of the Divisor in the Numerator of the Dividend for the Numerator of the Quotient*) being so far from their lowest Terms, that their values are seldom or never perceivable without reducing.

Rule.

Why Decimals must not be divided as Vulgar Fractions.

For instance, Suppose $\frac{5}{10}$ is to be divided by $\frac{25}{100}$ in Vulgar Fraction ways, according to that Rule, the Quotient is $\frac{500}{250}$, so that in this Vulgar Fraction way of Dividing Decimals, here is not only 2 Cyphers, one in the Numerator and the other in the Denominator superfluous; but when they are cut off the Quotient is $\frac{10}{25}$, which before you can know the value of, you must divide 50 by 25, according to the Rules foregoing, which was all that was required at first, if to be divided Decimal way; so that all that has been done thus far to dividing these Decimals as Vulgar Fractions is found unnecessary.

$$\frac{25}{100}\overline{)}\,\frac{5}{10}\,(\,\frac{500}{250}$$

2. Therefore the shortest and best way of dividing Decimals is to divide only the Numerator of the Dividend by that of the Divisor, adding a Cypher or Cyphers (when too little to be divided, as in the last Example above) both to the Numerator and Denominator.

3. And also divide the Denominator of the Dividend by that of the Divisor, and the Quotients will be the respective Numerators and Denominators of the Quotient required.

Thus $\frac{5}{10}$ divided by $\frac{25}{100}$ (by adding 1 Cypher to 5, and another to 10, making the Numerator of the Dividend 50, and the Denominator 100) the Quotient will be $\frac{2}{1}$ or 2; but more of this in the Demonstration following.

4. And by the same Rule may Division of Vulgar Fractions be performed (though not so good a way) as in the Vulgar Fractions, agreeing with the foresaid Decimals, to divide $\frac{1}{2}$ by $\frac{1}{4}$, because 2 the Denominator of the Dividend cannot be divided by 4, the Denominator of the Divisor, therefore I add a Cypher to 2, and 1 in the Dividend making it $\frac{10}{20}$, and then the 20 divided by 4 quotes 5, and the 10 by 1 quotes 10: So is the Quotient $\frac{10}{5}$ or 2 as before.

$$\frac{1}{4}\overline{)}\,\frac{10}{20}\,(\,\frac{10}{5}\text{ or }2$$

5. But.

5. But becauſe the Numerator and Denominator of the Dividend cannot always be divided by thoſe of the Diviſor without Remainers not being certain as Decimals; it is therefore more exact, and is uſed in dividing Vulgar Fractions, to Multiply the Numerators and Denominators of the Dividend and Diviſor croſs, as by the Rule foregoing; but more of this in the Denominator following.

The demonſtration of Diviſion of Fractions.

In demonſtrating this, I ſhall ſhew firſt the reaſon of the Rule for Vulgar Fractions, and then for that of Diviſion of Decimals.

1. *To augment the value of any Fraction is nothing but to Multiply, or add to its Numerator, or to take from, or divide the Denominator*; Becauſe a Fraction is nearer or farther from a Unit, as the Numerator is nearer or farther from the value of the Denominator; for $\frac{1}{1}$, $\frac{2}{2}$, $\frac{3}{3}$, *&c.* are each a Unit; but then in comparing 2 Fractions, reſpect muſt be had to the Denominators; for the leſs that is (when there is the ſame difference between the Numerators and Denominators) the greater is the Fraction thus: $\frac{2}{3}$ is more than $\frac{7}{11}$, though there be the ſame difference between the Numerators and Denominators, becauſe $\frac{2}{3}$ wants but $\frac{1}{3}$ of a Unit, whereas $\frac{7}{11}$ wants $\frac{4}{11}$ of a Unit; now $\frac{1}{11}$ of any thing being more than $\frac{1}{13}$, therefore that which wants $\frac{4}{11}$ of being a Unit, wants more than that which wants but $\frac{1}{13}$ of being ſo. And hence it follows

2. *That to make a Fraction leſs is to encreaſe or add to the Denominator.*

For as the Fraction (for Example) $\frac{3}{4}$ of a yard is made twice ſo much if the Numerator 3 is multiplied by 2, *viz.* $\frac{6}{4}$, or $1\frac{2}{4}$ yard; ſo is that ſame $\frac{3}{4}$ made but half ſo much, *viz.* $\frac{3}{8}$ (or 1 quarter and $\frac{1}{2}$ Quarter of a yard) if the Denominator 4 be multiplied by 2.

3. Now from theſe 2 Actions thus made out, the reaſon of the Rule for dividing Vulgar Fractions will appear. For admit $\frac{2}{3}$ is to be divided by $\frac{1}{4}$, if it were divided by a Unit the Quotient would be $\frac{4}{3}$; and therefore ſince it is to be divided but by $\frac{1}{4}$ of 1, the Quotient muſt be 4 times as much as $\frac{4}{3}$.

The Reason of Division. 4. For as in whole Numbers, so in Fractions, by how much the less the value of the Divisor is by so much the more will the Quotient be; which in our Example, (according to the first Axiom) is $\frac{16}{5}$ or 4 times $\frac{4}{5}$, which is a plain reason why the Numerator of the Dividend is multiplied by the Denominator of the Divisor to give the Numerator of the Quotient.

$\frac{3}{4}) \frac{4}{5} (\frac{16}{5}$

5. Now the reason why the Denominator of the dividend is to be multiplied by the Numerator of the Divisor is this: Suppose the $\frac{4}{5}$ were to be divided by $\frac{3}{4}$; by the 4th step, the Quotient will be but $\frac{1}{3}$ part of $\frac{16}{5}$ because $\frac{3}{4}$ (this new Divisor) is 3 times as much as $\frac{1}{4}$ (the former Divisor) therefore to make the Quotient $\frac{16}{5}$, to be $\frac{1}{3}$ less, is (according to the second Axiom) to Multiply its Denominator by 3 the Numerator in this second Example, and the product is the Denominator of the Quotient sought.

$\frac{3}{4}) \frac{4}{5} (\frac{16}{15}$

6. So that in short by multiplying the Numerator of the dividend by the Denominator of the Divisor (as *per* the third and fourth step we make the Quotient proportionably greater than the dividend as the Divisor (taking the Numerator for 1) is less than Unity: and by multiplying the Denominator of the Dividend by the Numerator of the Divisor, we again make the Quotient proportionably less as the said Numerator is greater than 1, as *per* the first and second and fifth steps.

The Demonstration of Division of Decimal Fractions. 7. The reason of the Rule for the dividing of one Decimal by another is this: When the Decimal places in the Numerator of the Dividend are less or equal to those of the Divisor there is Cyphers added to make the Division more exact as in dividing .4 by .25, there can no error proceed from placing Cyphers to the Right hand the 4 that so 25 may be had therein, because (as is said before) *A Cypher toward the Right hand in a Decimal neither encreaseth nor diminisheth the value thereof*, $\frac{4}{10}$ the dividend given, being as much as .40, *viz.* $\frac{40}{100}$ or $\frac{400}{1000}$ because though 'tis true the Numerator is encreased by adding the Cyphers, yet the Denominator encreasing at the same time as much, makes the Decimal to retain the same value.

8. Now the most rational way of dividing one Fraction by another (as is declared in the relation of Division foregoing) is to divide the Numerator of the Dividend by that of the Divisor for the Numerator of the Quotient, and likewise the Denominators for that of the Quotient,

Quotient, so in the Example above, in dividing $\frac{4}{10}$ by $\frac{25}{100}$, I make $\frac{4}{10}$ into $\frac{400}{1000}$ that so I may have the Divisor 25 the oftner therein, whereby the Quotient will be more exact than if I should only divide 40 by the 25, so that 400 divided by 25 gives 16 for the Quotient, and the Denominator 1000 divided by the other which is 100, the Quotient is 10; so it appears that the Quotient is $\frac{16}{10}$, or 1.6. For this is the ground of that Rule for pricking off Decimals from the Quotient, which says the Number so separated must be equal to the difference between the Decimal places in the Dividend and Divisor, which is evident; because that difference is always equal to the Number of Cyphers in the Denominator of the Quotient, by which Denominator, if the Numerator of the Quotient be divided, that Quotient will always shew the true value of the Fraction, which Division is always performed without working, because any Number is divided by 10, 100, 1000, &c. by cutting off as many places in the Dividend, as there are Cyphers in the Divisor; so is 16 divided by 10, by cutting off 6, and the Quotient is 1.6.

$$\frac{.25}{100}\Big)\ \frac{.400}{1000}\ \Big(\frac{16}{10}\Big\}\ \text{or } 1.6$$

And where the Quotient is of little value as in case of dividing a Decimal by a whole Number as in the Margent, the reason is the same, the Rules in Numeration of Decimals being observed for reading a Decimal Fraction without its Numerator, which will shew the reason of putting Cyphers toward the Left hand in such a Quotient, as here the Quotient $\frac{198}{100000}$ is expressed, without the Denominator 100000, by making the point to possess the hundred thousands place.

A second Example.

$$\frac{146}{1}\Big)\ \frac{29000}{100000}\ \Big(\frac{198}{100000}\Big\}\ \text{or } .00198$$

The reason why Decimals are divided as whole Numbers is shewn in the next Chapter but one.

CHAP. VII.

The Rule of Proportion by Decimals.

THis Rule being only Multiplication and Division, which are taught already; I might upon that account pass it by, but because Proportions are used in the Calculation of Interest, (which will be treated on afterwards) I shall give some Examples for the satisfaction of some, who may want the assistance of the same.

A Rule for stating your Question.

There being always 3 Numbers given to find a fourth: 2 of which are of (or are reducible to) one Denomination; therefore place that of a contrary Denomination to the said 2, down first; then to the Left hand that (or in the first place in course of Writing) place that Number of the said 2 that are of one name, which has dependance on that already placed down, and then place the third Number next the Left hand, which being done, for performing the opperation, this is the Rule.

A Rule for working any Question.

This is the same with that in whole Numbers, for if you multiply the second and third Numbers together, and divide the product by the first Number, the Quotient is the fourth proportional Number.

Example 1.

If $1\frac{1}{4}$ Yards of Cloath cost 1 Ponnd 3 Shillings, what will a Piece of $24\frac{1}{2}$ Yards cost at that rate.

The parts of a Yard reduced into the Decimals of a Yard, and the Shillings into the Decimal of a Pound, as in the 9, 10, 11, 12, 13, 14 and 15 Rules in Reduction foregoing, the Numbers when stated will stand thus:

Yards

Yards *l.* *Yards*
125 . 1.15 :: 245
1.15

1225
245
245

l.
125)28.8750(22.54 Answer or *l.* 22
250 10 *s.* 9½ *d.*

317
250

675
625

500
500

0

Example 2.

C. Q. lb.
If 112 : 3 : 25 of Sugar coſt 75 *l.* 17 *s.* 6½ *d.* what will 112 Pound coſt at that rate?

l. *l.* *C.*
The Decimal of a Hundred that 3 : 25 is, is .9732, and the 17 *s.* 6½ in the Decimal of a Pound is .877 therefore the work by the Rule is ſtated and worked thus:

```
C.        L.        C.
12.9732.  75.877.   1
                1
                ------------
                                l.
12.9732)75.8770000(5.848 Answer, or l. 5. 16:
                  ...:.  11⅓.
                ------------
                1101100
                ------------
                 632440
                ------------
                1135120
```

Another way of Division.

Note, That in this way of dividing, I have omitted putting down the products, having deducted the same as I multiplied, thus (in the first Figure 5) I say, 5 times 2 in the Divisor is 10, from 10 in the Dividend rests 0, and 5 times 3 is 15, and 1 carried is 16, from 17 rests 1, and 5 times 7 is 35, and 1 carried is 36, from 37 rests 1, and carry 3, 5 times 9 is 45, and 3 is 48, from 48 in the Dividend (borrowing 4) and the rest is 0, and 5 times 12 is 60, and 4 is 64, from 75 rests 11, which you see I put down, and the remainer is 110110, and so of the rest of the work.

The Demonstration of this Rule of Proportion.

The reason of Multiplication, and Division of Decimals have been made plain before; it only here remains to prove that Multiplying and Dividing, as *per* the last Rule above, will give a fourth proportional Number, that is, one that bears such proportion to the third, as the first does to the second; for whatever Number does so, is the true Number sought.

1. Admit .3 is in proportion to 2.7, as 12 is to a fourth Number unknown; for which fourth Number I put (*u*) then the proportion stands thus:

.3. 2.7 :: 12. *u*

2. And

2. And by the 16 of the 6 Euclid the Product of 3 into (*n*) the first and fourth is equal to the Product of 2.7 and 12 the 2 and 3 Numbers, thus:

.3 *n* is equal to 32.4

3. Now it follows, that if the Product of *n* by .3 is equal to 32.4, then (*n*) is equal to 32.4 divided by .3 thus:

n equal to $\frac{32.4}{.3}$

Now 32.4 divided by .3 gives the fourth proportional Number, *viz.* 108, which is the Answer to the Question. And if the second, third and fourth steps are compared, you will find that this Number 108 is discovered in the very same way, as *per the Rule for working any Question*, foregoing.

CHAP. VIII.

The Reason why the opperations in the Rules foregoing by Decimals, are performed as in those of whole Numbers.

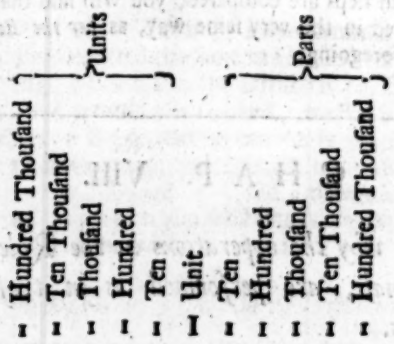
HAving in the Chapters before shewed the Nature of Fractions in General, and the Relation Decimals have to Vulgar Fractions, both in the Nature of the Fractions themselves and in their opperations, I shall here shew the Relation Decimals have to whole Numbers, and why they are wrought alike.

That Decimals should be wrought as whole Numbers, seems very unaccountable to some, who have not well considered the matter: I shall therefore give the Reason why they are so: which proceeds *from the Degrees of Figures places, and the Nature of the Denominators of Decimals.*

In

In Decimal Fractions the Unit being (as has been ſufficiently explained) divided into 10 parts, and each 10th part into a 10, each 100th part into 10, *&c.* makes the Subdiviſion of the parts to have the ſame decreaſe or increaſe that whole Numbers have; which is the chief Reaſon why the Adding, Subſtracting, Multiplying, Dividing, *&c.* of them agrees exactly with whole Numbers. For inſtance,

There is the ſame Geometrical proportion between 1 tenth part of a Unit and a Unit, as there is between a Unit and 10; for 10 Tenth parts is 1, and Ten Units is 10; ſo that if you ſuppoſe a Unit divided into 100000 parts the encreaſe of every place from the Right hand toward the Left, is by 10 not only till you come to a Unit; but until you come to 100000 Units, *&c. ad infinitum.*



Some Authors have ſo confounded Fractions and whole Numbers with the uſeleſs Terms of decreaſing and encreaſing Numbers, as though they were quite different in their Numeration: whereas there is nothing more plain than that the value of each of theſe Numbers above Unity, does gradually decreaſe (according to the places) to the 1 hundred thouſandth part of a Unit; and on the contrary the value of each Number encreaſeth gradually, according to the place from one hundred thouſand part of a Unit, or lower to 100000, *&c.* Units or higher; ſo that there is no manner of difference in the encreaſe or decreaſe of decimal Parts and Units, they making together as it were one infinite line, of which a Unit is the Center.

And

And every ten Decimal parts in one and the ſame place let the place be were it will, making one of the parts in the next place towards the Left hand, juſt as 10 in any place of whole Numbers makes one in the next place toward the Left hand: It muſt needs follow that there being no difference between the encreaſe or decreaſe of Decimal places and thoſe of whole Numbers, there can be none in the manner of Adding, Subſtracting, *&c.* of Decimals and thoſe of whole Numbers.

2. As to the ſeparation of places for Decimals from the Totals, Remainers, Products, and Quotients after the work of Addition, Subſtraction, Multiplication, or Diviſion is ended, the Reaſon is purely from the Number of Cyphers that the Denominators would conſiſt of if wrote down; which being always a Unit with Cyphers towards the Right hand, divides any Number only by a point of ſeparation of ſo many places, as the ſaid Denominator has Cyphers in it: But this having been ſo fully diſcuſſed in the Relations and Demonſtrations foregoing, I need to ſay no more thereof in this place.

CHAP. IX.

Containing the Excellent uſe of Decimals in General, and that they exceed any other Fraction.

1. THE Excellency of Decimals does ſufficiently appear (if there were nothing elſe to be ſaid for them) from their Nature being like whole Numbers, as in the laſt Chapter has been Explained, upon which account it is impoſſible there ſhould ever be any kind of Fraction invented ſo excellent as Decimal; becauſe no other Fraction can be expreſſed without the Denominators, and wrought as though they were Integers; theſe alone retaining the ſame proportion between their degrees or places that whole Numbers do, and conſequently none can be wrought as whole Numbers but theſe, And

Decimal Fractions the beſt.

1. Becauſe wrought as whole Numbers.

2. *From the Nature of the Denominator. How made preferable to Duodecimals in Measuring.*

2. In almost all operations by Fractions there is occasion to divide some Number by the Denominator, which in Decimals only is such as may be performed without work (as I have shewed above) I know there are some that put a great value upon Duodecimals, in the Mensuration of all manner of Work, as Wainscot, Painting, Plaistering, *&c.* but let but the Rule wherewith the Dimentions are taken, *viz.* the Yard, Foot, *&c.* be divided into 100 parts, according to the Nature of a Decimal Fraction, and the work is done by an easie Multiplication of one Denomination, whereas in other ways of Measuring work, there are many tedious Reductions and Divisions, which are avoided by the Decimal way.

3. *In dividing the lines on Mathematical Instruments*

3. Another thing wherein the Excellent use of Decimals appears, is their aptness for dividing any Measure into parts, which parts agree, so exactly with those of whole Numbers, that the parts serve either as Units themselves, or as parts of a Unit, as the Divisions on Diagonal and other Scales, and Instruments of singular use in all the parts of the Mathematick; all or most of which are Decimally divided.

4. *In the precise and easie Calculation of Tables.*

4. The Excellency of Decimal Fractions above all others that can possibly be thought on, appears in their usefulness in the precise and speedy Calculation of all manner of Tables; and more especially in those two excellent ones, of Compound Interest and of Logarithms; which if they were to be calculated by any other Fractions but Decimals, would not only be four or five times as long in perfecting; but when done, would be abundantly more troublesome in the using of those Numbers when Calculated, in all which, besides in things of lesser moment, I have very much experienced the exceeding usefulness of Decimals.

5. *In computing Interest especially Compound.*

5. Decimals are of admirable use in the calculating the Interest due for any Summ of Money, which may be found of 5 Summs of Money, by Decimals sooner than that of 1 Summ by Reduction; which I shall therefore shew more particularly in

CHAP.

CHAP. X.

Concerning Intereſt of Money.

IN Diſcourſing on this Subject, I ſhall ſhew you

1. What Principal Money is.
2. What Intereſt is both Simple and Compound.
3. What the Rate of Intereſt is.
4. What Diſcount of Money is.
5. How to find the Simple Intereſt of any Summ.
6. To find the Compound Intereſt of any Summ.
7. To find the true Diſcount of any Summ for any time.
8. Give you 2 Tables, the one of Simple, the other of Compound Intereſt.
9. Shew the uſe of the Table of Simple Intereſt.
10. Of that of Compound Intereſt
11. How to Calculate any of the Numbers in the Compound Intereſt Table.

1. Principal Money is the Money that is put out to Intereſt, which is probably called ſo; becauſe it is always the Stock or Summ which begets or produces Intereſt. *Principal what.*

2. So that Intereſt Money, or Uſury is the Summ paid to the Lender of the Principal by the Borrower, for his Intereſt therein; which is Simple or ſingle Intereſt, when the Intereſt of the Summ actually received by the Borrower is only paid. *Intereſt Simple and Compound what.*

But Compound Intereſt, is the Intereſt not only of the Original Principal Lent; but alſo of ſuch Intereſt as is not paid when due, being added to the Principal firſt Lent; as if I have 100*l.* at Intereſt for a Year, for which I am to receive 6*l.* but that not being paid when due, I reckon Compound Intereſt for the ſecond Year, *i. e.* the Intereſt of the *l.* 100, and of the 6 Pounds that ſhould have been paid for Intereſt the firſt Year.

3. The rate of Intereſt is the Summ that is paid for Intereſt of *l.* 100 for 1 Year; for it is from that Rate, Reaſon or Proportion that the Intereſt of any other Summ for any other time is Computed. *Rate of Intereſt what.*

Diſcount, what it is. 4. The Diſcount of Money, is the abating of part of the Principal for paying the Remainer before it would grow due; as if I have *l.* 100 due to me at the end of 1 Year; but having preſent occaſion for Money, I diſcount at the rate of 6 *per Cent. per Ann.* ſo that I muſt receive but *l.* 94: 6: 9½ preſently, to ſatisfie my Debt of *l.* 100 due to me a Year hence. How this is done will appear in the 7th. head of this Chapter.

To find the Simple Intereſt of any Summ for 1 Year. 1. Rule. 5. The way to find the Simple Intereſt, or increaſe of any Summ for any time; as ſuppoſe of 520 for 1 Year, is thus: ſay

As *l.* 100, is to 100, and the Intereſt thereof for a Year, ſo is 520 (or any other Summ) to the Summ it is amounted, or encreaſed unto in that time.

See the Work.

Prin.	*P. and Int.*	*Prin.*
100.	106::	520.
		106
		312
		52

Anſwer = *l.* 551.20

For any Number of Years. 2. Rule. Here you ſee that the *l.* 520 is encreaſed to 551: 4 *s.* in 1 Year, at the Rate of *6 per Cent. per Ann.* or if you would know the Simple Intereſt for any Number of Years, firſt Multiply the Rate by the Number of Years, and add the Summ to 100 *l.* then

As 100 is to that Summ Total,

So is the Principal given, to the amount required.

Example

Example 1.

What is the amount or encreaſe of *l.* 520 for 10 Years at 6 *per Cent per Ann.* Simple Intereſt?

According to the Rule 10 times the Rate is 60 *l.* which added to 100 makes 160, then

Prin. P. and Int. Prin.
As 100. 160 :: 520.
160

312
52

l. 832.00 Anſwer

Here you ſee that *l.* 520 in 10 Years will encreaſe to *l.* 832 at the rate aforeſaid; and the difference between *l.* 832 and *l.* 520, the Principal is *l.* 312, the Intereſt alone.

Intereſt for 4th parts of a Year. 3. Rule.

Again, If you would know the Simple Intereſt of 520 *l.* or other Summ for half or quarter of a Year: Take half or a quarter of the Rate, and add to 100 *l.* working then as before.

Example 2.

What is the Intereſt of *l.* 520 for $\frac{1}{4}$ of a Year?

100. 101.5 :: 520
520

2030
50.75

527.800

Here

Interest for any Number of Days. 4. Rule.

Here it is plain, that the amount of *l.* 520 for a quarter of a Year is *l.* 527.8 or the Interest is *l.* 7: 16: 0

Lastly, If you would find the Simple Interest of any Summ for any time, that is not the fourth part of a Year, Multiply .000164383 (which is the Interest of 1 *l.* for 1 day) by the days for which you would know the Interest, and that Product by the Summ given, and that last product is the Answer.

Example 3.

What is the Interest of *l.* 120: 6: 0 for 211 days?

.000164383 } Multiply
211 }

164383
164383
328766

.034684813 } Multiply
120.3

1040544 39
69369626
34684813

4.172583 0039 Answer.

Here you see the Answer by the Rule is *l.* 4: 3: 5¼

A Rule to find the Compound Interest of any Summ.

6. To find the Compound Interest of any Summ: You must have an operation in the Golden Rule for each of the Years the Summ is forborn; the first is to be wrought as in the 1 Rule of the last head, and the fourth Number in that must be the third Number in the second Operation; the fourth in the second must be the third in the third, *&c.* the 2 first Numbers in each operation being the same, as the amount of *l.* 520: 4 Years forborn is *l.* 656: 9: 9, thus:

l. 100.

	l.	l.	l.	l.
For the amount the first Year, as	100.	106 :	520.	551.2
For the second Year,	100.	106 :	551.2.	584.272
For the third Year,	100.	106 :	584.272 :	619.32832
For the fourth Year,	100.	106 :	619.32832 :	656.488

Here you see that if *l.* 100 require 106, *l.* 520 will require *l.* 551.2 at the end of 1 Year, and that will require *l.* 584.272 the second Year, and that *l.* 619.32832 the third, and that requires *l.* 656.488 the fourth Year.

As for the other things in Compound Interest, as the present worth of any Summ due any numbers of Years hence; the amount of Annuities, *&c.* I shall shew how to Calculate the same under the 11th. General head following.

7. To find the Discount to be allowed for paying any Summ before due; some only deduct the Interest, as if I have *l.* 520 due to me at the end of 1 Year; they usually deduct the Interest for that time, which by the first Rule to the 5th. General head of this Chapter is *l.* 31 : 4 : 00, and being deducted from the *l.* 520 leaves *l.* 488 : 16 : 00 to be paid presently: but the true method of finding the Discount upon *l.* 520, that is but due a Year hence to know what is payable presently, is by this Proportion. *The Discount how to find.* *Rule.*

As *l.* 106 is to 100 : so is *l.* 520, to *l.* 490 : 11 : 3$\frac{3}{53}$, which fourth number is the Summ to be paid presently.

So likewise the Discount of any other Summ, for any other time is found, by first finding the Interest of *l.* 100, for the time that the Money is paid before due: then

As the Summ of that Interest and *l.* 100, is in Proportion to *l.* 100 : so is any other Summ whatever to the Summ to be paid presently to satisfie the Debt, Discount being allowed of *l.* 6 *per Cent. per Ann.* for prompt payment. *General Rule.*

8. *The Tables of Simple and Compound Interest follow.*

Simple

Simple Interest at 6 per Cent.

Princi-pal.	One Day,				Two Days,				Three Days,				Four Days.			
	l.	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*	*l.*	*s.*	*d.*	*q.*
l. 2	0	0	0		0	0	0		0	0	0	1	0	0	0	1
3	0	0	0		0	0	0	1	0	0	0	1	0	0	0	2
4	0	0	0		0	0	0	1	0	0	0	2	0	0	0	2
5	0	0	0		0	0	0	1	0	0	0	2	0	0	0	3
6	0	0	0	1	0	0	0	2	0	0	0	3	0	0	1	
7	0	0	0	1	0	0	0	2	0	0	0	3	0	0	1	
8	0	0	0	1	0	0	0	2	0	0	1		0	0	1	1
9	0	0	0	1	0	0	0	3	0	0	1		0	0	1	2
10	0	0	0	1	0	0	0	3	0	0	1	1	0	0	1	2
20	0	0	0	3	0	0	1	2	0	0	2	1	0	0	3	
30	0	0	1		0	0	2	1	0	0	3	2	0	0	4	2
40	0	0	1	2	0	0	3		0	0	4	2	0	0	6	1
50	0	0	2		0	0	3	3	0	0	5	3	0	0	7	3
60	0	0	2	1	0	0	4	3	0	0	7		0	0	9	2
70	0	0	2	3	0	0	5	2	0	0	8	1	0	0	11	
80	0	0	3	1	0	0	6	1	0	0	9	2	0	1	0	2
90	0	0	3	2	0	0	7		0	0	10	2	0	1	2	1
100	0	0	4		0	0	7	3	0	0	11	3	0	1	3	3
200	0	0	7	3	0	1	3	3	0	1	11	2	0	2	7	2
300	0	0	11	1	0	1	11	2	0	2	11	2	0	3	11	1
400	0	1	3	3	0	2	7	2	0	3	11	1	0	5	3	1
500	0	1	7	3	0	3	3	2	0	4	11		0	6	6	3
600	0	1	11	2	0	3	11	1	0	5	11		0	7	10	2
700	0	2	3	3	0	4	7	1	0	6	10	3	0	9	2	2
800	0	2	7	2	0	5	3	1	0	7	10	2	0	10	6	1
900	0	2	11	2	0	5	11		0	8	10	2	0	11	10	
1000	0	3	3	2	0	6	6	3	0	9	10	1	0	13	1	3

A TABLE of Simple Interest at 6 per. Cent.

Principal	Five Days. l. s. d. q.	Six Days. l. s. d. q.	Seven Days. l. s. d. q.	Eight Days. l. s. d. q.
s.14	0 0 0	0 0 0	0 0 0	0 0 0 1
15	0 0 0	0 0 0	0 0 0	0 0 0 1
16	0 0 0	0 0 0	0 0 0 1	0 0 0 1
17	0 0 0	0 0 0	0 0 0 1	0 0 0 1
18	0 0 0	0 0 0 1	0 0 0 1	0 0 0 1
19	0 0 0	0 0 0 1	0 0 0 1	0 0 0 1
l. 1	0 0 0	0 0 0 1	0 0 0 1	0 0 0 1
2	0 0 0 1	0 0 0 2	0 0 0 2	0 0 0 2
3	0 0 0 2	0 0 0 3	0 0 0 3	0 0 1
4	0 0 0 3	0 0 1	0 0 1	0 0 1 1
5	0 0 1	0 0 1 1	0 0 1 1	0 0 1 2
6	0 0 1 1	0 0 1 2	0 0 1 3	0 0 1 3
7	0 0 1 1	0 0 1 3	0 0 2	0 0 2 1
8	0 0 1 2	0 0 2	0 0 2 1	0 0 2 2
9	0 0 1 3	0 0 2 1	0 0 2 2	0 0 2 3
10	0 0 2	0 0 2 1	0 0 2 3	0 0 3 1
20	0 0 4	0 0 4 2	0 0 5 2	0 0 6 1
30	0 0 5 3	0 0 7	0 0 8 1	0 0 9 2
40	0 0 7 3	0 0 9 2	0 0 11	0 1 0 2
50	0 0 9 3	0 0 11 3	0 1 1 3	0 1 3 3
60	0 0 11 3	0 1 2 1	0 1 4 2	0 1 6 3
70	0 1 1 3	0 1 4 2	0 1 7 1	0 1 10
80	0 1 3 3	0 1 7	0 1 10	0 2 1 1
90	0 1 5 3	0 1 9 1	0 2 0 3	0 2 4 2
100	0 1 7 3	0 1 11 2	0 2 3 2	0 2 7 2
200	0 3 3 2	0 3 11 1	0 4 7 1	0 5 3 1
300	0 4 11	0 5 11	0 6 10 3	0 7 10 2
400	0 6 6 3	0 7 10 3	0 9 2 2	0 10 6 1
500	0 8 2 2	0 9 10 1	0 11 6	0 13 1 3
600	0 9 10 1	0 11 10	0 13 9 2	0 15 9 1
700	0 11 6	0 13 9 2	0 16 1 1	0 18 5
800	0 13 1 3	0 15 9 1	0 18 4 3	1 1 0 2
900	0 14 9 2	0 17 9	1 0 8 2	1 3 8
1000	0 16 5 1	0 19 8 3	1 3 0	1 6 3 2

A TABLE of Simple Interest at 6 per Cent.

Principal	9 Days, l. s. d. q.	10 Days, l. s. d. q.	11 Days, l. s. d. q.	12 Days, l. s. d. q.
£.11	0 0 0	0 0 0	0 0 0	0 0 0 1
12	0 0 0	0 0 0 1	0 0 0 1	0 0 0 1
13	0 0 0 1	0 0 0 1	0 0 0 1	0 0 0 1
14	0 0 0 1	0 0 0 1	0 0 0 1	0 0 0 1
15	0 0 0 1	0 0 0 1	0 0 0 1	0 0 0 1
16	0 0 0 1	0 0 0 1	0 0 0 1	0 0 0 1
17	0 0 0 1	0 0 0 1	0 0 0 1	0 0 0 1
18	0 0 0 1	0 0 0 1	0 0 0 1	0 0 0 1
19	0 0 0 1	0 0 0 1	0 0 0 1	0 0 0 2
l. 1	0 0 0 1	0 0 0 1	0 0 0 2	0 0 0 2
2	0 0 0 3	0 0 0 3	0 0 0 3	0 0 0 3
3	0 0 1	0 0 1	0 0 1 1	0 0 1 1
4	0 0 1 2	0 0 1 2	0 0 1 3	0 0 1 3
5	0 0 1 3	0 0 2	0 0 2 1	0 0 2 1
6	0 0 2 1	0 0 2 1	0 0 2 2	0 0 2 3
7	0 0 2 2	0 0 2 3	0 0 3	0 0 3 1
8	0 0 2 3	0 0 3 1	0 0 3 2	0 0 3 3
9	0 0 3 1	0 0 3 2	0 0 4	0 0 4 1
10	0 0 3 2	0 0 4	0 0 4 1	0 0 4 3
20	0 0 7	0 0 7 3	0 0 8 3	0 0 9 2
30	0 0 10 2	0 0 11 3	0 1 1	0 1 2 1
40	0 1 2 1	0 1 3 3	0 1 5 1	0 1 6 3
50	0 1 5 3	0 1 7 3	0 1 9 2	0 1 11 2
60	0 1 9 1	0 1 11 2	0 2 2	0 2 4 2
70	0 2 0 3	0 2 3 3	0 2 6 1	0 2 9 1
80	0 2 4 1	0 2 7 2	0 2 10 2	0 3 1 3
90	0 2 8	0 2 11 2	0 3 3	0 3 6 2
100	0 2 11 2	0 3 3 2	0 3 7 1	0 3 11 1
200	0 5 11	0 6 6 3	0 7 2 3	0 7 10 2
300	0 8 10 2	0 9 10 1	0 10 10	0 11 9 3
400	0 11 10	0 13 0 3	0 14 5 2	0 15 9 1
500	0 14 9 2	0 16 5 1	0 18 1	0 19 8 3
600	0 17 9	0 19 8 3	1 1 8 1	1 3 8
700	1 0 8 2	1 3 0	1 5 3 3	1 7 7 1
800	1 3 8	1 6 3 3	1 8 11	1 11 6 3
900	1 6 7 2	1 9 7	1 12 6 2	1 15 6
1000	1 9 7	1 12 10 2	1 16 2	1 19 5 1

A TABLE of Simple Interest at 6 per Cent.

Principal	13 Days. l.	s.	d.	q.	14 Days. l.	s.	d.	q.	15 Days. l.	s.	d.	q.
19	0	0	0		0	0	0	1	0	0	0	1
10	0	0	0	1	0	0	0	1	0	0	0	1
11	0	0	0	1	0	0	0	1	0	0	0	1
12	0	0	0	1	0	0	0	1	0	0	0	1
13	0	0	0	1	0	0	0	1	0	0	0	1
14	0	0	0	1	0	0	0	1	0	0	0	1
15	0	0	0	1	0	0	0	1	0	0	0	1
16	0	0	0	1	0	0	0	1	0	0	0	2
17	0	0	0	1	0	0	0	2	0	0	0	2
18	0	0	0	2	0	0	0	2	0	0	0	2
19	0	0	0	2	0	0	0	2	0	0	0	2
11	0	0	0	2	0	0	0	2	0	0	0	2
2	0	0	1		0	0	1		0	0	1	1
3	0	0	1	2	0	0	1	2	0	0	1	3
4	0	0	2		0	0	2	1	0	0	2	1
5	0	0	2	2	0	0	2	3	0	0	3	
6	0	0	3		0	0	3	1	0	0	3	2
7	0	0	3	2	0	0	4		0	0	4	1
8	0	0	4	1	0	0	4	2	0	0	4	3
9	0	0	4	3	0	0	5		0	0	5	2
10	0	0	5	1	0	0	5	2	0	0	5	3
20	0	0	10	1	0	0	11		0	0	11	3
30	0	1	3	2	0	1	4	3	0	0	15	3
40	0	1	8	2	0	1	10		0	1	1	2
50	0	2	1	2	0	2	3	3	0	2	5	2
60	0	2	6	3	0	2	9	1	0	2	11	2
70	0	2	11	3	0	3	2	3	0	3	5	1
80	0	3	5		0	3	8	1	0	3	11	1
90	0	3	10	1	0	4	1	3	0	4	5	1
100	0	4	3	1	0	4	7	1	0	4	11	1
200	0	8	6	2	0	9	2	2	0	9	10	1
300	0	12	9	3	0	13	9	2	0	14	9	2
400	0	17	1		0	18	5		0	19	8	3
500	1	1	4	2	1	3	0		1	4	7	3
600	1	5	7	3	1	7	7	1	1	9	7	
700	1	9	10	3	1	12	2	2	1	14	6	1
800	1	14	2	1	1	16	9	3	1	19	5	2
900	1	18	5	2	2	1	5		2	4	4	2
1000	2	2	8	3	2	6	0	1	2	9	3	3

A TABLE of Simple Interest at 6 per Cent.

Principal	30 D. or 1 M.				2 Months,				3 Months,				4 Months.			
	l.	s.	d.	q.	l.	s.	d.	q.	l.	s.	d.	q.	l.	s.	d.	q.
5	0	0	0	1	0	0	0	2	0	0	0	3	0	0	1	0
6	0	0	0	1	0	0	0	2	0	0	1	0	0	0	1	1
7	0	0	0	1	0	0	0	3	0	0	1	1	0	0	1	2
8	0	0	0	2	0	0	0	3	0	0	1	1	0	0	1	3
9	0	0	0	2	0	0	1		0	0	1	2	0	0	2	0
10	0	0	0	2	0	0	1	1	0	0	1	3	0	0	2	1
11	0	0	0	2	0	0	1	1	0	0	2	0	0	0	2	2
12	0	0	0	3	0	0	1	2	0	0	2	0	0	0	2	3
13	0	0	0	3	0	0	1	2	0	0	2	1	0	0	3	0
14	0	0	0	3	0	0	1	2	0	0	2	2	0	0	3	1
15	0	0	0	3	0	0	1	3	0	0	2	3	0	0	3	2
16	0	0	0	3	0	0	1	3	0	0	2	3	0	0	3	3
17	0	0	1		0	0	2		0	0	3	0	0	0	4	0
18	0	0	1		0	0	2		0	0	3	1	0	0	4	1
19	0	0	1		0	0	2	1	0	0	3	2	0	0	4	2
11	0	0	1	1	0	0	2	1	0	0	3	2	0	0	4	3
2	0	0	2	1	0	0	4	3	0	0	7	0	0	0	9	2
3	0	0	3	2	0	0	7		0	0	10	2	0	1	2	1
4	0	0	4	3	0	0	9	2	0	1	2	1	0	1	6	3
5	0	0	5	3	0	0	11	3	0	1	5	3	0	1	11	2
6	0	0	7		0	1	2	1	0	1	9	1	0	2	4	2
7	0	0	8	1	0	1	4	2	0	2	0	3	0	2	9	1
8	0	0	9	2	0	1	6	3	0	2	4	2	0	3	1	3
9	0	0	10	2	0	1	9	1	0	2	8	0	0	3	6	2
10	0	0	11	3	0	1	11	3	0	2	11	2	0	3	11	1
20	0	1	11	2	0	3	11	1	0	5	11	0	0	7	10	3
30	0	2	11	2	0	5	11		0	8	10	2	0	11	10	0
40	0	3	11	1	0	7	10	3	0	11	10	0	0	15	9	1
50	0	4	11		0	9	10	2	0	14	9	2	0	19	8	3
60	0	5	11		0	11	10	0	0	17	9	0	1	3	8	0
70	0	6	10	3	0	13	9	3	1	0	8	2	1	7	7	1
80	0	7	10	2	0	15	9	1	1	3	8	0	1	11	6	3
90	0	8	10	2	0	17	9	0	1	6	7	2	1	15	6	0
100	0	9	10	1	0	19	8	3	1	9	7	0	1	19	5	2
200	0	19	8	3	1	19	5	1	2	19	2	0	3	18	10	3
300	1	9	7		2	19	2	0	4	8	9	1	5	18	4	1
400	1	19	5	2	3	18	10	3	5	18	4	1	7	17	9	3
500	2	9	3	3	4	18	7	2	7	7	11	1	9	17	3	1
600	2	19	2		5	18	4	1	8	17	6	1	11	16	8	2
700	3	9	0	2	6	18	1	0	10	7	1	2	13	16	2	0
800	3	18	10	3	7	17	9	2	11	16	8	2	15	15	7	1
900	4	8	9	1	8	17	6	1	13	6	3	3	17	15	0	3
1000	4	18	7	2	9	17	3	1	14	15	10	3	19	14	6	1

A TABLE of Simple Interest at 6 per Cent.

Principal.	5 Months, l. s. d. q.	6 Months. l. s. d. q.	7 Months. l. s. d. q.	8 Months. l. s. d. q.
£5	0 0 1 2	0 0 1 3	0 0 1 4	0 0 2 1
6	0 0 1 3	0 0 2 0	0 0 2 2	0 0 2 3
7	0 0 2 0	0 0 2 2	0 0 3 0	0 0 3 1
8	0 0 2 1	0 0 2 3	0 0 3 1	0 0 3 3
9	0 0 2 3	0 0 3 1	0 0 3 3	0 0 4 1
10	0 0 3 0	0 0 3 2	0 0 4 1	0 0 4 3
11	0 0 3 1	0 0 4 0	0 0 4 2	0 0 5 1
12	0 0 3 2	0 0 4 1	0 0 5 0	0 0 5 2
13	0 0 3 3	0 0 4 3	0 0 5 1	0 0 6 0
14	0 0 4 0	0 0 5 0	0 0 5 3	0 0 6 2
15	0 0 4 1	0 0 5 1	0 0 6 0	0 0 7 0
16	0 0 4 3	0 0 5 2	0 0 6 2	0 0 7 2
17	0 0 5 0	0 0 6 0	0 0 7 0	0 0 8 0
18	0 0 5 1	0 0 6 1	0 0 7 2	0 0 8 2
19	0 0 5 2	0 0 6 3	0 0 7 3	0 0 9 0
£1	0 0 5 3	0 0 7 0	0 0 8 1	0 0 9 2
2	0 0 11 3	0 1 2 1	0 1 4 2	0 1 6 3
3	0 1 5 3	0 1 9 1	0 2 0 3	0 2 4 2
4	0 1 11 3	0 2 4 2	0 2 9 1	0 3 1 3
5	0 2 5 3	0 2 11 2	0 3 5 2	0 3 11 1
6	0 2 11 2	0 3 6 2	0 4 1 3	0 4 8 3
7	0 3 5 2	0 4 1 3	0 4 10 0	0 5 6 5
8	0 3 11 1	0 4 8 3	0 5 6 1	0 6 3 3
9	0 4 5 1	0 5 4 0	0 6 2 2	0 7 1 1
10	0 4 11 0	0 5 11 0	0 6 10 3	0 7 10 3
20	0 9 10 1	0 11 10 0	0 13 9 2	0 15 9 1
30	0 14 9 2	0 17 9 0	1 0 8 2	1 3 8 0
40	0 19 8 3	1 3 8 0	1 7 7 1	1 11 6 3
50	1 4 7 3	1 9 7 0	1 14 6 1	1 19 5 1
60	1 9 7 0	1 15 6 0	2 1 5 0	2 7 4 0
70	1 14 6 1	2 1 5 0	2 8 4 0	2 15 2 3
80	1 19 5 1	2 7 4 0	2 15 2 3	3 3 1 2
90	2 4 4 2	2 13 3 0	3 2 1 2	3 11 0 0
100	2 9 3 3	2 19 2 0	3 9 0 2	3 18 10 3
200	4 8 7 2	5 18 4 0	6 18 1 0	7 17 9 3
300	7 7 11 1	8 17 6 1	10 7 1 2	11 16 8 2
400	9 17 3 1	11 16 8 2	13 16 2 0	15 15 7 1
500	12 6 6 3	14 15 10 3	17 5 2 2	19 14 6 1
600	14 15 10 2	17 15 0 3	20 14 3 0	23 13 5 0
700	17 5 2 2	20 14 3 0	24 3 3 2	27 12 4 0
800	19 14 6 1	23 13 5 0	27 12 4 0	31 11 2 3
900	22 3 10 0	26 12 7 1	31 1 4 2	35 10 1 2
1000	24 13 1 3	29 11 9 1	34 10 5 0	39 9 0 2

A TABLE of Simple Interest at 6 per Cent.

s.	9 Months. l.	s.	d.	q.	10 Months, l	s.	d.	q.	11 Months, l	s.	d.	q.	1 Year. l.	s.	d.	q.
5	0	0	2	3	0	0	3	0	0	0	3	1	0	0	3	3
6	0	0	3	1	0	0	3	2	0	0	4	0	0	0	4	1
7	0	0	3	3	0	0	4	1	0	0	4	2	0	0	5	0
8	0	0	4	1	0	0	4	3	0	0	5	1	0	0	5	3
9	0	0	4	3	0	0	5	1	0	0	5	3	0	0	6	2
10	0	0	5	1	0	0	5	3	0	0	6	2	0	0	7	1
11	0	0	5	3	0	0	6	2	0	0	7	0	0	0	8	0
12	0	0	6	1	0	0	7	0	0	0	7	3	0	0	8	3
13	0	0	6	3	0	0	7	2	0	0	8	2	0	0	9	2
14	0	0	7	2	0	0	8	1	0	0	9	0	0	0	10	0
15	0	0	8	0	0	0	8	3	0	0	9	3	0	0	10	3
16	0	0	8	2	0	0	9	1	0	0	10	2	0	0	11	2
17	0	0	9	0	0	0	10	0	0	0	11	0	0	1	0	1
18	0	0	9	2	0	0	10	2	0	0	11	3	0	1	1	0
19	0	0	10	0	0	0	11	0	0	1	0	1	0	1	1	3
1.1	0	0	10	2	0	0	11	3	0	1	1	0	0	1	2	2
2	0	1	9	1	0	1	11	3	0	2	2	0	0	2	4	3
3	0	2	8	0	0	2	11	2	0	3	3	0	0	3	7	1
4	0	3	6	2	0	3	11	1	0	4	4	0	0	4	9	2
5	0	4	5	1	0	4	11	2	0	5	5	0	0	6	0	0
6	0	5	4	0	0	5	11	0	0	6	6	0	0	7	2	2
7	0	6	2	2	0	6	10	3	0	7	7	0	0	8	4	3
8	0	7	1	1	0	7	10	3	0	8	8	0	0	9	7	1
9	0	7	11	3	0	8	10	2	0	9	9	0	0	10	9	2
10	0	8	10	2	0	9	10	1	0	10	10	0	0	12	0	0
20	0	17	9	0	0	19	8	3	1	1	8	1	1	4	0	0
30	1	6	7	2	1	9	7	0	1	12	6	2	1	16	0	0
40	1	15	6	0	1	19	5	2	2	3	4	2	2	8	0	0
50	2	4	4	2	2	9	3	3	2	14	3	0	3	0	0	0
60	2	13	3	0	2	19	2	0	3	5	1	0	3	12	0	0
70	3	2	1	2	3	9	0	2	3	15	11	1	4	4	0	0
80	3	11	0	0	3	18	10	3	4	6	9	2	4	16	0	0
90	3	19	10	3	4	8	9	1	4	17	7	3	5	8	0	0
100	4	8	9	1	4	18	7	2	5	8	5	3	6	0	0	0
200	8	17	6	1	9	17	3	0	10	16	11	2	12	0	0	0
300	13	6	3	2	14	15	10	2	16	5	5	2	18	0	0	0
400	17	15	0	0	19	14	6	1	21	13	11	1	24	0	0	0
500	22	3	10	3	24	13	1	3	27	2	5	1	30	0	0	0
600	26	12	7	1	29	11	9	1	32	10	11	1	36	0	0	0
700	31	1	4	2	34	10	5	0	37	19	5	1	42	0	0	0
800	35	10	1	2	39	9	0	2	43	7	11	0	48	0	0	0
900	39	18	10	3	44	7	8	0	48	16	5	0	54	0	0	0
1000	44	7	8	0	49	6	3	2	54	4	10	3	60	0	0	0

A TABLE of Compound Interest at 6 per Cent.

Years.	The Amount of 1 Pound.	The present worth of 1 pound.	The Amount of 1 pound Annuity.	The present worth of 1 pound Annuity.	The Annuity that 1 l. will Purchase.
1	1.06	.943396	1.00000	0.943396	1.06000
2	1.1236	.889996	2.06000	1.833392	.54543
3	1.19101	.839619	3.18360	2.673012	.37411
4	1.26247	.792093	4 37461	3 465105	.28859
5	1.33833	.747258	5.63709	4.212363	.23739
6	1.41851	.704960	6.97531	4 917324	.20336
7	1.50363	.665057	8.39383	5.582381	.17913
8	1.59384	.627412	9.89746	6.209792	.16103
9	1.68947	.591898	11.49131	6.801691	.14702
10	1.79084	.558391	13.18079	7.360086	.13586
11	1.89829	.526787	14.97164	7.886873	.12679
12	2.01219	.496969	16.86994	8 383843	.11927
13	2.13292	.468839	18.88213	8.852682	.10296
14	2.26091	.442300	21.01506	9.294983	.10758
15	2.39655	.417265	23.27596	9.712248	.10296
16	2.54035	.393646	25.67252	10.105903	.09895
17	2.69277	.371364	28.21287	10 477266	.09544
18	2.85433	.350343	30.90565	10.827602	.09235
19	3.02559	.330512	33.75999	11.158115	.08962
20	3.20713	.311804	36.78559	11.469920	.08718
21	3.39956	.294154	39.99272	11.764200	.08500
22	3.60353	.277505	43.39228	12.041466	.08304
23	3.81975	.261797	46.99582	12.303377	.08127
24	4.04893	.246978	50.81557	12.550356	.07967
25	4.29187	.232998	54.86451	12.783354	.07822
26	4.54938	.219810	59.15638	13.003164	.07690
27	4.82234	.207367	63.70576	13.210532	.07569
28	5.11178	.195630	68.52810	13.406162	.07459
29	5.41838	.184556	73 63979	13.590719	.07357
30	5.74349	.174110	79.05818	13.764829	.07264
31	6.08807	.164255	84.80167	13.929166	.07179
32	6.46035	.154956	90.88977	14.084050	.07100
33	6.84068	.146186	97.34316	14.230227	.07027
34	7.25098	.137946	104.18375	14.367666	.06960
35	7.68605	.130105	111.43477	14.531583	.06881
40	10.28563	.097222	154.76196	15.046300	.06646
45	13.76458	.072660	212.74299	15.455666	.06470
50	18.42003	.054287	290.33382	15.761883	.06344
55	24.65005	.040566	394.16748	15.990566	.06253
60	32.98719	.030313	533.11981	16.161450	.06187

A TABLE of Compound Interest at 10 per Cent.

Years.	The Amount of 1 Pound.	The present worth of 1 pound.	The Amount of 1 pound Annuity.	The present worth of 1 pound Annuity.	The Annuity that 1 l. will Purchase.
1	1.1	90909	1.	0.90909	1.1
2	1.21	.826446	2.1	1.73553	.57619
3	1.331	.751314	3.31	2.48685	.40211
4	1.4641	.683013	4.641	3.16986	.31947
5	1.61051	.620921	6.1051	3.79078	.26379
6	1.77156	.564574	7.71561	4.35526	.2296
7	1.94871	.513158	9.48717	4.86841	.20545
8	2.14358	.466507	11.43588	5.33492	.18744
9	2.35794	.424097	13.57947	5.75901	.17364
10	2.59374	.385543	15.93742	6.14456	.16274
11	2.85311	.350494	18.53116	6.49506	.15396
12	3.13842	.31863	21.38428	6.81366	.14679
13	3.45217	.289664	24.52271	7.10335	.14077
14	3.79749	.263331	27.97498	7.36668	.13574
15	4.17724	.239392	31.77248	7.60608	.13147
16	4.59497	.217629	35.94972	7.82371	.12781
17	5.05447	.197844	40.5447	8.02155	.12466
18	5.55991	.179858	45.59917	8.20141	.12192
19	6.1159	.163508	51.15909	8.36492	.11954
20	6.72749	.148643	57.27499	8.51356	.11745
21	7.40024	.13513	64.00249	8.64869	.11562
22	8.14027	.122846	71.40274	8.77154	.114
23	8.9543	.111678	79.54302	8.88329	.11257
24	9.84973	.101525	88.49732	8.98474	.11129
25	10.8347	.092196	98.34705	9.07704	.11016
26	11.91818	.083905	109.18176	9.16094	.10915
27	13.10999	.076277	121.09994	9.23722	.10825
28	14.41099	.069343	134.20993	9.30656	.10745
29	15.86309	.063039	148.63092	9.3696	.10672
30	17.4494	.057308	164.49402	9.42691	.10607
31	18.49636	.052098	181.94342	9.479	.10558
32	19.60614	.047361	200.43978	9.52636	.10497
33	20.78251	.043055	220.04593	9.56941	.10449
34	22.02946	.03914	240.82844	9.60856	.10407
35	23.35123	.035582	262.85790	9.64413	.10369
36	24.7523	.032347	286.20914	9.67657	.10334
37	26.23744	.029406	310.96144	9.70598	.10302
38	27.81168	.026733	337.19888	9.73271	.10274
39	29.48038	.024302	365.01057	9.75701	.10249
40	31.24921	.022092	394.49095	9.77911	.10225

The uſe of the Table of Simple Intereſt.

This Table ſerves whether the Intereſt is to be Calculated for a certain number of Days, or even Twelfths or fourth parts of a Year, which thoſe Tables cannot, which are ſaid to be calculated for 1, 2, 3, *&c.* Months, and at the ſame time never ſhew what they mean by thoſe Months; it is well known our Months are different in length, and as for their even Twelfth parts of a Year, there is no ſuch time taken notice of in any Kalender or Computation of time: therefore I think where the Intereſt Tables are for Months; there ought ſome mention to be made what Months they mean, as *January*, *February*, *&c.* becauſe they differ much, or elſe to call ſo many days a Month, which I have done for the more eaſie computing the exact Intereſt for any Number of Days; but I have ſaid more of this in my Supplement to *Comes Commerci*: I ſhall proceed here to ſhew how to find the Intereſt for any Number of Years, Days, or Quarters of Years.

Propoſition. 1.] *To find the Intereſt of any Summ for any Number of Months, or Days*, as from any Day of one Month to any of another Month: as the Intereſt of *l.* 500 from *January* 21, to *October* 11th. following.

The intire Months in this time are 8, of which there are 4 that have 31 Days therein, and 1 that has but 28; which is 2 ſhort of 30, which 2 deducted from the 4, there reſts 2 Days above 8 Months, and thoſe 2 added to the 11 Days in *January*, and the 10 in *October* the Summ is 8 Months and 23 Days. So the Intereſt by the Table of

	l.	*s.*	*d.*
l. 500 for 8 Months of 30 Days each, is —	19	14	06½
and for 15 and 8, or 23 Days — —	1	17	09½
The Summ or Anſwer —	21	12	03¾

And that you may the better know the Days in each Month, and compute the odd Days above 30, take the ſame as follows.

January 31	*April* 30	*July* 31	*October* 31
February 28	*May* 31	*Auguſt* 31	*November* 30
March — 31	*June* 30	*September* 30	*December* 31

A ſecond Example of the laſt Propoſition, being the moſt uſual Caſe.

Admit you would know the Intereſt of *l.* 500 from *April* 24 to *October* the 5th following. The intire Months in this time are 5, of which there are 3 that have 31 days, which 3 added to the 7 odd Days in *April*, make 10, and thoſe 4 in *October*, are in all 5 Months and 14 Days; now the Intereſt

Of *l.* 500 for 5 Months of 30 Days each is — *l.* 12: 6: 6: 3
for 14 Days——— *l.* 1: 3: 0: 0

The Summ is — *l.* 13: 9: 6 ¾

Prop. 2.] *To find the Simple Intereſt of any Summ for a Year, or* 1, 2, *or* 3 *quarters, or for any Number of Years*, as of *l.* 360: 12: 00 for 1 Year at *6 per Cent.* is thus:

l. 300 for 1 Year is — *l.* 18: 00: 00
l. 60 for 1 Year — *l.* 3: 12: 00
12 *s.* is — *l.* 0: 00: 08¾

Anſwer — *l.* 21: 12: 08¾

Here you ſee the Intereſt of *l.* 360: 12: for a Year is *l.* 21: 12: 8¾, ¼ of which is *l.* 5: 8: 2 the Intereſt of *l.* 360: 12: 00 for ¼ of a Year; half thereof is *l.* 10: 16: 4 ½ the Intereſt thereof for ½ a Year, and 3 quarters of the *l.* 21: 12: 8¾ is *l.* 16: 4: 6¾ the Intereſt for 3 Quarters of a Year.

If the Intereſt of the *l.* 360: 12: 0 were deſired for any Number of Years, it is but multiplying the *l.* 21: 12: 8¾ by that Number, and the product is the Anſwer, as the Intereſt for 5 Years is *l* 108: 03: 07½ for 9 Years *l.* 194: 14: 6¾, *&c.*

To

The uſe of the Tables of Compound Intereſt.

Caſe 1. To find the Amount of any Summ forborn any Number of Years, as if a Teſtator bequeath to a Legatee *l* 500 to be paid him at the end of 21 years, with the utmoſt improvement of Intereſt at 6 *per Cent.* The Queſtion is what muſt then be paid.

Caſe 2. To find the preſent worth of any Summ due at the end of any Number of years to come. As ſuppoſe a Legatee is minded to ſell for preſent Money, the Summ of *l.* 1699: 15: 7¼ which was left him payable at the end of 21 years, what is it worth preſently.

Caſe 3. To find the Amount of any Annuity forborn any Number of Years: as if an Infant has an Annuity or Rent charge of 30 *l per Ann.* payable out of the Rent of certain Lands, to be paid him at the end of 21 years, with the utmoſt improvement for that time by the perſon then in poſſeſſion, what will that Annuity be then worth?

Caſe 4 To find the preſent worth of the Reverſion,

1. Of an Eſtate in Land, } in Fee
2. Of an Eſtate in Houſes, }
3. Of an Eſtate in Land, } for a time limited
4. Of an Eſtate in Houſes, }

Caſe 5. To find the Annuity that any Summ will purchaſe, either of Land or Houſes in preſent, or in Reverſion, wherein is ſhewed to Fine off Rent of Land or Houſes, *&c.*

The Anſwer to Caſe 1.

Look in the Table of Compound Intereſt at 6 *per Cent.* and againſt 21 in the Collum of Years you ſhall find *l.* 3.39956 which is the Amount of 1 *l.* for 21 Years, and being multiplied by 500 the Product is *l.* 1699.78 or *l.* 1699: 15: 7½.

See the Work.

3.39956	
500	
1699.78000	

Anſwer to Caſe 2.

The ſolving this Caſe proves the truth of the laſt, and it is done by the ſecond Table from that of Years, for againſt 21 Years in the Table of 6 *per Cent.* you have in the Collumn of [the preſent worth of 1 *l.*] .294154 which multiply by the Principal 1699.78 and the Product will be *l.* 500, the preſent worth of 1699.78 due 21 Years hence. *Proof of the laſt Caſe.*

Anſwer to Caſe 3.

In the third Column from that of years in the Table of *6 per Cent.* you have againſt 21 years, *l.* 39.99272 the Amount of 1 *l.* Annuity forborn 21 Years, which multiplied the *l.* 30 the Annuity given, the Product will be *l.* 1199. 15: 7½. See the operation where the value of the Decimals .781 *l.* is (the 60 being not conſiderable) 15 *s.* 7½ *d.*

39.99272
30.

1199.78160

Anſwers and Examples to Caſe 4.

The Common value of Land, or Houſes for Ever, or for Lives, or Years.

The value of an Eſtate in Land, Fee-Simple is commonly about 20 Years Purchaſe. And of an Eſtate in Houſes if well Built, is but worth in Fee about 12 Years Purchaſe; becauſe of the great Charge that attends the maintaining and upholding thereof. Alſo in the Purchaſing of Leaſes 7 Years, is commonly reckoned Equivalent to 1 Life, 14 to 2, and 21 Years to 3 Lives; but 3 Lives in a Leaſe are generally much more advantageous to the Purchaſer than 21 Years, though he pay a Years Purchaſe for Renewing a Life when it falls.

The Purchaſe of an Eſtate in Land.

	Years.	*Months.*	
For 1 Life, or 7 Years is	5 :	7	Purchaſe.
For 2 Lives, or 14 Year —	9 :	3	
For 3 Lives, or 21 Years —	11 :	9.	

The Purchaſe of an Eſtate in Houſes.

	Years.	*Months.*	
For 1 Life, or 7 Years is —	4 :	10	Purchaſe.
2 Lives or 14 Years —	7 :	4	
3 Lives, or 21 Years —	8 :	7	

For

For they that Purchaſe Land expect but 6 *per Cent.* for their Money, but thoſe that lay out their Money in the Purchaſe of Houſes expect 10 *per Cent.* for their Money, for the reaſon aforeſaid. *When to reckon 6, and when 10 per Cent in Purchaſing.*

This being premiſed, I ſhall ſhew the method of Purchaſing the Reverſion of any Eſtate, as

1. Admit I have 15 Years to come in a Leaſe of Land, for which I pay 80 *l. per Ann.* which I am willing to Purchaſe in Fee, after the Expiration of my Leaſe; What is ſuch an Eſtate worth in preſent Money? *An Examp. of the purchaſe of the Reverſion of Land in Fee.*

To anſwer this, I ſuppoſe the Land in Fee to be worth 20 Years Purchaſe, *i. e.* 20 times 80 Pounds or *l.* 1600.

Then I conſider what is the preſent worth of *l.* 80 *per Ann.* to continue 15 Years, which (by the help of the 4th Column from that of Years in the Compound Intereſt Table of 6 *per Cent.* I find *l.* 823 : 00 : 4½.

For againſt 15 Years in that Column is the preſent worth of 1 *l.* Annuity to continue 15 Years. } *l.* 9.712248

Which Multiply by —— 80

and the Product is — *l.* 776.979840

1600.0

which is the preſent worth of *l.* 80 *per Ann.* to continue 15 Years, which deducted from the *l.* 1600 the rem. is *l.* 823 02016 which muſt be given preſently for the Reverſion of the Land in Fee after the Expiration of the Leaſe.

2. Or if this *l.* 80 *per Ann.* be in Houſes, then the Reverſion in Fee Simple after 15 Years is expired, is worth in preſent Money *l.* 351:10:3 *Of Houſes in Fee.*

For the value of 80 *l. per Ann.* in Fee, is here 12 Years Purchaſe, *i. e.* 12 times 80, or 960 *l.*

And the preſent worth of 1 *l. per Ann.* at 10 *per Cent.* in the Compound Intereſt Table, againſt 15 Years is — } *l.* 7.60608

Which multiplied by the Annual Rent 80

The product is —— *l.* 608.48640

960

which product is the preſent worth of *l* 80 *per Ann.* in Houſes to continue 15 Years, which deducted from the Purchaſe in Fee, *l.* 960, the remainer is — } *l.* 351.5136 which muſt be given preſently for the Reverſion of that Eſtate in Houſes.

3. But

3. But if this Tenant is minded to augment the time in his Leaſe, as ſuppoſe he would Purchaſe with preſent Money 20 Years next after the 15 that is in his Leaſe, what ſhall he give.

A Rule for purchaſing the Reverſion of Land or houſes for a time limited.

Add the 15 and the 20 together, and they make 35 Years, then find by the 4th Column in the Compound Intereſt Table (of 6 *per Cent.* if the Eſtate be in Land, or of 10 *per Cent.* if it be in Houſes) the preſent worth of 1 *l. per Ann.* to continue 35 Years, and Multiplying that by 80, you have the preſent worth of 80 *l. per Ann.* to continue 35 Years reſpectively.

Then find by the very ſame method the preſent worth of *l.* 80 to continue but 15 Years, and deduct that from the preſent worth for 35 Years, and the remainer is the Summ to be paid preſently for 20 Years Reverſion, after the 15 Years Leaſe is expired. See the Work.

15 more 20 is = 35 Years (ſuppoſe this to be Land).

Example for Land.

The preſent worth of 1 *l. per Ann.* to continue 35 Years is *l.* 14.531583
Multiply by — Years 80
and the preſent worth of 80 *l. per Ann.* to continue 35 Years is — 1162.526640
now the preſent worth of *l.* 80 *per Ann.* to cont. 15 years is 776.97984
which deducted the Remainer is the Summ to be paid preſently for 20 years Annuity of 80 *l.* after the expiration of 15 years; which is — *l.* 385.5468

Or if the Eſtate be in Houſes.

Example for Houſes.

Then by the Compound Intereſt Table of 10 *per Cent.*
The preſent worth of 1 *l. per Ann.* to contin. 35 years is *l.* 9.64413
Multiply by years 80
and you'l find the preſent worth of *l.* 80 *per Ann.* to continue 35 years — *l.* 771.53040
the preſent worth of 80 *l. per Ann.* to cont. 15 years is *l.* 608.4864
which deducted, the remainer is *l.* 163 : 00 : 10½
which is the Summ to be paid preſently for an Annuity of *l.* 80 Rent of Houſes, for 20 Years next after the 15 Years is expired. — 163.044

4. *A* hath a Leaſe of a Houſe of 50 *l. per Ann.* to continue 15 years, and *B* hath one of 40 *l. per Ann.* to continue 35 years, and they agree to make an exchange; The Queſtion is what one muſt give the other in preſent Money?

To

To solve this, find the present worth of *A* his Lease, of 90 *l. per Ann.* to continue 15 years, which is — *l.* 380: 6: 1

Also the present worth of *B* his 40 *l. per Ann.* to continue 35 years is — *l.* 385: 15: 3½

Here it appears that *A* must give *B* *l.* 5: 9: 2½ in hand, the present worth of his Estate being so much more than that of *A*'s. Difference *l.* 5: 9: 2½

Answers and Examples to Case 5.

To find what Annuity to continue any Number of years, any Summ will Purchase, is performed by help of the 5th Column, from that of Years.

Example 1.] There is a Lease of a House to be Lett for 31 years, of *l.* 60 *per Ann.* and *l.* 200 Fine; but a Tennant for the advantage of his Wife in case of his own death, would pay the less Rent, and *l.* 500 Fine: The Question is what Rent he must pay?

A Rule to find off Rent of Houses.

Here the Money advanced by the Tennant in the Fine is *l.* 300, therefore the Annuity that *l.* 300 will Purchase to continue 31 years, must be deducted out of the Annuity 60 *l.* and the Remainer must be paid and the Fine *l.* 500.

By the Table of 10 *per Cent.* *l.* 1 will Purchase *per Ann.* for 31 years — *l.* 10558

Which Multiplied by — 300

The Product is — *l.* 31.67400

Which *l.* 31.674 deducted from the Annuity 60 *l.* the remainer is 28.326, or *l.* 28: 06: 6½, which is the Rent to be paid for 31 years, the Purchaser paying *l.* 500 Fine.

How a Fine may be lessen'd by a greater Rent.

Examp. 2.] There is a Lease of a House to be Lett for 31 years, for *l.* 28: 6: 6½ *per Ann* and *l.* 500 Fine; but the Lessee being a Trader newly set up, and having occasion for present Money, is willing to pay the greater Rent, so that he may pay but *l.* 200 Fine; How must his Rent be encreased, to make the Bargain Equivalent to the said *l.* 28: 6: 6½ Rent, and *l.* 500 Fine?

Here the difference, or abatement of the Fine being *l.* 300, therefore the Annuity that *l.* 300 will Purchase, to continue 31 years at 10 *per Cent.* *i.e.* *l.* 31: 13: 5½ being added to the proposed Annual Rent, *l.* 28: 6: 6½, the Summ is *l.* 60 the Rent, to be paid for 31 years and *l.* 200 Fine; which proves the last Question.

Examp.

Examp. 3.] There is a Leaſe of a Houſe for 31 years of *l.* 28: 6: $6\frac{1}{4}$ *per Ann.* Rent, and *l.* 500 Fine; but the Leaſee is willing to pay 60 *l. per Ann.* Rent, that he may pay a leſs Fine: The Queſtion is what Fine he ought to pay?

In this Caſe the difference of the Rent is *l* 31: 13: $5\frac{3}{4}$: ſo that it is plain the Leaſee muſt pay ſo much the leſs Fine, as that Summ is, which will Purchaſe *l.* 31: 13: $5\frac{3}{4}$ *per Ann.*

Now to know what Summ will Purchaſe, or the preſent worth of that Annuity, to continue 31 years; look in the 4th Column from that of Years, and againſt 31 you'l find 9.479, which is the preſent worth of 1 *l. per Ann.* to continue 31 years, and being multiplied by the Annual Rent, *l.* 31: 13: $5\frac{3}{4}$, or 31 674, the product is *l.* 300; which deducted from the Fine propoſed, *i. e. l.* 500, the remainer is *l.* 200, and ſo much Fine muſt be paid, and *l.* 60 *per Ann.* Rent, to anſwer the demand.

To Purchaſe Annuities in Reverſion.

Examp. 4.] A Gentleman having received his Fortune which is *l.* 1500, and alſo a profliſe of maintainance for 5 years *Gratis*; he is minded to Purchaſe an Annuity with the ſaid *l.* 1500 to continue 7 years to commence after the Expiration of the 5; the Queſtion is what Annuity to continue that 7 years, the *l.* 1500 will Purchaſe?

To anſwer this, firſt find the amount of *l.* 1500 for 5 years Compound Intereſt, at 10 *per Cent.* which is *l.* 2415: 15: $3\frac{1}{2}$.

Then find what Annuity *l.* 2415: 15: $3\frac{1}{2}$ will Purchaſe to continue 7 years, which is *l.* 496: 6: $4\frac{1}{2}$ *per Ann.* the anſwer; found by multiplying the 5th *Tabular* N° from that of years, in the Table of 10 *per Cent.* againſt 7 years, *i. e.* .20545 by *l.* 2415: 15: $3\frac{1}{2}$, or *l.* 2415.765.

Note, That though I have done this at 10 *per Cent.* yet Annuities are moſt properly computed at 6 *per Cent.* by the 1ſt Table of Compound Intereſt.

To Calculate any of the Numbers in the Tables of Compound Intereſt.

1. *The Column next that of Years is made thus, for* 6 per Cent.
   1. As 100. to 106:: ſo 1. to 1.06 the 1ſt. N°.
   2. As 100. 106:: 1.06. 1. 1236 the 2d. N°. and ſo the reſt.
2. *The ſecond Column from that of Years is made thus.*
   1. As 106. 100:: 1. .943396 — the 1ſt. N°.
   2. As 106. 100:: .943396. .889996 — the 2d N°. *&c.*

3. *The*

3. *The Third Column from that of Years is made thus:*
   1. The first Number is always 1.
   2. The second in this Column is the Summ of the first in the first and third Columns, *&c.*
4. *The Fourth Column from that of Years is thus made:*
   1. The first is always the same with the first in the second Column.
   2. The second in this, is the Summ of the first in this and the second in the second Column, *&c.*
5. *The Fifth Column from that of Years is thus Calculated:*
   Divide a Unit by any of the Numbers in the fourth Column, and the Quotient is the Respective N°. in this fifth Column.
   1. The first Number in this, is the first in the first Column.
   2. As 1.833392. 1:: 1. .54543 the second Number in this fifth Column, *&c.*

CHAP. XI.

Of the Extraction of the Square and Cube Roots.

Sect. 1. *The Extraction of the Square-Root.*

1. THE Extraction of the Square-Root is the finding a Geometrical mean proportional between 1 and the Number given, *i. e.* it is the finding out such a Number, as being multiplied by it self produces the Number given.

2. Hence it follows, that every Number cannot have its Root Extracted, because every Number cannot be produced by Multiplying another Number in it self; but all Numbers are either compleat or imperfect.

3. A compleat Square-Number is one that can have its Root extracted without a Remainer, being the Product of any Number multiplied in it self, as 4, 9, 16, 25, *&c.* *Compleat and imperfect Square Numbers what.*

4. An imperfect Square-Number is one that cannot have its Root extracted without a remainer, as not being the product of any Number whatsoever multiplied in it self, as 2, 3, 5, 6, 7, 8, 10, 11, 12, *&c.*

What the Root and the Square of any Number is.

5. The Number so to be multiplied by it self is called the Root' and the Product arising from the Multiplication is called the Square' as the Line *a b*, *c d*, *a d*, or *b c* is the Root, or side of the Square, which is here 3, and the 9 little Squares is the Product of 3 (any of the sides) multiplied in it self, which is the Square of the side or Root; the use of all which will appear in the next Chapter.

6. *A* TABLE *of the Squares of all the 9 Digits or Roots take as follows, which will be useful in the Work of Extraction following.*

Roots —	1	2	3	4	5	6	7	8	9
Squares —	1	4	9	16	25	36	49	64	81

An Example of Extracting the Root of a Compleat Square Number.

7. If you would find the Square-Root of any whole Number as of 1550025, you must first make a point over Units place, and so over every second toward the Left Hand as you see in the Example following; where observe, that you will always have as many places in the Root as there are points, and the 4 points divides as it were the Number given into 4 parts or branches.

2. Then consider what Number being multiplied by it self will produce a Square next to, and less than the first branch toward the Left Hand, which by the foregoing Table of Roots and their Squares is 1, which therefore put in the Root as in a Quotient.

. . . . *The Root.*
The Square. 1550025 (1245.

Divisors
22) 55 — 1. Divid.
244) 1100 — 2. Divid.
2485) 12425 — 3. Divid.

0 remainer.

3. Square

3. Square (or Multiply by it ſelf) the figure ſo put in the Quotient.

4. Deduct that Square from the ſaid firſt branch (1) toward the left hand, and the remainer is (0).

5. Bring down the next branch 55 (as you ſee in the Example) which muſt have been placed toward the Right Hand of the remainer had there been any.

6. Double the Figure (1) laſt put in the Root, and put that 2 like a Diviſor whereby to divide the laſt branch 55.

7. See always how often you can have the figure ſo double (as here 2) in all the Figures except that in Units place of the dividend (here 5) and put the anſwer in the Root (in this Example 2) which put alſo toward the Right Hand the Diviſor 2.

8. Multiply the Diviſor by the ſaid Figure laſt put in that Root as here 2 times 2 is 4, from 5 reſt 1, and 2 times 2 is 4, from 5 reſts 1, ſo there remains (11).

9. To that remainer (11 bring down the next branch (00) and you have (1100) for a new dividend.

10. Then double the Figure in Units place of your laſt Diviſor, and if that double exceeds a Digit, add the 10 to the Figure in tens place of your laſt Diviſor, as twice 2 in the laſt Diviſor is 4, ſo is the new Diviſor 24.

11. By this 24 divide the ſecond Dividend (except Units place) which is 110, as here 24 is had in 110, 4 times, which 4 put in the Root, and alſo in Units place of the Diviſor 24 making it 244, and having deducted 4 times 244 from 1100, the remainer is 124.

12. To this remainer 124 bring down the laſt branch 25, and it will make 12425 for the 3 dividend.

13. Then double the Figure in Units place of the laſt Diviſor (4) and it makes 8.

14. Then divide 1242 in the Dividend (being all but Units place) by 248, and the Quotient is 5, which put alſo in Units place of the laſt Diviſor, then Multiply 2485 by 5 (laſt put in the Root) deducting the Product from the Dividend 12425, and the remainer is (0) ſo is the Square-Root 1245, which Work may be proved by Multiplying the Root 1245 in it ſelf, for if the Work is right it will produce the Number given to have its Root extracted, *viz.* 1550025. *How to prove the Extraction of the Square-Root.*

Note, That if any dividend (excluding Units place) cannot be divided by the Diviſor; you muſt put a Cypher in the Root, as alſo in Units place of the Diviſor, and ſo proceed to bring down the next branch or part of the Square N° given.

Example 2.

What is the Square-Root of 144057642?

Root.

144057642 (12002

Diviſors { (22) 44 — 1ſt. Dividend.
{ (24002) 057642 2, 3 and 4 Dividends.
9638 — Remains.

Here you ſee this Queſtion is wrought juſt as the laſt, but only that whereas the Diviſor (24) cannot be had in the part brought down, *viz.* (05) therefore I put (0) in the Root and Diviſor, and bring down the next Branch (76) and ſo divide 57 by 240, putting the anſwer (0) in the Root, and to the Right Hand the 240, then to the 576 I bring the next Branch 42, and ſo divide 5764 by 2400, and the Quotient is 2, which I put in the Root, and to the Right Hand the Diviſor, *&c.*

8. *To Extract the Square-Root of a Decimal.*

Decimals to have an even Number of places before Extraction.

This is done the very ſame way with whole Numbers, only obſerving that if you have not an even Number of places in the Decimal given, to make them ſo by adding a Cypher, or Cyphers according as you would have the Root to conſiſt of places, for inſtance, What is the Square-Root of .781?

Example 3.

.78100000 (.8837

168) 1410

1763) 6600

17667) 131100

7431 Remains.

Note,

Note, That if your Decimals given have Cyphers toward the Left Hand next the point; you muſt for every 2 Cyphers there put one Cypher in the Quotient or Root, and ſo Extract the Root of the reſt as is taught in whole Numbers.

How to deal with Decimals that have Cyphers next the point.

Example 4.

What is the Root of .00071? Anſwer .0266. See the Work.

.00071000 (.0266

46) 310

526) 3400

244 Remains.

9. So that if you would have a more Exact Root of any Imperfect Square Number, as that in the ſecond Example above, you need only add ſo many couple of Cyphers toward the Right Hand the Number given, as you would have Decimal places in the Root, and work as follows, and you will have the Anſwer near enough the truth.

Example 5.

144057642.000000. (12002.401=R.

22) 44.

24002) 057642

240044) 963800

24004801) 36240000

12235199 Remains.

To Extract the Root of an imperfect Square Number, near the truth, by putting Cyphers toward the the Right Hand the Number given.

Note,

Note, That the Decimal places being an even Number, a point will always fall over Units place of the whole Number as it ought to do.

And alſo Note, That if the Decimal in the fourth Example were joined with a whole Number, as 371 (for inſtance) the pointing and Root will be found as follows.

Example 6.

What is the Square-Root of 371.00071000?

To know how many Decimals muſt be in the Root.

Note, That as many points as are over the Decimal parts given, ſo many Decimal places muſt you have in the Root.

371.00071000 (19.2613

29) 271

382) 1000

3846) 23607

38521) 53110

385223) 1458900

303231 Remains.

How to Extract the Square-Root of a Vulgar Fraction.

10. The Square-Root of a Vulgar Fraction is extracted by reducing it firſt to its loweſt Terms, and then the Roots of the Numerator and Denominator are thoſe of the Anſwer. But if the Roots of both cannot be taken, the beſt way is to reduce the Vulgar Fraction to a Decimal, and then work as in the third and fourth Examples foregoing.

Sect.

SECT. 2.

The Extraction of the Cube-Root.

A Cube is a solid body, having six equal Geometrical Squares for its bounds, and to Extract the Cube-Root of any Number is to find the side of one of those Squares; so that the Cube-Root of any Number multiplied by it self, produces the content of one of the 6 Superficies's, or Geometrical Squares; which Content or Square being again multiplied by the side of one of the Squares, produces the Content of the whole Cube. *Definition.*

For Instance, to find the Cube of the Root 4, the Square of 4 is 4 times 4, or 16, and 4 times 16 is 64, which is the Cube of 4; so is 4 the Cube-Root, and 64 the Cube.

II. Hence it follows that every Number cannot have its Cube-Root Extracted without a Remainer; because every Number cannot be produced by Multiplying another in it self, and that Product again by the Root: So that

III. Cube Numbers are either (as Squares) compleat or imperfect Cubes; for a compleat Cube Number, is that which can have its Root extracted without a Remainer, as 8, 27, 64, 125, *&c.* But an imperfect Cube Number is that whose side or Root can never be exactly known, as 3, 4, 5, 6, 7, 9, 10, and infinite others. *Compleat imperfect Cube-Numbers what.*

A TABLE *of the Cubes of the* 9 *Digits, or single Roots, which will be useful in Extracting the Cube-Root, take as follows.*

Roots —	1	2	3	4	5	6	7	8	9
Squares —	1	4	9	16	25	36	49	64	81
Cubes —	1	8	27	64	125	216	343	512	729

IV. Admit it were required to find the Cube-Root of the compleat Cube Number, 192978125? *To find the Cube-Root of a compleat Cube Number.*

In order to perform this you must first point the Numbers into Branches, beginning always at Units place, and proceeding to make a point over every third, after the first, as you see in the Example following.

The

The Cube Number.

1929781123 (1245 = *The Cube Root.*

33) 929 — first Reſolvend, or Dividend.

Add { 3 — the treble Root (1)
3 — the treble Square of the Root (1)

Summ — 33 the first Diviſor, whereby to Divide the first Reſolvend.

Add { 8 — the Cube of 2 laſt put in the Root,
12 — the Squ. of that 2 mult. by the laſt treb. R.
6 — the laſt treb. Square of the R. mult. by 2

The Summ — 728 — the ſubtrahend to be taken from the Ref.

4356) 201781 — the ſecond Reſolvend.

Add { 36 — the treble Root (12)
432 — the treble Square of that Root.

Summ — 4356 — the 2d Diviſ. for the 2d Reſolvend.

Add { 64 — the Cube of 4
576 — the Square of 4 in the treble Root.
1728 — the treb. Sq. of the Root multipl. by 4

Summ — 178624 — the ſubtrah. to take from the laſt Ref.

46165) 23157125 — the third Reſolvend.

Add { 372 — the treble Root (124)
46128 — the treble Square of that Root.

Summ — 461652 the 3d Div. for the 3d Reſolvend.

Add { 125 — the Cube of 5 laſt put in the Root
9300 — the Sq. of 5 mult. by the laſt treb. R.
230640 — the treb. Sq. of the R. mult. by 5.

Summ — 23157125 — the Subt. to take from the laſt Ref.

(0) Remains. I

I have been ſo particular in this Example, that I ſhall need to give fewer directions here for performing the Operation; for you ſee I begin with conſidering what Number being Cubed will be equal to, or next leſs than the firſt branch (1) which I find to be (1), and therefore put one in the Root; ſo that Deducting the Cube of the Figure (1) from the firſt branch (1) the Remainer is (0), but if it had been ſomething, I would bring down the next branch 929 to the Right hand thereof.

2. Here by the Work you may ſee that every Reſolvend or Dividend muſt be Divided by the Summ of 3 times the then Root, and three times the Square of the Root added together.

3. In Dividing thereof you are to obſerve the ſame Rule as in the Extraction of the Square-Root, *i. e.* to take no Notice of the Units place of the Reſolvend, in ſeeing how often the Diviſor may be had in it.

4. Obſerve that firſt the Cube of the Figure (2) laſt put in the Root (ariſing from the ſaid Diviſion) ſecondly the Square of that 2 multiplied by the treble Root, and thirdly the treble Square of the Root multiplied by the ſaid Figure put laſt in the Root (which is here 2) do all 3 together make a Subtrahend, or Number to be Deducted from the laſt preceding Dividend or Reſolvend.

5. To the Remainer bring down the next branch (which is here 781) and ſo go on with the treble Root, &c. as before, for what follows is done juſt in the ſame method as that preceding.

6. Obſerve that if the Diviſor cannot be had in the Reſolvend (excluding the Units place of that Reſolvend) then you are to put a Cypher in the Root, and bring down the next branch to the laſt Reſolvend, and make a new Diviſor of the whole Root with the Cypher laſt put therein.

Example 2.

What is the Cube-Root of 8242409?
See the Work.

An Example of extracting the Root of an imperfect Cube-Number.

8242409 (202

0242 = Reſolvend the firſt.

Add { 6 = the treble Root (2)
12 = the treble Square of the Root (2)

126 = the Summ or Diviſor, to Divide the Reſ.

12060) 242409 = the laſt Reſolvend, and the next branch.

60 = the treble Root (20)
1200 = the treble Square of the Root.

12060 = the Summ or Diviſor, to Divide the laſt R.

Add { 8 = the Cube of 2.
240 = the Square of 2 in the treble Root.
2400 = the treble Square of the Root in 2.

Summ = 242408 = the Subtrahend, to take from the laſt Reſ.

1 Remainer.

7. Here according to the 6th Rule the 2 firſt Figures of the Reſolvend 242, *viz.* 24 cannot be Divided by the Diviſor 126, I therefore put a Cypher in the Root, bringing down the next branch (409) to the laſt Reſolvend 242, and ſo proceed to form a New Diviſor from the treble Root (20), and the treble Square thereof.

V. For Extracting the Root more exactly of any imperfect Cube-Number; you muſt place 3, 6, or 9, *&c.* Cypher to the Right hand the Number given, to have the Root Extracted, and ſo proceed as with the Numbers, and for every 3 Cyphers, one Decimal place muſt be in the Root as in the Operation following.

Example

Example 3.

8252.409000000 (20.208

First Divisor 126 } 02524 09 — the 1st and 2d Resolv. 252 and 252409
the 2d — 12060 }

6 the treble Root 2.
12 the treble Square of the Root.

Summ — 126 Divisor the first.

Add { 60 the treble Root 20
1200 the treble Square of that Root.

An Example of a Mixt Number.

Summ — 12060 Divisor the second.

8 the Cube of 2 last put in the Root.
240 — the Sq. of 2 mult by the last treble Root.
2400 — the treble Square of the Root mult. by 2.

The Summ — 242408 the 1st subtr. to take from the last Resolv.

3d Divisor 1224726 } 10001000000 { the third and fourth Resolv.
4th — 122418060 } { 10001 and 10001000000.

606 the treble Root 202.
122412 the treble Square of that Root.

Summ — 1224726 Divisor the third.

Add { 6060 treble Root.
12241200 treble Square of the Root.

Summ — 122418060 Divisor the fourth.

Add { 512 — the Cube of 8 last put in the R.
387840 — the Sq of 8 in the tr. R. (6060)
97929600 — the tr. Sq. of the R. mult. by 8

Summ — 97968389 12 — the 2d sub. to take from the last R.

204161088 Remainer.

Notes on the Work above.

Note, From the foregoing Work, That because the first Divisor (126) cannot be had in the 2 first places of the first Resolvend or Dividend, (252) I therefore put (0) in the Root, and bring down the next branch 409; so is the second Resolvend 252409, and the second Divisor whereby to Divide that second Resolvend is the treble of the Root 20, and treble Square thereof 1200, *viz.* 12060, and the like I have done with the third and fourth Resolvends and Divisors, Cyphers falling in the Quotient or Root.

Note, Also that you might yet Extract the Cube-Root of the last Number nearer the truth, by placing 3 Cyphers to the Right hand the last Remainer 204161088, and so proceeding as in the first Example is directed.

Cube-Root of Decimals

VI. To Extract the Cube-Root of a Decimal, work as you did for a whole Number; but observe,

1. That your Decimals must either consist of 3, 6, 9, or 12, *&c.* places to be made so, by adding Cyphers to the Right hand, if there be occasion; so that in a Mixt Number, as that in the last Example, a point will always fall over Units place of the whole Number.

Two Rules for the Root of Decimals.

2. For every 3 Cyphers toward the Left hand, next the point in the Decimal given, you must put one Cypher in the Root, and Extract the Root of the Remaining Decimal, as if there were no Cyphers, thus the Cube-Root of .0001728 is .055, and something more; the Root of .00001728 is .025, and a little more, the Root of .000001728 is .0055, and a small matter more, see the Examples.

Example 4.

An Example of the 2 last Rules.

Cube-Numb.

.000172800 (.055 *Cube-Root.*

125

765) 47800

15

75

765

125

375

375

41375

Remains — 6425

Example

Example 5.

Cube-Numb.

.000017280 (.025 *Cube-Root.*

126) 9280

6

12

126

125

150

60

7625

Remains — 1655

Example 6.

Cube Number.

.000000172800 (.0055 *Cube-Root.*

125

765) 47800

15

75

765

125

375

375

41375

Remains — 6425

Note,

Note, That I could have made each of these Roots nearer the truth by putting 3 Cyphers to the Right hand of each Remainer, and so proceeding to Extract as before.

CHAP. XII.

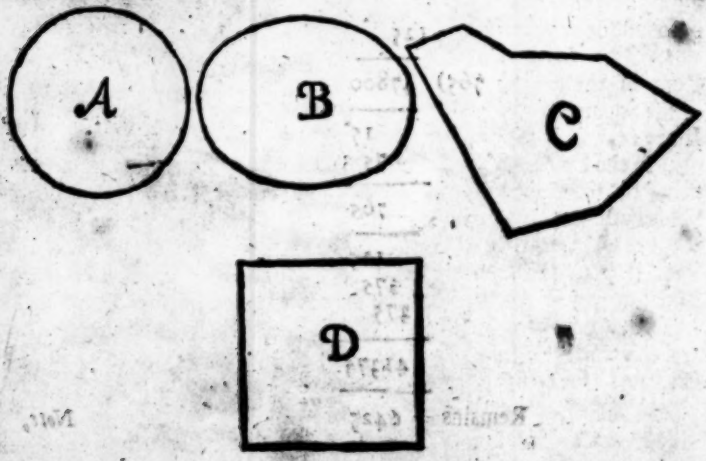
The use of the Extraction of the Square and Cube-Roots.

SECT. 1.

The use of the Extraction of the Square-Root.

1. *To reduce any Superficies to a Geometrical Square.*

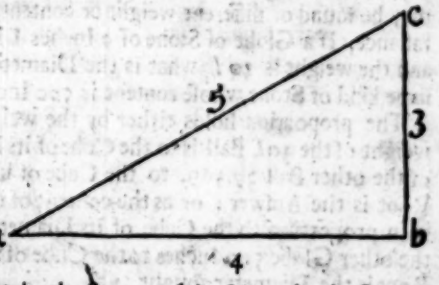
First use. IF there were no other use of the Extraction of the Square-Root than this, it were to be much esteemed by all the Students in the Mathematicks: For suppose a Circle as *A*, an Elipsis, or Oval as *B*, or an irregular Multangle as *C*, the Square-Root of the Content of any of them multiplied in it self gives the Content of the Geometrical-Square *D*, which Square is (near enough) equal to any of the other Figures.



II. Another use of the Extraction of the Square-Root is the making that most Excellent Table of Logarithms, useful in all parts of the Mathematicks, which Table is made by Extracting the Square-Root of the Square-Root continually of the Number whose Logarithm is sought, until such time as the Root is so little, that it has as many Cyphers in the Decimal part next the point, as are equal to the Number of places you intend the Logarithms to consist of, &c. *Second use.*

Which Table of Logarithms when with much trouble made (there requiring above 20 several Extractions, and as many Multiplications to make the Logarithm of one Number) is not only useful in Surveying, Dyalling, Navigation, Astronomy, &c. but in the Extraction of the Square and Cube-Roots of Numbers, thereby much facilitating the Work; for to find the Square-Root of any Number by the Logarithms, is nothing but to take ½ the Logarithm of the Square-Number given, and you have the Logarithm of the Root, and the Cube-Root is found by taking ⅓ of the Logarithm of the given Number, &c. the excellent use of which Table of Logarithms I have only just hinted at, to shew the great use of the Extraction of the Square-Root whereby (among other admirable uses) that Table is made.

III. A third and material use of the Extraction of the Square-Root is by having any two sides of a Right Angled Triangle as (*a b c*) given, the third may be found, for instance. *Third use.*



1. The Square-Root of the Summ of the Squares of the Lines *a b*, and *b c* is equal to the Line *a c*.

2. If from the Square of the Line *a c* you take the Square of the Line *a b*, the Square-Root of the Remainer is the Line *b c*.

3. If from the Square of the Line *a c* you take the Square of the Line *b c*, the Square-Root of the Remainer is the line *a b*.

These three Cases do answer several propositions in the Mathematicks, and the truth thereof is Demonstrated by *Euclid*, in the 47, 1 of his Elements, for if the bace (*a b*) be 4, and the perpendicular (*b c*) be 3, the Hypothenuse (*a c*) will be 5, &c.

IV.

Fourth use. IV. By the Content of a Circle, its Diameter may be found by the Extraction of the Square Root, for

As .7854 is in proportion to 1, so is the Area or Content of any Circle to the Square of the Diameter of that Circle, which Square having the Root extracted gives the Diameter; this is demonstrated by *Euclid* in the 2, 12 of his Elements.

There are many more uses; but none so material as those I have mentioned, and that may be gathered therefrom.

SECT. 2.

The use of the Extraction of the Cube-Root.

I. To reduce any solid body to a Cube; this is performed by extracting the Cube Root of the Content of any solid body; for that Root is the side of a Cube equal to the solid, thus if a long or round piece of Timber, or Stone were 1929781125 Inches solid; the side of Cube equal thereto, is 1245, as appears in the first Example of the Extraction of the Cube-Root.

Second use. II. The content or weight, and side of the Cube or Diameter of any solid being given the side of the Cube or Diameter of any other may be found of different weight or content, in Feet, Inches, *&c.* for instance, If a Globe of Stone of 9 Inches Diameter, is in content 381, and the weight is 30 *l.* what is the Diameter of another Globe of the same kind of Stone, whose content is 500 Inches, and the weight 39.369.

The proportion holds either by the weight or content, for as the weight of the 30 *l.* Ball is to the Cube of its Diameter; so is the weight of the other Ball 39.369, to the Cube of its Diameter, whose Cube-Root is the Answer; or as the content of the Globe 381 Inches solid, is in proportion to the Cube of its Diameter 9; so is the content of the other Globe 500 Inches to the Cube of its Diameter, whose Cube-Root is the Diameter sought.

The truth of this is demonstrated by *Euclid* 18, 12 Elements, and by the same Reason the measure of long, round or square solids, or the Concavity of Guns, *&c.* being compared are easily discovered.

FINIS.